Managerial incentives in oligopoly: The Hicks conjecture revisited

Kaniška Dam† Alejandro Robinson-Cortés‡

April 20, 2018

Abstract

We analyze an incentive contracting model under oligopolistic competition to show that the strength of managerial incentives depends on the nature of competition in the product market. The main objective of the paper is to formalize the Hicks conjecture which asserts that increased competition in the product market induces firms to elicit higher managerial efforts by offering more high-powered incentives. Following earlier literature, we show that the conjecture is refuted when firms set quantities simultaneously. Our main contribution is to show that the conjecture is validated under sequential quantity competition à la Stackelberg. The threat of entry induces incumbents to offer stronger incentives which imply lower managerial slack. Our model offers interesting testable implications.

1 Introduction

The Hicks conjecture (Hicks, 1935), which postulates that product market competition makes the internal organization of a firm more efficient by reducing managerial slack, is based on the Darwinian view of organizations. Firms that survive and perform well in a competitive environment are presumed to have solved governance problems by adapting the optimal organizational structure. There is a plethora of empirical evidence that supports this view (Van Reenen, 2011). For example, by analyzing around 670 U.K. firms, Nickell (1996) concludes that TFP growth rate is positively associated with competition, either measured by increased number of competitors or by lower rents. Both Kole and Lehn (1999) and Palia (2000) assert that CEOs receive more high-powered incentives in deregulated industries. Karuna (2007) finds a positive relationship between the degree of product substitutability and stock option payment of CEOs. Cuñat and Guadalupe (2009) show that increased competition as measured by greater import penetration increases the sensitivity of CEO pay to performance.¹

¹Although a major portion of the empirical literature supports the Hicks conjecture, there are findings which suggest a negative association between competition and managerial incentives. For example, Aggarwal and Samwick (1999) in a cross-
The main objective of the present paper is to provide a unified framework that relates the degree of product market competition with the internal organization of a firm—namely, the structure of managerial incentives. We consider a quantity setting oligopolistic industry with a fixed number of identical risk neutral incumbent and entrant firms. All firms are initially inefficient, and hire a risk neutral manager apiece whose principal task is exert non-verifiable R&D effort to bring down the constant marginal cost of production (process innovation). While the final realization of the marginal cost remains private information, at each firm, the owners/shareholders offer a publicly observable incentive contract to the manager. The effort elicited by a manager can thus be viewed as the ‘power of the managerial incentive contract’.

We analyze two distinct post-entry games. In the first one, new firms are allowed to set quantities simultaneously along with the incumbents. In the second post-entry game, a sequential quantity setting oligopoly, the entrant firms behave as Stackelberg followers who choose managerial contracts and set quantities after observing the aggregate quantity set by the incumbents. Our goal is to compare the effect of entry on the optimal managerial contracts in the incumbent firms in the two post-entry games. Our main result is that it is optimal for the incumbents to provide less high-powered incentives when the new firms set quantities simultaneously, while it is optimal to strengthen incentives if the new firms set quantities afterwards, as Stackelberg followers. The crux of our analysis is that managerial effort is beneficial in two ways in a competitive environment. First, steeper incentives that induce each manager to exert higher effort directly increase the likelihood of securing a cost reduction. Moreover, they also change the beliefs of the rival firms about the true cost realization of a given firm. That is, even if a manager fails to achieve the cost target, her effort is valuable in as much it makes the rivals believe that the cost reduction has been attained. Therefore, a change in the degree of competition works through two channels. It changes the expected value of the cost reduction (‘value-of-cost-reduction’ effect), and it also affects the expected marginal profit of effort conditional on the realized cost (‘marginal-profitability-of-effort effect’). The focus of our analysis is on how these two effects differ in the two distinct entry models that we consider.

In the first game, in which all incumbents and entrants compete simultaneously à la Cournot, the entry of new firms reduces the equilibrium output of every incumbent, regardless of its cost realization, and hence, the expected value of a cost reduction is lower. Also, the entry of new firms implies a decrease in the expected marginal profit of managerial effort when the market is sufficiently competitive. Thus, the marginal benefit of effort, which is the sum of the expected value of cost reduction and the expected marginal profit of managerial effort, decreases following the entry of new firms. Since the marginal costs of incentive provision remain unaffected by entry in our model, each incumbent firm elicits lower managerial effort by offering less high-powered incentives in the post-entry equilibrium. By contrast, in the second game in which entrants are Stackelberg followers, there is a reversal of the above negative association—entry of new firms makes it optimal for the incumbents to strengthen managerial incentives. Since the entrants’ optimal contracting and production decisions are directly affected by the incumbents' output, which they take as given, the entry of new firms implies an increase in the market size and the size of the cost reduction for the incumbents. In particular, we show that the low-cost incumbents benefit more from entry as it increases the expected value of cost reduction for them. Moreover, an incumbent’s expected marginal profit of effort increases as the number of entrants grows. As a result, sectional study of relative performance evaluation show that the ratio of own-firm to rival-firm pay-performance sensitivity decreases as the products become closer substitutes.

---

We assume that both the incumbents [in the pre-entry stage] and the entrants [in the post-entry stage] play Cournot games among themselves (as in Daughety, 1990).
entry of new firms incentivizes the incumbents to reduce cost, and hence, to provide stronger managerial incentives. Our analysis sheds light on the precise channels through which the entry of new firms affects the incumbents optimal contracting decisions, which have not been thoroughly analyzed in the previous literature.

The effect of product market competition on managerial incentives may be assessed empirically in two distinct ways. First, the association between competition and incentives may be viewed as a result of cross-sectional variations in the market structure, which compares corporate governance across markets with different degrees of competition, e.g. number of firms. According to our results, firms in more competitive markets that are otherwise identical offer weaker managerial incentives. Second, conforming to the Darwinian view of organizations, the effect of product market competition on governance practices may be viewed as a reaction to sudden changes in the competitive environment within a given industry, such as an increase in the number of firms due to industry-specific deregulation. While our model of simultaneous quantity competition among the incumbents and entrants conforms to the first view, the model of sequential competition is close in spirit to the second view.

Our results also indicate that important aspects of an industry, such as the time to build production capacity, affect the interplay between product market competition and managerial incentives. In particular, our model of simultaneous competition applies to industries in which entrant firms act in a similar fashion as the incumbents when they enter the industry, as is the case when no significant time is required to build capacity. By contrast, our model of sequential competition fits better to industries in which the incumbents enjoy a first-mover advantage; they can adjust their contracting and production decisions once they know that new firms will enter the market, but they require ample time to build capacity and start operating. According to our results, firms in industries, where the new market participants require “time to build”, strengthen managerial incentives following the entry of new firms, while the reverse holds in industries in which production capacities can be adjusted instantaneously.

The paper is organized as follows. In Section 2, we review the related literature. We outline the building blocks of our model in the following section. In Section 4, we develop the model of simultaneous quantity competition. In the next section, we analyze the case of sequential quantity competition. In Section 6, we offer a set of testable implications. We conclude in Section 7. All proofs are relegated to the Appendix.

2 Related literature

We have mentioned earlier that there has been a plethora of empirical evidence confirming the positive association between competition and managerial incentives. However, the attempt of underpinning the Hicksian view that product market competition is a powerful disciplining force on managers in incentive theoretic models has not been an easy task, and has produced mixed results. The main reason for the ambiguous impact of product market competition on managerial incentives is that increased competition invariably reduces equilibrium output and profits, thus diminishing the value of cost reduction, which is the so-called scale or output effect (Hermalin, 1994; Raith, 2003). Therefore, in order to counteract the negative effect of competition on output and profits, one needs to consider additional channels through

---

3For example, the Airline Deregulation Act of 1978 serves as a natural experiment to study the evolution of corporate governance in the U.S. airline industry (e.g. Kole and Lehn, 1997, 1999).
which product market competition may incentivize firms to enhance managerial performance.

Beyond the “value-of-cost-reduction effect”, Hermalin (1992) identifies two effects of product market competition on managerial incentives. CEOs in a firm typically receive a fraction of the firm’s expected profit. Thus, when more stringent competition implies lower expected profit, the managers tend to consume fewer “agency goods”, i.e., expend more effort. Second, the inherent riskiness of a firm varies with the competitive environment it operates in, and so does the actions of a CEO if he is not risk neutral. Higher volatility of firm’s profits may thus result in lower managerial effort. Overall, the effect of competition on managerial effort is ambiguous. The “value-of-cost-reduction effect”, is also present in Schmidt’s (1997) model. Moreover, Schmidt (1997) shows that if a firm is more likely to go bankrupt in a more competitive environment, the manager tends to work harder to avoid liquidation of the firm’s assets since liquidation implies a loss of reputation. In a framework of hidden information (about the realization of marginal costs), Martin (1993) assumes that the marginal productivity of managerial effort decreases in the number of active firms in a Cournot market, and hence, the equilibrium state-contingent incentive contracts are less high-powered as the number of firms grows. Raith (2003) analyzes a managerial incentive problem in a price-setting oligopoly with privately realized marginal costs, and shows that managerial incentives in a free-entry equilibrium are more high-powered in a more competitive market as measured by the degree of product substitutability, market size or the cost of entry. Piccolo, D’Amato, and Martina (2008) consider an environment similar to Martin (1993) to show that profit-sharing contracts improve productive efficiency relative to the cost-plus mechanisms, and establish an inverted-U shaped relationship between competition and managerial effort. Finally, analyzing managerial incentives in a Cournot oligopoly, Golan, Parlour, and Rajan (2015) establish a negative association between competition and managerial incentives. As the expected product market profit of each firm depends on the likelihood of achieving a low marginal cost in the rival firms, the observed profit as a signal of managerial effort becomes noisier, and hence, the cost of incentive provision magnifies in a more competitive environment. This effect points in the same direction as the standard scale effect implying the aforementioned negative association.

Our paper contributes to the above strand of literature, and is closely related to the works of Hermalin (1994) and Raith (2003). In our model, increased competition affects managerial incentives through the standard channel of ‘value-of-cost-reduction effect’, which is also present in all the aforementioned papers. However, we identify an additional channel through which greater degree of competition acts on incentives – namely, the ‘marginal-profitability-of-effort effect’, which, unlike the extant literature, is novel to our analysis. In the simultaneous quantity setting oligopoly, entry of new firms in general dampens both these effects, implying a negative association between competition and managerial incentives. By contrast, under sequential competition, entry strengthens each of these effects, and hence, there is a positive association between competition and incentives.

Hermalin (1994) assumes that more firms in a Cournot market implies an exogenous decrease in the slope of the inverse market demand [with the intercept remaining constant], and hence, an exogenous increase the market size of each firm, which, apart from the standard value-of-cost-reduction effect, is identified as a countervailing business stealing effect. He shows that if the market size changes sluggishly (rapidly) as the number of firms grows, the adverse scale effect is relatively stronger (weaker), and hence, there is a negative (positive) association between competition and incentives. Raith (2003) also

---

4 Another strand of literature (e.g. Fershtman and Judd, 1987; Sklivas, 1987) analyzes managerial incentive problems in Cournot oligopoly where a manager’s objective function is a linear combination of profits and revenues, but does not consider incentive problems arising from non-verifiability of managerial efforts.
considers an exogenous variations in market size which is one of the main sources of changes in the degree of market competition, and is key to a positive association between competition and incentives in his model. Our model of simultaneous quantity competition is a special case of Hermalin (1994); thus, it is not very surprising that we obtain a negative relation between competition and incentives. By contrast, in the sequential competition, the (expected) market size and the size of cost reduction for each incumbent firm is endogenous, and are influenced favorably by entry. Because market size and the size of cost reduction in general enhance both the value-of-cost-reduction and marginal-profitability-of-effort effects, we obtain a positive association between competition and incentives in the Stackelberg game.

Another view of the theoretical literature on the effect of product market competition on managerial incentives is to consider cost or output correlation across the competitive firms, which gives rise to yardstick competition. Hart (1983) analyzes a market consisting of both managerial and neoclassical firms. In a neoclassical firm, managerial effort is observable, and hence, the presence of such firms makes the provision of incentives to put in high effort less costly in the managerial firms. Consequently, competition reduces managerial slack. Scharfstein (1988) argues that Hart’s (1983) result crucially depends on the specification of discontinuous preferences of the manager over income, i.e., below a certain threshold manager’s utility tends to minus infinity, and hence, the result can be reversed, i.e., increased competition leads to greater managerial slack under continuous preferences. Nalebuff and Stiglitz (1983) show that with correlated output, competition reduces the cost of implementing relative performance evaluation contracts, and hence, there exists a positive association between competition and incentives.

3 The model

The economy lasts three dates, $t = 1, 2, 3$, and consists of two classes of risk neutral agents: $N$ ex-ante identical firms who compete in quantities in a market for a homogeneous goods, and $N$ ex-ante identical managers. The firms are divided in two groups – namely, a fixed subset $I$ of $n \geq 1$ incumbents and a subset $J$ of $m \geq 1$ entrants. A typical incumbent firm is denoted by $i$, and a typical entrant, by $j$. Often for convenience we will denote a generic firm (incumbent or entrant) by $k \in K = I \cup J$ with $|K| = N = n + m$ and $I \cap J = \emptyset$. Let $q_k$ denote the production of firm $k$. The inverse market demand is given by $P = 1 - Q$, where $Q$ denotes the aggregate industry output. Each firm $k$ operates on a constant-returns-to-scale production technology with marginal cost $c_k \in \{0, c\}$ where $0 < c < 1$. Initially, all firms have the inefficient technology, i.e., $c_k = c$ for all $k$. Each firm may hire a manager whose principal task is to exert non-verifiable R&D effort in order to lower marginal cost to 0. The probability that the marginal cost is reduced is given by $e_k$, which is the effort exerted by the manager of firm $k$. Each firm $k$ offers its manager a take-it-or-leave-it contract $(w_k(0), w_k(c))$ which is contingent on the realized marginal cost $c_k \in \{0, c\}$. Contracts are subject to limited liability of the managers. Managerial contracts are publicly observable, but the realized marginal costs remain private information of the firms. Every manager has the same effort cost function $\psi(e) = e^2/2$, and her outside option is normalized to 0.

The timing of events is as follows. At date 1, firms hire a manager apiece by offering publicly observable contracts $(w_k(0), w_k(c))$. At $t = 2$ each manager exerts non-verifiable effort $e_k$, and the marginal cost $c_k$ of each firm $k$ is privately realized. At $t = 3$ firms simultaneously compete in the product market by setting quantities, product market profits are collected, and the contracts are executed. Throughout, we assume that all firms, incumbents and entrants, produce a positive output in equilibrium.
regardless of their realized marginal cost.\textsuperscript{5} In our case, this is a conservative assumption since the incentives to attain the cost reduction would have been stronger otherwise.

In what follows, we will analyze two sorts of product market competition and the resulting incentive contracts. In the first kind, a \textit{simultaneous quantity setting oligopoly}, both the incumbents and the entrants compete simultaneously à la Cournot in the product market at date 3. In the second kind, a \textit{sequential quantity setting oligopoly}, the entrants set managerial contracts and quantities after observing the aggregate quantity of the incumbents. Thus, the above timeline is repeated twice. The incumbents behave as Stackelberg leaders who compete in a Cournot fashion, while the entrants choose quantities simultaneously after observing the aggregate output of the incumbents, which they take as given.

4 Managerial incentives in simultaneous oligopoly

In the simultaneous quantity setting oligopoly, the \( m \) entrant firms are allowed to set quantities simultaneously along with the \( n \) incumbents. Therefore, this market structure is nothing but a Cournot market with \( N = n + m \) firms with privately known marginal costs \( \{c_1, \ldots, c_n\} \) of the incumbents, and \( \{c_1, \ldots, c_m\} \) of the entrants. We solve the model by backward induction.

4.1 Product market competition

At \( t = 3 \), a representative firm \( k \in K \) sets quantity \( q_k \) to maximize its expected profit which is given by:

\[
q_k(1 - q_k - \mathbb{E}q_{-k} - c_k),
\]

where \( \mathbb{E}q_{-k} = \sum_{l \in K \setminus \{k\}} \mathbb{E}q_l \) is the aggregate expected output of the rivals of firm \( k \). Note that \( q_l \) for all \( l \neq k \) is a random variable because \( c_l \) is unknown to firm \( k \). Firm \( k \) takes as given the contracting variables \( e_k \) and \( e_l \) for all \( l \neq k \) at date 2. The first-order condition of the above maximization problem yields

\[
q_k(c_k, \mathbb{E}q_{-k}) = \frac{1 - c_k}{2} - \frac{1}{2} \mathbb{E}q_{-k}. \tag{1}
\]

The expected marginal cost at each firm \( k \) is given by \( \mathbb{E}c_k = c(1 - e_k) \) where \( e_k \) is the managerial effort at firm \( k \) chosen at \( t = 2 \). Therefore,

\[
\mathbb{E}q_k = \frac{1 - c(1 - e_k)}{2} - \frac{1}{2} \mathbb{E}q_{-k}. \tag{2}
\]

Summing the above over \( k \) we get

\[
\sum_{k=1}^{n+m} \mathbb{E}q_k = \frac{1}{n+m+1} \left[ (n+m)(1-c) + c \sum_{k=1}^{n+m} e_k \right]. \tag{3}
\]

Since \( \mathbb{E}q_{-k} = \sum_{k=1}^{n+m} \mathbb{E}q_k - \mathbb{E}q_k \), (2) can be written as

\[
\mathbb{E}q_k = \frac{1 - c(1 - e_k)}{2} - \frac{1}{2} \left( \sum_{l=1}^{n+m} \mathbb{E}q_l - \mathbb{E}q_k \right) = \frac{1 - c + c[(n+m)e_k - e_{-k}]}{n+m+1}, \tag{4}
\]

where \( e_{-k} = \sum_{l \in K \setminus \{k\}} e_l \). Thus, substituting for \( \sum_{l=1}^{n+m} \mathbb{E}q_l \) and \( \mathbb{E}q_k \) from (3) and (4) into (1), we obtain

\textsuperscript{5}This implies a restriction of the parameter space – namely, an upper bound on \( c \).
**Lemma 1** Given the privately realized marginal costs \( \{c_1, \ldots, c_n, c_1, \ldots, c_m\} \) and the managerial efforts \( \{e_1, \ldots, e_n, e_1, \ldots, e_m\} \), the quantity and profit of each firm in the Bayesian Cournot equilibrium are respectively given by:

\[
q_k(c_k, e_k, e_{-k}) = \frac{2 - (n + m + 1)c_k + (n + m - 1)c(1 + e_k) - 2ce_{-k}}{2(n + m + 1)}, \tag{5}
\]

\[
\pi_k(c_k, e_k, e_{-k}) = \left( \frac{2 - (n + m + 1)c_k + (n + m - 1)c(1 + e_k) - 2ce_{-k}}{2(n + m + 1)} \right)^2. \tag{6}
\]

Note that \( \pi_k(c_k, e_k, e_{-k}) \) is the expected market profit of firm \( k \) conditional on its realized cost, \( c_k \). It depends on \( e_k \) even when conditioning on \( c_k \) because the effort exerted by the manager at firm \( k \) pins down the beliefs of the rival firms about \( c_k \). These beliefs affect the rivals’ output decisions in the same way as \( e_{-k} \) affects that of firm \( k \), so the effort exerted by the manager at firm \( k \) is profitable beyond its cost realization. If the realized marginal costs were publicly observable, the product market profits would not depend on managerial efforts, instead they would depend on the observed numbers of high- and low-cost firms (e.g. Golan et al., 2015).

### 4.2 Equilibrium managerial efforts and incentives

At \( t = 2 \), each manager \( k \) chooses her effort \( e_k \) optimally, given the contracts \( w_k(0) \) and \( w_k(c) \) at firm \( k \). Because the realizations of marginal costs are independent, managerial contracts at each firm \( k \) are independent of the realizations of marginal costs at the rival firms. The optimal effort at firm \( k \) is given by:

\[
e_k = \arg\max_{\hat{e}_k} \left\{ \hat{e}_kw_k(0) + (1 - \hat{e}_k)w_k(c) - \frac{1}{2} \hat{e}_k^2 \right\} = w_k(0) - w_k(c). \tag{IC_k}
\]

The above is the incentive compatibility constraint of the manager at firm \( k \). We assume limited liability (non-negative income for the manager at each state of the nature), i.e.,

\[
w_k(c) \geq 0, \quad \text{and} \quad w_k(0) \geq 0. \tag{LL_k}
\]

Finally, the expected utility of the manager at each firm \( k \) must be at least as high as her outside option 0, i.e.,

\[
e_kw_k(0) + (1 - e_k)w_k(c) - \frac{1}{2} e_k^2 \geq 0. \tag{PC_k}
\]

The above is the participation constraint of the manager at firm \( k \). The optimal contracting problem at \( t = 1 \) at each firm \( k \) is solved in two stages (e.g. Grossman and Hart, 1983). First, firm \( k \) minimizes the expected incentive costs in order to implement a given level of effort subject to the aforementioned constraints, i.e.,

\[
C_k(e_k) = \min_{\{w_k(0), w_k(c)\}} e_kw_k(0) + (1 - e_k)w_k(c), \tag{Min_k}
\]

subject to \( \{IC_k\}, \{LL_k\} \) and \( \{PC_k\} \).

For the moral hazard problem to be non-trivial, the limited liability at \( c_k = c \) must bind, and hence, \( w_k(c) = 0 \) and \( w_k(0) = w_k \). The contract term \( w_k \) is the ‘bonus’ offered to the manager at firm \( k \) for succeeding in reducing the marginal cost. We will use \( w_k \) and \( e_k \) equivalently to refer as the ‘power of
incentives’ at firm $k$. Given that $e_k = w_k$, the manager’s expected utility reduces to $e_k^2/2$, and hence, $(PC_k)$ always holds. The value function, called the ‘incentive cost function’, of the above minimization problem is given by:

$$C_k(e_k) = C(e_k) = e_k^2$$

for all $k \in K$.

In the second stage, firm $k$ chooses the effort level $e_k$ in order to maximize the expected profits

$$\Pi_k(e_k, e_{-k}) \equiv e_k \pi_k(0, e_k, e_{-k}) + (1 - e_k) \pi_k(c, e_k, e_{-k})$$

net of its incentive costs $C(e_k)$, i.e.,

$$\max_{e_k} \Pi_k(e_k, e_{-k}) - C(e_k). \quad (\text{Max}_k)$$

Define by $\Delta \pi_k(e_k, e_{-k}) \equiv \pi_k(0, e_k, e_{-k}) - \pi_k(c, e_k, e_{-k})$ the ‘expected value of cost reduction’ of each representative firm $k$. The first-order condition of program $(\text{Max}_k)$ is given by:

$$\frac{\partial \Pi_k(e_k, e_{-k})}{\partial e_k} = C'(e_k)$$

$$\iff \Delta \pi_k(e_k, e_{-k}) + \left[ e_k \frac{\partial \pi_k(0, e_k, e_{-k})}{\partial e_k} + (1 - e_k) \frac{\partial \pi_k(c, e_k, e_{-k})}{\partial e_k} \right] = 2e_k. \quad (\text{FOC}_k)$$

At the optimal managerial effort, the marginal benefit of effort is equalized with the marginal incentive cost. The marginal benefit of effort is composed of two terms – namely, the expected value of cost reduction, and the expected ‘marginal profitability of effort’ (the term inside the square braces). On the right-hand-side of the above equation is the marginal incentive cost. Condition $(\text{FOC}_k)$ implicitly defines the best response (in effort) $e_k(e_{-k})$ of the manager at firm $k$. It is easy to show that

$$e_k(e_{-k}) = \alpha(n + m, c) - \beta(n + m, c) e_{-k}, \quad (\text{BR}_k)$$

where $\alpha(n + m, c)$ and $\beta(n + m, c)$ are positive constants. Because, $e_k(e_{-k})$ is a linear function, and $\partial e_k/\partial e_l < 0$ for any $l \neq k$, i.e., managerial incentives are strategic substitutes, the equilibrium of the effort choice game is unique and symmetric. Let $e_k = e(n + m)$ for all $k \in K$.

**Proposition 1** For all $n \geq 1$ and $m \geq 0$, the equilibrium managerial effort and bonus are unique, and are given by:

$$e(n + m) = w(n + m) = \frac{c[8(n + m) + c((n + m)^2 - 6(n + m) + 1)]}{2[4(n + m + 1)^2 + c^2(n + m - 1)^2]} \in (0, 1). \quad (\text{EC})$$

Moreover, there is $\bar{c} \in (0, 1)$ such that every firm produces strictly positive output regardless of its realized marginal cost provided that $c \in (0, \bar{c})$.

Our objective is to analyze how increased competition due to the entry of new firms in the Cournot market affects managerial incentives at the incumbent firms. Note that, in the simultaneous quantity setting game, the entrants compete with the incumbents in a Cournot fashion, and hence, analyzing the effect of entry on managerial incentives is equivalent to analyzing the effect of an increase in the number of firms $N$ in the Cournot market with fixed demand. The following proposition states the result.

---

6The property of strategic substitutability of managerial incentives, which is a key ingredient of our results, holds under very general market demand functions (e.g. Chalioti and Serfes, 2017).
Proposition 2 Entry of new firms implies that each incumbent elicits lower managerial effort, i.e., \( e(n + m') < e(n + m) \) by providing less high-powered incentives to its manager, i.e., \( w(n + m') < w(n + m) \) for all \( n \geq 1 \) and for any \( m' > m \geq 0 \).

The above result is a direct consequence of the fact that \( e'(N) < 0 \). To understand the intuition, consider any given common effort level \( e \) induced by the \( N = n + m \) firms. Substituting \( e_k = e \) and \( e_{-k} = (N - 1)e \), and using the expressions for \( \pi_k(0, e_k, e_{-k}) \) and \( \pi_k(e, e_k, e_{-k}) \), the first-order condition (FOC) reduces to:

\[
\frac{c[4 + c(N - 3) - 2c(N - 1)e]}{4(N + 1)} + \frac{c(N - 1)(1 - c + ce)}{(N + 1)^2} = \frac{2e}{c'(e)}
\]

The left-hand-side of the above equation, denote it by \( \Pi'(e) \), is the marginal benefit of effort evaluated at the common effort level \( e \), which is a strictly decreasing function. On the other hand, the right-hand-side is the marginal incentive cost at the common effort level \( e \), which is strictly increasing in \( e \). The intersection of \( \Pi'(e) \) and \( C'(e) \) determines the equilibrium managerial effort \( e(N) \) given in (EC). When the number of firms \( N \) increases, it affects the \( \Pi'(e) \) function through two channels. First, the standard 'output channel'. To see this, let \( q(c_k, e) \) denote the output of each firm \( k \) at the common effort level \( e \). Because \( \partial q(c_k, e)/\partial N < 0 \), and it does not depend on \( c_k \), we have

\[
\frac{d\Delta \pi(e)}{dN} = 2[q(0, e) - q(c, e)] \frac{dq(c_k, e)}{dN} < 0.
\]

Thus, the expected value of cost reduction decreases with \( N \) as more firms imply less output of each firm regardless of its cost realizations. This 'value-of-cost-reduction effect' would work under the same logic if the realizations of marginal costs would have been public knowledge. Due to the presence of privately realized marginal costs, an increase in the number of firms affects the marginal benefit of managerial effort through a second channel—namely, the 'marginal-profitability-of-effort' effect.\(^7\) Note that

\[
\frac{dE[\partial \pi(c_k, e)/\partial e_k]}{dN} = -\frac{c(1 - c + ce)}{(N + 1)^2} \frac{N - 3}{N + 1}
\]

The last expression is strictly positive for \( N < 3 \), and negative for all \( N \geq 3 \). Note that as the number of firms grows, the output of each firm at any realization of marginal cost decreases, but the marginal impact of managerial effort on output increases, i.e., \( \partial^2 q(c_k, e)/\partial e_k \partial N > 0 \). Even after the marginal costs have been realized, having elicited a higher effort is profitable because it affects the beliefs of the rival firms. When the market is more concentrated (less firms), the positive effect outweighs the negative one and \( dE[\partial \pi(c, e)/\partial e_k]/dN > 0 \). Nonetheless, the aggregate effect of an increase in the number of firms on the marginal benefit of effort is always negative as

\[
\frac{d\Pi'(e)}{dN} = \frac{d\Delta \pi(e)}{dN} + \frac{dE[\partial \pi(c_k, e)/\partial e_k]}{dN} = -\frac{c^2(N - 1)^2}{2(N + 1)^2} < 0.
\]

\(^7\)See Appendix A for details.

\(^8\)The expected marginal profitability of effort at the common effort level \( e \) is given by:

\[
E[\partial \pi(c_k, e)/\partial e_k] = \frac{\partial \pi(0, e) / \partial e_k}{e(1 - e) \partial \pi(c, e) / \partial e_k} = e \left( 2q(0, e) \frac{\partial q(0, e) / \partial e_k}{\partial e_k} \right) + (1 - e) \left( 2q(c, e) \frac{\partial q(c, e) / \partial e_k}{\partial e_k} \right)
\]

\[
= \frac{c(N - 1)}{N + 1} \left[ q(0, e) + (1 - e)q(c, e) \right],
\]

as \( \partial q(0, e) / \partial e_k = \partial q(c, e) / \partial e_k = c(N - 1)/(N + 1) \).
and hence, the $\Pi'(e)$ function shifts down as $m$, or equivalently, $N$ increases. On the other hand, the marginal incentive cost function $C'(e)$ remains unaffected by changes in the number of firms. Therefore, the equilibrium managerial effort in each firm decreases with the number of firms.

5 Managerial incentives in sequential oligopoly

Under sequential quantity setting oligopoly, first the incumbent firms follow the timing of events described in Section 3. After observing the aggregate quantities set by the incumbents, the entrants repeat the timing. We follow Daughety (1990), which is a generalization of the standard notion of Stackelberg competition, to model market competition in the current context. The $n$ incumbent firms (the “leaders”), after deciding on the publicly observable managerial contracts, compete à la Cournot in the product market at date 3. After observing the aggregate quantities set by the incumbents, the $m$ entrants (the “followers”) choose simultaneously the managerial contracts. Following the realization of the true marginal cost at each entrant firm, each of them sets quantity in a Cournot fashion by taking the aggregate incumbent output and that of the rival followers as given. Thus, the timing of events described earlier is repeated twice as depicted in Figure 1 – by the incumbents in the first phase, and then by the entrants in the second phase. As we mentioned earlier, we assume that in equilibrium all $m$ entrants decide to enter, i.e., regardless of the incumbents’ cost realizations, all entrants find it optimal produce a positive output in equilibrium. This rules out the possibility that the incumbents may deter entry.

![Figure 1: The timing of events in the sequential quantity setting oligopoly.](image)

Given the linear market demand, $P = 1 - Q$, let $Q_I = \sum_{i \in I} q_i$ and $Q_J = \sum_{j \in J} q_j$ be the aggregate incumbent and entrant outputs, respectively. Further, let the managerial effort and bonus vectors be denoted by $(e_i, e_f)$ and $(w_i, w_f)$, respectively for $i \in I$ and $j \in J$. At the quantity setting stage, each entrant $j$ takes $Q_I$ and $q_{-j} = \sum_{\ell \in f_j} q_\ell$, the quantities of the rival entrants, as given to solve

$$\max_{q_j} q_j(1 - Q_I - q_j - E q_{-j} - c_j).$$

The subgame that follows the stage of quantity setting by the incumbents is simply a Cournot game among the ex-ante identical $m$ (entrant) firms who face a residual demand $P = 1 - Q_I - \sum_{j \in J} q_j$. Therefore, the managerial effort in the symmetric subgame perfect equilibrium depends on the aggregate incumbent output $Q_I$ and the number of entrants $m$, and is analogous to the equilibrium managerial effort of the simultaneous quantity game in the previous section. Therefore,
Lemma 2 Given the aggregate output $Q_I$ of the incumbent firms, the equilibrium managerial effort of the entry subgame is unique and given by:

$$e_j(m, Q_I) = w_j(m, Q_I) = \frac{c[8m(1 - Q_I) + c(m^2 - 6m + 1)]}{2[4(m + 1)^2 + c^2(m - 1)^2]}.$$  \hspace{1cm} (EE)

Note that the above expression is analogous to $(EC)$ which is obtained by replacing the intercept of the demand function, 1 by that of the residual demand (faced by the entrants), $1 - Q_I$, and the total number of firms $n + m$ by the number of entrants $m$. Further, the higher the aggregate output of the incumbents, $Q_I$, the weaker is the managerial incentives provided in each incumbent firm. This is because when the aggregate output of the incumbents expands, the entrants face a reduced residual demand, and hence, it is optimal for each of them to offer weaker incentives to its manager. While setting the quantities, the incumbents take into account the aggregate best response of the entrants, $Q_I(Q_I)$ and anticipate the managerial efforts in the entrant firms, $e_j(m, Q_I)$. The aggregate best response of the entrants is given by:

$$Q_I(Q_I) = \kappa(m)(C_1 - C_2 Q_I),$$  \hspace{1cm} (7)\hspace{1cm}

where

$$\kappa(m) \equiv \frac{m(m + 1)}{4(m + 1)^2 + c^2(m - 1)^2}$$

is an increasing function in $m$, and $C_2 \equiv 4 + c^2$ and $C_1 \equiv C_2 - c(4 + c^2/2)$ are positive constants that depend on $c$. As the incumbent firms behave in a Cournot fashion, each incumbent firm $i$ solves the following maximization problem at date 3:

$$\max_{q_i} q_i(1 - q_i - Eq_{-i} - Q_J(q_i + Eq_{-i}) - c_i) \Leftrightarrow \max_{q_i} q_i(A(m) - B(m)(q_i + Eq_{-i}) - c_i),$$  \hspace{1cm} (8)

where $A(m) \equiv 1 - C_1 \kappa(m)$ and $B(m) \equiv 1 - C_2 \kappa(m)$. From the incumbents’ perspective, the entry of new firms implies two countervailing effects. On the one hand, more firms implies a lower market price, $A(m) < 1$. However, since the aggregate incumbents output diminishes the optimal effort and output of entrants, it also implies that the price is less responsive to the incumbents output, $B(m) < 1$. This gives them more leeway; they can increase output without reducing the equilibrium price too much. For reasons that will become clear below, it is convenient to consider these effects in a different but equivalent way. Note that the solution to (8) is equivalent to the solution to the following problem:

$$\max_{q_i} q_i(a(m) - (q_i + Eq_{-i}) - \theta(m)c_i),$$

where

$$a(m) \equiv \frac{A(m)}{B(m)} = \frac{1 - C_1 \kappa(m)}{1 - C_2 \kappa(m)}, \hspace{0.5cm} \theta(m) \equiv \frac{1}{B(m)} = \frac{1}{1 - C_2 \kappa(m)} \text{ with } a'(m), \theta'(m) > 0.$$

That is, from the perspective of each incumbent $i$, the entry of new firms is equivalent to an increase in the market size, $a(m) > 1$, and the size of the cost reduction, $\theta(m)c_i > c_i$. We depict this equivalence graphically in Figure 2 by analyzing the marginal revenue [derived from the residual demand faced by $i$] and marginal cost curves.
In Figure 2 the solid downward sloping line is the marginal revenue function derived from the residual demand \( [A(m) - B(m)E_{q_{-i}}] - B(m)q_i \) of incumbent \( i \) for \( m > 0 \). This marginal revenue function has a slope equal to \(-2B(m)\). The maximum price is represented by the point \( A(m) - B(m)E_{q_{-i}} \) and the market size is represented by \( a(m) - B(m)E_{q_{-i}} = A(m)/B(m) - E_{q_{-i}} \), and hence, the horizontal intercept of the marginal revenue function is given by \( [a(m) - E_{q_{-i}}]/2 \). If there were no entrants, we would have \( A(0) = B(0) = 1 \). Following the entry of at least one firm, we have \( A(m) < 1 \) and \( B(m) < 1 \). The solid horizontal line is the marginal cost of a high-cost incumbent \( i \). The equilibrium quantity \( q_i(c \mid m) \) is determined by the intersection of the marginal revenue and marginal cost of the high-cost incumbent \( i \) for a given number of entrant \( m \). The ‘normalized’ marginal revenue function that is derived from the ‘normalized’ residual demand \( a(m) - E_{q_{-i}} - q_i \) with \( a(m) > 1 \), and the ‘normalized’ marginal cost curve \( \theta(m)c > c \) are shown by the dashed lines. The normalized marginal revenue curve is steeper than the actual marginal revenue curve because it has a slope equal to \(-2\). These two normalized functions intersect at the same equilibrium output level \( q_i(c \mid m) \) of each high-cost incumbent. For each low-cost incumbent, the equilibrium quantity is given by \( q_i(0 \mid m) = [a(m) - E_{q_{-i}}]/2 \) because for such a firm \( i \), \( c_i = \theta(m)c_i = 0 \).

Given the marginal cost \( c_i \) of each incumbent firm \( i \), it’s managerial effort \( e_i \), and the aggregate managerial efforts \( e_{-i} = \sum_{i \in F \setminus \{i\}} e_i \) of the rival incumbents, the equilibrium output and profit of \( i \) are respectively given by:

\[
q_i(e_i, e_i, e_{-i} \mid m) = \frac{2a(m) - (n + 1)\theta(m)c_i + (n - 1)\theta(m)c(1 + e_i) - 2\theta(m)ce_{-i}}{2(n + 1)},
\]

\[
\pi_i(e_i, e_i, e_{-i} \mid m) = \left(\frac{2a(m) - (n + 1)\theta(m)c_i + (n - 1)\theta(m)c(1 + e_i) - 2\theta(m)ce_{-i}}{2(n + 1)}\right)^2.
\]

In the optimal contracting stage at date 1, each incumbent firm \( i \) solves the same maximization problem as \( (\text{Max}_k) \) (replace \( k \) by \( i \) everywhere). As in the case of simultaneous competition, the equilibrium managerial effort in each incumbent firm is unique and symmetric.
Proposition 3 The unique symmetric equilibrium managerial effort and incentives in each incumbent firm in the sequential quantity setting oligopoly are given by:

\[ e(n, m) = w(n, m) = \frac{\theta(m)c[8a(m)n + \theta(m)c(n^2 - 6n + 1)]}{2[4(n+1)^2 + \{\theta(m)c\}^2(n-1)^2]} . \] (EI)

Moreover, there exists \( \hat{c} \in (0, 1) \) such that every firm (incumbent or entrant) produces a positive output regardless of its realized cost, provided that \( c \in (0, \hat{c}) \).

In the following proposition, we analyze the effect of entry, i.e., an increase in \( m \) on the equilibrium managerial incentives in each of the incumbent firms, which is depicted in Figure 3.

Proposition 4 Entry of new firms induces each incumbent firm to elicit higher managerial effort, i.e., \( e(n, m') > e(n, m) \) by providing more high-powered incentives, i.e., \( w(n, m') > w(n, m) \) for all \( n \geq 1 \) and for any \( m' > m \geq 0 \).

![Figure 3: Equilibrium managerial effort as a function of the number of entrants m under simultaneous and sequential quantity setting oligopolies for a given number \( \bar{n} \) of incumbents.](image)

The proof of Proposition 4 consists in showing that the equilibrium effort elicited by the incumbents, \( e(n, m) \) given in (EI), is increasing in both the market size, \( a(m) \), and the size of the cost reduction, \( \theta(m) \), both of which are increasing functions of \( m \). To understand better the intuition, define by \( \Delta \pi_i(e_i, e_{-i} | m) = \pi_i(0, e_i, e_{-i} | m) - \pi_i(c, e_i, e_{-i} | m) \) the expected value of cost reduction of each incumbent firm \( i \).

The optimality condition for the contracting problem of incumbent \( i \) is given by:

\[
\frac{\partial \Pi_i(e_i, e_{-i} | m)}{\partial e_k} = C'(e_i)
\]

\[ \iff \Delta \pi_k(e_i, e_{-i} | m) + \left[ e_i \frac{\partial \pi_i(0, e_i, e_{-i} | m)}{\partial e_i} + (1 - e_i) \frac{\partial \pi_i(c, e_i, e_{-i} | m)}{\partial e_i} \right] = 2e_i . \] (FOC\(_i\))

In order to see why entry of new firms induces the incumbents to elicit higher managerial effort, we have to analyze how an increased number of entrants affects the two terms [evaluated at a common effort level \( e \) at the incumbent firms] in the left-hand-side of (FOC\(_i\)), i.e., the expected value of cost reduction and
the expected marginal profitability of effort at each incumbent firm. From (9) it follows that at a common effort level $e$,
\[
q(0, e \mid m) = \frac{2a(m) + \theta(m)c(n-1)(1-e)}{2(n+1)},
\]
\[
q(c, e \mid m) = \frac{2a(m) - \theta(m)c(n+1) + \theta(m)c(n-1)(1-e)}{2(n+1)}.
\]

Clearly, $q(0, e \mid m)$ is strictly increasing in both $a(m)$ and $\theta(m)$, and hence, in $m$. Entry of new firms induces the low-cost incumbents to produce more because both their market size and size of cost reduction increase. On the other hand, high-cost incumbents do not benefit from entry because they are harmed by the increased size of the cost reduction. Their optimal output is decreasing in the number of entrants:
\[
\frac{dq(c, e \mid m)}{dm} = \frac{2a'(m) - \theta'(m)c(2 + (n-1)e)}{2(n+1)} < 0
\]
since $2a'(m) - \theta'(m)c(2 + (n-1)e) < 0$ is equivalent to $c^2 + (4 + c^2)(n-1)e > 0$ which always holds for any $n \geq 1$. The effect of an increase in the number of entrants on the equilibrium output of both low- and high-cost incumbents is shown in Figure 4.

![Figure 4: Effect of an increase in the number of entrants from $m$ to $m'$ on the equilibrium outputs of low- and high-cost incumbents. Left panel: $A \equiv A(m) - B(m)\mathbb{E}q_{-i}$, $A' \equiv A(m') - B(m')\mathbb{E}q_{-i}$, $q_{c0} \equiv q_{i}(c_{i} \mid m)$ and $q'_{c0} \equiv q_{i}(c_{i} \mid m')$ for $c_{i} = 0, c$. Right panel: $a \equiv a(m) - \mathbb{E}q_{-i}$, $a' \equiv a(m') - \mathbb{E}q_{-i}$, $q_{c0} \equiv q_{i}(c_{i} \mid m)$ and $q'_{c0} \equiv q_{i}(c_{i} \mid m')$ for $c_{i} = 0, c$.](image)

The left panel of Figure 4 shows how an increase in the number of entrants affects the optimal outputs of low- and high-cost incumbents via a change in the marginal revenue. More entrants implies a flatter marginal revenue curve since its slope is $-2B(m)$. Clearly, $q_{i}(0 \mid m)$ increases with $m$. By contrast, the direction of the change in $q_{i}(c \mid m)$ following an increase in the number of entrants is a priori ambiguous because $B'(m) < A'(m) < 0$. The equilibrium output of each high-cost incumbent decreases in $m$, as drawn in the left panel of Figure 4, only if the intersection between the two marginal revenue curves is below the marginal cost $c$. In other words, it is difficult to determine the sign of $\frac{\partial q_{i}(c \mid m)}{\partial m}$ from the left panel. Nevertheless, one can easily show that the optimal output of each high-cost incumbent is indeed decreasing in the number of entrants by considering the alternative interpretation in terms of
the ‘normalized’ curves. In the right panel of Figure 4, we see that an increase in \( m \) induces both the market size and the size of the cost reduction to increase, i.e., \( a'(m) > 0 \) and \( \theta'(m) > 0 \). To show \( \partial q_i(c \mid m) / \partial m < 0 \), note that

\[
\text{sign} \left[ \frac{\partial q_i(c \mid m)}{\partial m} \right] = \text{sign} \left[ a'(m) - \theta'(m)c \right], \quad \text{where} \quad a'(m) - \theta'(m)c = -\frac{c^3 \theta(m)^2 \kappa'(m)}{2[1 - C_2 \kappa(m)]^2} < 0.
\]

Therefore, the entry of new firms strengthens the ‘value-of-cost-reduction effect’:

\[
\frac{d \Delta \pi(e \mid m)}{dm} = 2 \left[ q(0, e \mid m) \cdot \frac{dq(0, e \mid m)}{dm} - q(c, e \mid m) \cdot \frac{dq(c, e \mid m)}{dm} \right] > 0.
\]

An increase in the number of entrants also affects the marginal benefit of effort by changing the expected marginal profitability of effort. Since the marginal profitability of effort is the same for every incumbent regardless of its realized cost at a common effort level \( e \) [follows from (9)], we may write the expected marginal profitability of effort at any incumbent firm \( i \) as:

\[
\mathbb{E} \left[ \frac{\partial \pi_i(c_i, e \mid m)}{\partial e_i} \right] = 2 \left[ eq(0, e \mid m) + (1 - e)q(c, e \mid m) \right] \left( \frac{\partial q_i(c_i, e \mid m)}{\partial e_i} \right),
\]

where the term in square braces is the expected optimal output of each incumbent. We show that \( \mathbb{E} \left[ \partial \pi_i(c_i, e \mid m) / \partial e_i \right] \) is an increasing function of \( m \). First, note that the marginal productivity of effort is increasing in \( m \), \( \partial^2 q_i(c_i, e \mid m) / \partial e_i \partial m > 0 \). That is, regardless of its cost realization, an incumbent’s optimal output is higher if its manager elicits more effort. As mentioned above, this is because the elicited effort pins down the beliefs of the rival firms over having attained the cost reduction. Therefore, a sufficient condition for the ‘marginal-profitability-of-effort effect’ to be positive is that the expected incumbent output is increasing in the number of entrants. Showing this is not straightforward since only the output of low-cost incumbents is increasing in \( m \). Namely, the expected output is increasing in the number of entrants if the probability of attaining the cost reduction is sufficiently high. At a common effort level \( e \), this is equivalent to

\[
e > \frac{c^2}{2(4 + c^2)}.
\]

The difficulty of showing the above inequality at the equilibrium, \( e = e(n, m) \), is that both sides are increasing functions of \( c \). Since the upper bound \( \hat{e} \) does not have a closed form solution (see Appendix B), it is simpler to verify that the condition holds numerically, by doing an extensive search in the parameter space.\(^9\)

Given that both the ‘value-of-cost-reduction effect’ and the ‘marginal-profitability-of-effort effect’ are increasing in the number of entrants, the aggregate effect of an increase in the number of entrants on the marginal benefit of effort is positive. That is, at a common effort level \( e \),

\[
\frac{d[\partial \Pi_i(e \mid m) / \partial e_i]}{dm} = \frac{d \Delta \pi(e \mid m)}{dm} + \mathbb{E} \left[ \frac{\partial \pi(c_i, e)}{\partial e_i} \right] > 0.
\]

Therefore, at a common effort level \( e \), the left-hand-side of (FOC) shifts up following an increase in \( m \), but the right-hand-side, i.e., the marginal incentive cost remains unchanged, and hence, the symmetric

\(^9\)Despite the fact that we prove this claim numerically, the proof of Proposition 4 is fully analytical. For more details on this and the other claims below that we show numerically, see Appendix C.
equilibrium effort in each incumbent firm increases following the entry of new firms. The crucial difference between the simultaneous and sequential models is that in the sequential model, the entry of new firms affects the incumbents’ output decision by altering their effective market size and the size of the cost reduction. In the simultaneous oligopoly, by contrast, entry directly alters the number of firms in the market, and leaves the market size and the size of cost reduction unaffected.

6 Empirical implications

6.1 Equilibrium product market price and profits

In this section, we briefly discuss two additional empirical implications of our model. Namely, we focus on the effect that the entry of new firms has on the equilibrium expected market price and the incumbents expected market profits. Rather than dealing with the analytic complexity of these two equilibrium objects, we simplify the analysis by analyzing them numerically. We use a granular grid of the model’s parameters to validate the two claims below.¹⁰

Claim 1 Under both market structures, simultaneous and sequential oligopolies, the equilibrium expected market price decreases with the number of entrants. However, it is higher when the entrants set quantities along with the incumbents simultaneously than when they are Stackelberg followers.

According to Claim 1, the expected market price is decreasing in the number of entrants, whatever may be the nature of competition, simultaneous or sequential. This is unsurprising since more firms implies a higher aggregate output. However, the price decreases more rapidly in the number of entrants when they set quantities as Stackelberg followers. This is because in the simultaneous game all firms are symmetric, so the expected equilibrium outputs of the incumbents and entrants are equal. By contrast, in the sequential game, a higher output by an incumbent translates into a lower output by all the entrants, both directly through diminishing the price and indirectly through the reduction in the managerial incentives at each entrant firm. This can be seen in Figure 5. Whatever may be the number of incumbents, the expected market price is lower when entrants produce sequentially. Even though the plots are for a fixed value of c, we show numerically that the same holds in all the parameter space.

Claim 2 The equilibrium expected market profits of each incumbent decrease with the number of entrants when these set quantities along with the incumbents simultaneously. By contrast, the equilibrium expected market profits of each incumbent are non-decreasing when the entrants set quantities afterwards as Stackelberg followers.

According to Claim 2, the expected market profits behave qualitatively similar to the managerial effort in equilibrium (cf. Proposition 4): the higher the number of entrants, the higher the expected market profits of the incumbents. This result goes in line with our finding in the previous section; namely, with the fact that the incumbents equilibrium expected output is increasing in the number of entrants. By strengthening managerial incentives, incumbents counteract the effect that the entry of new firms has on the market price. Interestingly, the size of the relative change in profits is small when compared to that undergone when entrants set quantities along with the incumbents simultaneously. This can be seen

¹⁰For more details on the numeric implementation of the model, see Appendix C.
in Figure 6. While equilibrium profits are clearly decreasing in the number of firms when entrants set quantities simultaneously, they increase only slightly when entrants are Stackelberg followers. Indeed, incumbent profits seem to be flat graphically. Nonetheless, we show numerically that they are not; they are strictly increasing in the number of entrants over the entire parameter space.

Figure 6: *Equilibrium expected product market profits of each incumbent in the simultaneous and sequential games*

### 6.2 Nature of industry competition

The main insight of our stylized model is the juxtaposition of Proposition 2 with Proposition 4, i.e., when the entrants are able to set quantities simultaneously, the incumbent firms offer less high-powered managerial incentives as the number of firms grows, whereas when the entrants act as Stackelberg followers, the result is reversed. How one can relate these contrasting results to the nature of competition that prevails in different industries?
In an important contribution to the literature on oligopolistic competition, Kreps and Scheinkman (1983) analyze a two-stage game where firms simultaneously choose their production capacities in the first stage, and compete in prices in the stage that follows. They show, under very general conditions, simultaneous capacity pre-commitment followed by price competition yield Cournot outcome. By contrast, Allen, Deneckere, Faith, and Kovenock (2000) show that when firms choose capacities sequentially, i.e., the entrant firms choose capacities after observing the capacities set by the incumbents, and then the firms compete in prices, the subgame perfect equilibrium yields the Stackelberg outcome. An entrant firm cannot instantly adjust its capacity as it requires time to build, which is the crux of Allen et al.’s (2000) analysis. Thus, the Cournot outcome, which is the result of simultaneous capacity pre-commitment both by the incumbents and entrants [followed by price competition], is based on a no-time-to-build assumption. This disparity in the time taken to build capacity implies the following result.

**Implication 1** In an industry where production capacity can be adjusted instantaneously, entry of new firms implies that the incumbents would provide less high-powered incentives to their managers. By contrast, if the production capacity requires ‘time to build’, then entry of new firms induces the incumbents to provide more high-powered managerial incentives.

Production capacities can be built almost instantaneously due to the possible presence of low sunk costs in industries, e.g. services and technology. Therefore, our results imply that the Hicks conjecture is refuted in such industries. On the other hand, in industries such as manufacturing where capacity requires time to build, the conjecture is validated.

### 7 Conclusion

In a quantity-setting oligopoly the implications of competition for managerial incentives may be quite different depending on the nature of competition. In a simple model of strategic interaction among firms we show that when the incumbent and entrant firms compete à la Cournot, the incumbent firms offer weaker incentives to their managers, which imply greater managerial slack, following the entry of new firms in the market. When the incumbents, on the other hand, behave as Stackelberg leaders by setting their quantities prior to entry, managerial incentives are stronger in the incumbent firms in the post-entry equilibrium. This result is obtained because entry of new firms induces each entrant to elicit lower managerial effort because the entrants compete in a Cournot fashion. This raises the expected cost at each entrant firm, and hence, lowers the aggregate production by the entrants. As a result, the effective market size and the size of cost reduction for the incumbents are larger which induce them to offer more high-powered incentives to their managers. Thus, in a sequential quantity-setting game the well-known Hicks conjecture is validated.

The sequential quantity-setting oligopoly model considers only two phases of Cournot competition (first, among the incumbents, and then among the entrants). As the model can be derived from the ‘base model’ analyzed in Appendix A, adding more phases of sequentiality would be a trivial extension as it would have retained the same underlying structure, yet it would deliver interesting testable implications. In recent years many countries have taken a gradual approach to deregulation, e.g. the Japanese electricity market, as markets have been deregulated over several phases. Our model would deliver clear predictions on the effect of competition on managerial incentives at different phases of competition as entry of new
firms will alter the nature of Cournot competition in the last phase and will have spillover effects by altering the market size and the size of cost reduction, and hence, the power of managerial incentives in the firms competing in the earlier phases.

Although managerial activities in the current model include only cost-reducing R&D efforts, the model can easily be extend to other settings such as product innovation and price competition, and our would results remain valid. This is because of the general conclusion of such models in the literature that the nature of strategic effects of managerial tasks, i.e., whether the activities are strategic substitutes or complements, translates into the same nature of strategic effects of managerial incentives (see Chalioti and Serfes, 2017). In this sense, our model also contributes to the existing literature on the effect of product market competition on incentives to innovate (e.g. Bester and Petrakis, 1993; Belleflamme and Vergari, 2011).

Appendices

A The base model

Both the simultaneous and sequential quantity setting models have the same underlying structure. In this section, we analyze a more general version of the simultaneous quantity competition model with a fixed number \( |K| = N \) of firms, called the base model, and derive the results in Sections 4 and 5 as special cases. Let \( P = A - BQ \) be the inverse market demand with \( A, B > 0 \). The marginal cost of a representative firm \( k \) is given by \( c_k \), with \( c_k \in \{0, c\} \) with \( c \in (0, \bar{c}) \). The upper bound \( \bar{c} \) is such that all firms produce a positive outputs in equilibrium regardless of their realized marginal costs. We will prove that such bound exists. Define by \( a = A/B \) the market size of each firm, and \( \theta = 1/B \) so that \( \theta c \) represents the size of cost reduction of each firm. The following lemma describes the main results of the base model.

**Lemma 3** Let \( N \geq 1 \) and \( a > \theta c \).

(a) The equilibrium effort of the base model is symmetric and unique across all firms. It is given by:

\[
e(N) = \frac{\theta c[8aN + \theta c(N^2 - 6N + 1)]}{2[4(N + 1)^2 + \theta^2 c^2(N - 1)^2]}.
\]

Moreover, the second order condition associated with the individual firm maximization problem in a Bayesian Cournot equilibrium is satisfied for every firm if all of them produce a positive quantity in equilibrium;

(b) If \( a \) and \( \theta \) are independent of \( c \), then there is \( \bar{c} \in (0, a/\theta) \) such that, if \( c \in (0, \bar{c}) \), every firm produces a strictly positive quantity of output and elicits strictly positive level of managerial effort in a symmetric equilibrium, regardless of its realized marginal cost;

(c) The equilibrium effort in (EC) is decreasing in the number of firms, and increasing in the market size and the size of cost reduction. That is, for every \( N \geq 1 \),

\[
\frac{de(N)}{dN} < 0, \quad \frac{de(N)}{da} > 0, \quad \text{and} \quad \frac{de(N)}{d(\theta c)} > 0.
\]
Proof
(a) Once all the contracts are observed and marginal costs are privately realized, each firm $k$ solves

$$\max_{q_k} q_k[A - B(q_k + E q_{-k}) - c_k].$$

The first-order condition of the above maximization problem is given by:

$$A - 2Bq_k - B E q_{-k} - c_k = 0$$
$$\iff 2Bq_k = A - c_k - B E q_{-k}$$
$$\iff q_k(c_k, E q_{-k}) = \frac{A - c_k}{2B} - \frac{1}{2} E q_{-k}. \quad (10)$$

Taking expectation in the above equation, we get

$$E q_k = \frac{A - E c_k}{2B} - \frac{1}{2} E q_{-k} = \frac{A - c(1 - e_k)}{2B} - \frac{1}{2} E q_{-k}. \quad (11)$$

Summing the above over $k$ we get

$$\sum_{k=1}^N E q_k = \frac{N(A - c)}{2B} + \frac{c}{2B} \sum_{k=1}^N e_k - \frac{N - 1}{2} \sum_{k=1}^N E q_k$$
$$\iff \sum_{k=1}^N E q_k = \frac{1}{B(N + 1)} \left[ N(A - c) + c \sum_{k=1}^N e_k \right]. \quad (12)$$

On the other hand, (11) can be written as

$$E q_k = \frac{A - c(1 - e_k)}{2B} - \frac{1}{2} \left( \sum_{l=1}^N E q_l - E q_k \right)$$
$$\iff \frac{1}{2} E q_k = \frac{A - c(1 - e_k)}{2B} - \frac{1}{2B(N + 1)} \left[ N(A - c) + c \sum_{k=1}^N e_k \right]$$
$$\iff E q_k = \frac{A - c + c(N e_k - e_{-k})}{B(N + 1)}, \quad (13)$$

where $e_{-k} = \sum_{l \in K \setminus \{k\}} e_l$, and hence, $E q_{-k} = \sum_{l=1}^N E q_l - E q_k$. Thus, substituting for $\sum_{l=1}^N E q_l$ and $E q_k$ from (12) and (13) into (10), we obtain the quantity and profit of each firm in the Bayesian Cournot equilibrium, which are respectively given by:

$$q_k(c_k, e_k, e_{-k}) = \frac{2A - (N + 1)c_k + (N - 1)c(1 + e_k) - 2ce_{-k}}{2B(N + 1)}$$
$$= \frac{2a - (N + 1)e_k + (N - 1)e(1 + e_k) - 2e e_{-k}}{2(N + 1)}, \quad (14)$$

$$\pi_k(c_k, e_k, e_{-k}) = \left( \frac{2a - (N + 1)e_k + (N - 1)e(1 + e_k) - 2e e_{-k}}{2(N + 1)} \right)^2. \quad (15)$$
The expression in (14) is obtained by using the facts that \( a \equiv A/B \) and \( \theta \equiv 1/B \). At date 1, each firm \( k \) chooses the optimal managerial incentives to solve

\[
\max_{e_k} e_k \pi_k(0, e_k, e-\_k) + (1 - e_k) \pi_k(e, e_k, e-\_k) - e_k^2.
\]

(16)

The expected value of cost reduction, \( \Delta \pi_k(e_k, e-\_k) \) := \( \pi_k(0, e_k, e-\_k) - \pi_k(e, e_k, e-\_k) \) of firm \( k \) is given by:

\[
\Delta \pi_k(e_k, e-\_k) = \frac{\theta c[4a + c(N - 3) + 2c(N - 1)e_k - 4c e\_k]}{4(N + 1)}.
\]

(17)

Also, note that

\[
e_k \cdot \frac{\partial \pi_k(0, e_k, e-\_k)}{\partial e_k} + (1 - e_k) \cdot \frac{\partial \pi_k(e, e_k, e-\_k)}{\partial e_k} = \frac{\theta c(N - 1)[a - c + c(Ne_k - e-\_k)]}{(N + 1)^2}.
\]

(18)

Using the expressions (17) and (18), the first-order condition of the maximization problem in (16) is given by:

\[
\Delta \pi_k(e_k, e-\_k) + e_k \cdot \frac{\partial \pi_k(0, e_k, e-\_k)}{\partial e_k} + (1 - e_k) \cdot \frac{\partial \pi_k(e, e_k, e-\_k)}{\partial e_k} = 2e_k
\]

\[\iff\]

\[\theta c[8aN + c(N^2 - 6N + 1) + 6c(N^2 - 1)e_k - 8c Ne\_k] = 2e_k.\]

(FOC\(_k^\prime\))

Condition (FOC\(_k^\prime\)) implicitly defines the best response (in effort) \( e_k(e-\_k) \) of the manager at firm \( k \), which is given by:

\[
e_k(e-\_k) = \frac{\theta c[8aN + c(N^2 - 6N + 1)]}{2[4(N + 1)^2 - \theta^2 c^2(3N + 1)(N - 1)]} - \frac{4\theta^2 c^2 Ne\_k}{4(N + 1)^2 - \theta^2 c^2(3N + 1)(N - 1)}.
\]

(BR\(_k^\prime\))

In a symmetric equilibrium, we have \( e_k = e(N) \) for all \( k \), and hence, \( e-\_k = (N - 1)e(N) \). Substituting these into (FOC\(_k^\prime\)) yields the symmetric equilibrium effort level which is given by (EC).

Next, we show that the second order condition is satisfied for every firm if all of them produce a positive output in equilibrium. Note that the second-order condition of firm \( k \)'s maximization problem (16) is given by:

\[
2 \left( \frac{\partial^2 \Delta \pi_k}{\partial e_k^2} \right) + e_k \cdot \frac{\partial^2 \Delta \pi_k}{\partial e_k^2} + \frac{\partial^2 \pi_k}{\partial e_k^2} - 2 \leq 0.
\]

\[\iff\]

\[\theta^2 c^2 (N - 1) \left( \frac{1}{N + 1} + \frac{\theta^2 c^2 (N - 1)^2}{2(N + 1)^2} \right) - 2 \leq 0.\]

(SOC\(_k\))

Note that (SOC\(_k\)) holds if \( N = 1 \), and it is equivalent to

\[
\theta c \leq \frac{2(N + 1)}{\sqrt{(N - 1)(3N + 1)}} \quad \text{for } N \geq 2.
\]

(SOC\(_k^\prime\))

From (14), it follows that \( q_k(c, \cdot) = q_k(0, \cdot) - \theta c/2 \), so \( q_k(c, \cdot) > 0 \) for all \( k \) implies

\[
\theta c < \frac{2}{N} \sum_k q_k(0, \cdot).
\]

(19)

The upper bound on \( \theta c \) in (19) is lower than the one in (SOC\(_k^\prime\)) since, by construction, \( \sum_k q_k(0, \cdot) < 1 \), and \( N(N + 1)/\sqrt{(N - 1)(3N + 1)} > 1 \) for each \( N \geq 2 \).
(b) We prove the existence of \( \bar{c} \in (0, a/\theta) \) such that \( c \in (0, \bar{c}) \) implies \( q_k(c_k, \cdot) > 0 \) in equilibrium for every \( k \in K \) and \( c_k \in \{0, c\} \). Fix \( N \geq 1 \). Write \( e(N, c) \equiv e(N) \). From (14), see that the symmetric equilibrium production of a high-cost firm \( q^*(c) \) is lower than that of a low-cost firm and satisfies:

\[
q^*(c) = \frac{2(a-\theta c) - \theta c(N-1)e(N, c)}{2(N+1)} > 0 \iff f(N, c) \equiv \frac{2(a-\theta c)}{\theta c(N-1)} - e(N, c) > 0. \tag{20}
\]

Note that

\[
\lim_{c \to 0} f(N, c) = \infty,
\]

\[
f(N, a/\theta) = 0 - \frac{a^2(N+1)^2}{2[4(N+1)^2 + \theta^2 c^2(N-1)^2]} < 0.
\]

Therefore, by Intermediate Value Theorem, there is \( c_0 \in (0, a/\theta) \) such that \( f(N, c_0) = 0 \). If \( c_0 \) is unique, then take \( \bar{c} = c_0 \). Otherwise, take \( \bar{c} = \min\{c_0\} \). Next, we prove that \( e(N) > 0 \), which is equivalent to the following:

\[
8aN + \theta c(N^2 - 6N + 1) > 0. \tag{21}
\]

Given that \( a > \theta c \), we have

\[
8aN + \theta c(N^2 - 6N + 1) > 8\theta c + \theta c(N^2 - 6N + 1) = \theta c(N+1)^2 > 0,
\]

which proves (21) for all \( N \geq 1 \).

(c) Fix \( N \geq 1 \). Differentiating (EC) with respect to \( N \) we obtain

\[
\frac{\partial e(N)}{\partial N} = -\frac{2\theta c(N^2 - 1)[8b^2(a-\theta c) + \theta^2 c^2(2a-\theta c)]}{[4b^2(N+1)^2 + \theta^2 c^2(N-1)^2]^2}.
\]

The above expression is negative for \( a > \theta c \). Next,

\[
\frac{\partial e(N)}{\partial a} = \frac{8\theta cN}{2[4b^2(N+1)^2 + \theta^2 c^2(N-1)^2]} > 0.
\]

Finally, use the fact that \( e(N) > 0 \) to verify \( \partial e(N)/\partial (\theta c) > 0 \). Note that

\[
\frac{\partial e(N)}{\partial (\theta c)} > 0 \iff e(N) < \frac{4aN + \theta c(N^2 - 6N + 1)}{2\theta c(N-1)^2}.
\]

Because of the interior solution condition in (20), it suffices to show

\[
\frac{2(a-\theta c)}{\theta c(N-1)} < \frac{4aN + \theta c(N^2 - 6N + 1)}{2\theta c(N-1)^2}
\]

\[
\iff 4(a-\theta c) + \theta c(N-1)^2 > 0.
\]

The last inequality holds since \( a > \theta c \).

This completes the proof of Lemma 3. \( \square \)
B Proofs

Proof of Proposition 1

Write \( N = n + m \). Note that the inverse demand function is given by \( P = A - BQ \) with \( A = B = 1 \). Therefore, \( a = \theta = 1 \). The proof directly follows from Lemma 3(a)-(b). \( \square \)

Proof of Proposition 2

Write \( N = n + m \) and \( N' = n + m' \). Clearly, \( m' > m \) implies \( N' > N \), and hence, the proof directly follows from Lemma 3(c). \( \square \)

Proof of Lemma 2

The proof directly follows from Lemma 3(a)-(b) with \( A = 1 - Q_I \), \( B = 1 \), and \( N = m \). \( \square \)

Proof of Proposition 3

The maximization problem of each incumbent \( i \) is given by:

\[
\max_{q_i} q_i (1 - q_i - \mathbb{E}q_{-i} - Q_I(q_i + \mathbb{E}q_{-i}) - c_i)
\]

\[
\iff \quad \max_{q_i} q_i \left[ \left( 1 - C_1 \kappa(m) \right) - \left( 1 - C_2 \kappa(m) \right) (q_i + \mathbb{E}q_{-i}) - c_i \right]
\]

Therefore, setting \( a \equiv a(m) = \frac{A(m)}{B(m)} \), \( \theta \equiv \theta(m) = 1/B(m) \) and \( N = n \) it follows from Lemma 3(a) that

\[
e(n, m) = \frac{\theta(m)c[8a(m)n + \theta(m)c(n^2 - 6n + 1)]}{2[4(n + 1)^2 + \{\theta(m)c\}^2(n - 1)^2]}.
\]

Recall that the subgame played by the entrants is equivalent to the base model with \( B = 1 \), \( a = A/B = 1 - Q_I \) and \( \theta = 1/B = 1 \). We cannot apply the bound in Lemma 3(b) directly since \( a \) depends on \( c \) (\( Q_I \) is an equilibrium object that depends on the model’s parameters). Obtain the equilibrium output of a high-cost entrant by replacing \( a = 1 - Q_I \) and \( \theta = 1 \) in (14):

\[
q_j(c | Q_I) = \frac{2(1 - Q_I - c) - c(m - 1)e_I(m, Q_I)}{2(m + 1)},
\]

where \( e_I(m, Q_I) \) is the optimal effort of the entrants given in (EE). Since low-cost entrants produce more than high-cost ones in equilibrium, the interior solution condition is equivalent to \( q_j(c | Q_I^*) > 0 \), where \( Q_I^* \) is the total output of the incumbents in the symmetric equilibrium. Note that \( q_j(c | Q_I) \) is decreasing.
in $Q_t$ since

$$\frac{\partial q_j(c \mid Q_t)}{\partial Q_t} < 0$$

$$\iff 2 - \frac{4c^2m(m-1)}{4(m+1)^2 + c^2(m-1)^2} > 0$$

$$\iff m^2(4-c^2) + m(8-c^2) + 4 + c^2 > 0.$$

Hence, a high-cost entrant produces the least when all incumbents have low costs. Let $q_i(0)$ be the optimal output of a low-cost incumbent, so the interior solution for each entrant $j$ requires $q_j(c \mid \sum_{i \in J} q_i(0)) > 0$. By (22), this is equivalent to

$$\sum_i q_i(0) < 1 - c - \frac{c(m-1) \cdot e_j(\sum_i q_i(0))}{2}.$$  \hspace{1cm} (23)

Note that an incumbent $i$'s best response to the expected aggregate output of the rival incumbents is given by:

$$q_i(\mathbb{E}q_{-i}) = \frac{1}{2}[a(m, c) - \theta(m, c)c_i] - \frac{1}{2}\mathbb{E}q_{-i}.$$  

From (9), the equilibrium output of a low-cost incumbent is given by:

$$q_i(0) = \frac{2a(m, c) + \theta(m, c)c(n-1)(1-e(n, m, c))}{2(n+1)}.$$  \hspace{1cm} (24)

Therefore, since $e(n, m) \to 0$, $a(m, c) \to 1$, and $\theta(m, c) \to m + 1$, as $c \to 0$ because $\lim_{c \to 0} \kappa(m, c) = m/[4(m+1)]$, 

$$\lim_{c \to 0} \sum_i q_i(0) = \frac{n}{n+1} < 1 = \lim_{c \to 0} 1 - c - \frac{c(m-1) \cdot e_j(\sum_i q_i(0))}{2}.$$  

Hence, according to (23), there exists $\bar{c}_j > 0$ such that every entrant produces a positive output in equilibrium, provided $c \in (0, \bar{c}_j)$. Furthermore, $\bar{c}_j < a/\theta = 1 - Q_t^*$ since (23) also implies

$$c < c + \frac{c(m-1) \cdot e_j(\sum_i q_i(0))}{2} < 1 - \sum_i q_i(0) \leq 1 - Q_t^*.$$  

Finally, we characterize the interior solution condition of the incumbents. From (14), the interior solution condition for incumbents, $q_i(c) > 0$, is equivalent to

$$e(n, m, c) < \frac{2(a(m, c) - \theta(m, c)c)}{\theta(m, c)c(n-1)}.$$  \hspace{1cm} (25)

Since $e(n, m, c) \to 0$ as $c \to 0$, while the right-hand-side of (25) tends to 0, there exists $\bar{c}_f > 0$ such that all incumbents produce a positive output in equilibrium, provided $c \in (0, \bar{c}_f)$. Moreover, $\bar{c}_f < a(m, c)/\theta(m, c)$ since (25) does not hold if $c > a(m, c)/\theta(m, c) = 1 - \kappa(m, c)$. Define $\hat{c} = \min\{\bar{c}_f, \bar{c}_i\}$ to obtain the appropriate bound. By Lemma 3(a), every firm’s second order condition of the optimal contracting problem is satisfied if $c \in (0, \hat{c})$.  \hspace{1cm} □
Proof of Proposition 4

We first establish that \( \kappa(m) \) is strictly increasing in \( m \). Note that

\[
\kappa'(m) = \frac{(4 + c^2)(m + 1)^2 - 4c^2m^2}{4(m + 1)^2 + c^2(m - 1)^2}.
\]

The numerator of the above expression is strictly positive if and only if

\[
\frac{4 + c^2}{4c^2} \frac{m}{m + 1} > \left( \frac{m}{m + 1} \right)^2.
\]

Note that \( h(c) \) is strictly decreasing on \([0, 1]\) with \( \min\{h(c)\} = h(1) = 5/4 > 1 \). The right-hand-side of the above inequality is always strictly less than 1 for \( m \geq 1 \). Hence, \( \kappa'(m) > 0 \). Next,

\[
a(m) = \frac{1 - C_1 \kappa(m)}{1 - C_2 \kappa(m)} \quad \implies \quad a'(m) = \frac{(C_2 - C_1) \kappa'(m)}{[1 - C_2 \kappa(m)]^2} = \frac{c(8 + c^2) \kappa'(m)}{2[1 - C_2 \kappa(m)]^2} > 0,
\]

\[
\theta(m) = \frac{1}{1 - C_2 \kappa(m)} \quad \implies \quad \theta'(m) = \frac{C_2 \kappa'(m)}{[1 - C_2 \kappa(m)]^2} = \frac{(4 + c^2) \kappa'(m)}{[1 - C_2 \kappa(m)]^2} > 0.
\]

It follows from Lemma 3(c) that \( e(n, m) \) is strictly increasing in both \( a(m) \) and \( \theta(m)c \), and hence, the proposition. \( \square \)

C Numerical implementation

In order to show that the incumbents expected output at equilibrium is increasing in the number of entrants, and the validity of Claims 1 and 2 in Section 6.1, we compute the model numerically. We define a grid over the parameter space \((c, n, m) \in \mathcal{C} \times \mathcal{N} \times \mathcal{M}\), where \( \mathcal{C} \) is a grid of \((0, 1)\) with each cell of size \( c^* = 7.6893e-6 \), \( \mathcal{N} = \{1, 2, \ldots, 50\} \), and \( \mathcal{M} = \{0, 1, \ldots, 50\} \). For each \((n, m) \in \mathcal{N} \times \mathcal{M}\), we solve numerically for the upper bound of \( c \), both in the simultaneous and sequential games, given by \( \hat{c}(n, m) \) and \( \hat{c}(n, m) \), respectively. We then show the validity of the claims in every point in the parameter grid with \( c \leq \hat{c}(n, m) \) and \( c \leq \hat{c}(n, m) \), accordingly. The cell size \( c^* \) corresponds to \( 0.5 \min_{(n, m) \in \mathcal{N} \times \mathcal{M}} \{\hat{c}(n, m), \hat{c}(n, m)\} \).

References


