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Executive compensation and competitive pressure in the product market: How does firm entry shape managerial incentives?[☆]

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ABSTRACT

Motivated by empirical evidence, we develop an incentive contracting model under oligopolistic competition to study how incumbent firms adjust managerial incentives following deregulation policies that enhance competition. We show that firms elicit higher managerial effort by offering stronger incentives as an optimal response to entry, as long as incumbent firms act as production leaders. Our model draws a link between an industry-specific feature, the time needed to build production capacity, and the effect that product market competition has on executive compensation. We offer novel testable implications regarding how this industry-specific feature shapes the incentive structure of executive pay.

1. Introduction

There is a plethora of empirical evidence that supports the Hicksian view (Hicks, 1935) that executive compensation tends to be more performance-sensitive in more competitive environments (e.g. Nickell, 1996; Van Reenen, 2011). A series of empirical studies have used industry-specific regulatory reforms to analyze the effect of competition on executive pay (Crawford, Ezzell, and Miles, 1995; Hubbard and Palia, 1995; Kole and Lehn, 1999; Palia, 2000; Cuñat and Guadalupe, 2009a; Dasgupta, Li, and Wang, 2017). These studies focus on how deregulation policies that increase competition in the product market affect the structure of managerial incentive contracts. The main takeaway from this literature is that, following a deregulation policy that intensifies product market competition, firms reduce managerial slack by increasing executive compensation and strengthening its pay-performance sensitivity.

Our objective in this paper is to explain the nature of the aforementioned empirical regularity, and to offer new insights into how executive pay is shaped by industry-specific features. First, we provide a simple model of oligopolistic competition with firm entry that shows why incumbent firms find it optimal to reduce managerial slack when competition rises because of deregulation. Then, we use our model to derive novel empirical implications regarding the *time to build production capacity* in an industry. Our model shows that this industry-specific feature is a crucial factor when analyzing the effect that firm entry has on executive compensation. According to our model, the relationship observed in the empirical studies obtains in industries in which the time to build capacity is such that incumbents act as production leaders and entrants as followers. This result goes in line with the empirical literature given that existing studies focus on industries in which it takes time to build production capacity, such as banking, manufacturing, and the airline industry.

The question of how product market competition shapes managerial incentives is far from being new in the literature.[1] Notwithstanding, our approach is novel in that we analyze it explicitly in a framework of firm entry. Because incumbent firms anticipate (and accommodate) future entry with relaxing regulation, we use a standard model of sequential quantity-setting oligopoly, in which entrant firms choose their managerial contracts and quantities after observing those of the incumbents. Our focus is on the strategic response of incumbents regarding managerial incentive pay as they foresee the entry of new firms. In line with the empirical literature, our main finding is that it is optimal for incumbents to strengthen incentive pay and reduce managerial

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[1] The notion that monopoly, and market power in general, are detrimental to managerial efficiency dates back to Smith (1776, Book 1, Chapter 11), and has a long tradition in the literature (Leibenstein, 1966; Hart, 1983; Scharfstein, 1988).

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slack when they foresee the entry of new firms into the product market. Moreover, we show that the strength of the managerial incentives offered by incumbents is increasing in the number of entrants—higher competitive pressure leads to steeper incentives and lower managerial slack.

Our model incorporates managerial incentive contracts into the Stackelberg quantity competition framework proposed by Daughety (1990). There is a fixed number of incumbents and a set of potential entrants with more entrants meaning greater competitive pressure on the incumbent firms. Both incumbents (in the pre-entry stage) and entrants (in the post-entry stage) play Cournot games among themselves; entrants take the aggregate output of incumbents as given. All firms are initially inefficient, and each hires a risk neutral manager whose principal task is to exert non-verifiable R&D effort to bring down the constant marginal cost of production, what is often termed "process innovation". We assume that the final realizations of marginal costs are private information among firms, and that incentive contracts are publicly observable. Hence, even though the marginal costs of rival firms are unknown, each firm observes a signal of how likely every other firm is to reduce its marginal cost.

The crux of our model is that managerial effort is beneficial to incumbents in two ways. First, steeper incentives that induce each manager to exert higher effort directly increase the likelihood of cost reduction (value-of-cost-reduction effect). Second, they also alter the beliefs of the rival firms about the true cost realization of a given firm (marginal-profitability-of-effort effect). Even if a manager fails to achieve the cost target, her effort is profitable in as much as it makes the rivals believe that a cost reduction has actually been attained. More intensified product market competition affects each of these two effects through the market size and the effective size of cost reduction. As the entrants' optimal contracting and production decisions are negatively affected by the aggregate incumbent output, the entry of new firms implies an increase in both market size and the effective size of cost reduction for incumbents. In turn, this implies both a higher expected value of cost reduction and expected marginal profitability of effort, which makes it optimal for the incumbents to elicit higher managerial effort by strengthening incentives. It is worth noting that, even in the absence of the marginal-profitability-of-effort effect, a growing number of entrants strengthens the value-of-cost-reduction effect. Such case arises, for example, when marginal costs are public information and managerial effort is unprofitable beyond cost reduction.

The key to our main result is that incumbent firms are able to strategically pre-commit to managerial contracts, which in turn determine technological efficiency endogenously. The general intuition goes in line with the seminal works of Fudenberg and Tirole (1984) and Bulow, Geanakoplos, and Klemperer (1985). In a standard entry model, when an incumbent and an entrant compete in quantities (strategic substitutes), lowering the marginal cost of the incumbent decreases the entrant's total profits (since the incumbent's optimal output increases). Hence, when costs are endogenously determined, incumbents find it optimal to behave more aggressively in cost-reduction activities. In our framework, this corresponds to incumbents offering stronger managerial incentives which are observed by the entrant firms. Thus, by making a commitment to be more aggressive, the incumbents push the entrants into a more passive posture. This is an example of the 'top-dog' strategy, according to the terminology proposed by Fudenberg and Tirole (1984). This sort of aggressive or accommodating behavior on behalf of the incumbent firms does not emerge under simultaneous competition because the incumbents fail to reap such benefits due to the lack of pre-commitment to any investment strategy. By contrast, under strategic complementarity, e.g. price competition, the aforementioned result is reversed because the incumbent firms would commit to a strategy of 'underinvestment' (weakened managerial incentives) after which the entrants would optimally respond by lowering their prices. Fudenberg and Tirole (1984) call such underinvestment strategy to avoid stoking competition 'puppy-dog ploy'.

The paper is organized as follows. In Section 2, we review the related literature. In Section 3, we outline the model. In Section 4, we solve for the equilibrium and present our main results. In Section 5, we present testable implications of our model. In Section 6, we analyze two extensions, hierarchical entry and price competition. We conclude in Section 7. All proofs are relegated to Appendix B, most of which follow from Result 1 in Appendix A.

2. Related literature and our contribution

The astounding rise in both the level and incentive component of executive compensation packages over the past three decades is often attributed to changes in industry configurations. The idea is based on the Darwinian view of organizations, which states that, in order to survive and perform well, firms must solve governance problems by adapting their structure of managerial incentive contracts as product market competition rises. As mentioned in the previous section, several studies have exploited regulatory reforms to analyze how product market competition shapes the incentive structure of the executive compensation packages. Kole and Lehn (1999), and Palia (2000) study how the introduction of the Airline Deregulation Act in 1978 has altered the structure of the incentive contracts offered to CEOs in the U.S. airline industry. Crawford et al. (1995), Hubbard and Palia (1995), and Cuñat and Guadalupe (2009a), analyze the changes in executive pay in the U.S. banking sector following an important regulatory reform that permitted interstate banking during the 1980s. In the context of international trade, Cuñat and Guadalupe (2009b) study the effect of changes in foreign competition on executive pay in the U.S. firms. Dasgupta et al. (2017) analyze the effect of industry-level tariff cuts on CEOs pay-performance sensitivity in the U.S. manufacturing sector. Overall, these studies confirm the view that one of the ways in which firms react to intensifying product-market competition is by increasing the pay-performance sensitivity of their executive compensation packages.[2]

We build on Daughety's (1990) Stackelberg leadership model by endogenizing firm technology via managerial incentive contracts.[3] However, Daughety (1990) does not consider the possibility of incumbents using (endogenous) cost-reducing R&D investments as a pre-commitment device for product-deterrence, as in Fudenberg and Tirole (1984) and Bulow et al. (1985). In our model, since the incumbents are able to pre-commit to strategic managerial incentive contracts, incumbent output is increasing in the number of entrants. Namely, a higher number of entrants implies a higher *expected marginal profitability of effort*. By contrast, in Daughety (1990), incumbent output is independent of the number of entrants.[4]

A couple of other papers also analyze the interaction between entry and R&D incentives in oligopoly with sequential moves. Etro (2004) considers a model of patent race where a monopolist leader faces a fringe of entrants. In the Stackelberg equilibrium

[2] In a related study, Karuna (2007) also finds a positive relationship between the degree of product substitutability and stock options granted to CEOs.

[3] Both the Stackelberg and Cournot settings of our model can be seen as special cases of a slightly more general model, which we refer to as the "base model". The base model may be of independent theoretical interest as it provides a simple method for analyzing comparative statics on the number of firms in Stackelberg and Cournot models under cost uncertainty. See Appendix A for further details.

[4] We owe this observation to an anonymous referee.

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under free entry, the incumbent monopolist innovates more aggressively because any profitable innovative opportunity would be reaped by new entrants until entry dissipates profit. Ishida, Matsumura, and Matsushima (2011) analyze a two-stage Cournot competition with ex-ante cost asymmetry, whereas we consider ex-ante symmetry. As in the present paper, investment by one firm in process innovation is an instrument for pre-commitment to expand output in order to deter the output of its rivals. However, none of the aforementioned papers consider the possibility of endogenous production technology in *managerial* firms via optimal incentive contracts; instead, they focus on the direct effect of increased competition on R&D investment.

In agency theoretic models relating product market competition to managerial incentives, competing against more firms invariably reduces equilibrium output and profits.[5] In turn, this lowers the value of attaining a cost reduction and thus makes it optimal to offer weaker managerial incentives (the so-called *scale* or *output* effect). In a framework of hidden information (about the realization of marginal costs), Martin (1993) assumes that the marginal productivity of managerial effort decreases in the number of active firms in a Cournot market, and hence, the equilibrium state-contingent contracts provide weaker incentives as the number of firms grows. Golan, Parlour, and Rajan (2015) also analyze managerial incentives in a Cournot oligopoly. As the expected product market profit of each firm depends on the likelihood of achieving a low marginal cost in the rival firms, the observed profit as a signal of managerial effort becomes noisier, and hence, the cost of incentive provision magnifies in a more competitive environment. This effect points in the same direction as the standard scale effect implying a negative association between competition and incentives.

In order to counteract the negative effect of competition on managerial incentives due to lower product market profits, one thus requires to identify additional countervailing effects of product market competition on managerial incentives. The effect of competition on executive pay-performance sensitivity may be, in theory, non-monotonic. Hermalin (1992) models CEOs as receiving a fraction of the share-holder income. Because more intense competition erodes this income, managers tend to consume fewer "agency goods", i.e., expend more effort, as agency goods are assumed to be normal goods. Hermalin (1994) assumes that more firms in a Cournot market imply an exogenous decrease in the slope of the inverse market demand (with the intercept remaining constant), and hence, an exogenous increase in the market size of each firm is identified as a countervailing *business stealing effect*, apart from the standard value-of-cost-reduction effect. Schmidt (1997) shows that if a firm is more likely to go bankrupt in a more competitive environment, the manager tends to work harder to avoid liquidation of the firm's assets as liquidation implies a loss of reputation. The value-of-cost-reduction effect and the threat-of-liquidation effect do not often point in the same direction. Piccolo, D'Amato, and Martina (2008) build on Martin (1993), and identify an *agency effect*. In their model, profit-sharing contracts improve productive efficiency, which points in the direction opposite to the standard scale effect. Thus, they obtain an inverted-U relationship between competition and managerial effort. Raith (2003) analyzes a managerial incentive problem in a price-setting oligopoly with horizontal differentiation and privately realized marginal costs. He establishes a positive association between competition and managerial incentives by showing that in a free-entry equilibrium managerial incentives increase due to a higher degree of product substitutability, market size or lower cost of entry. Wu (2017) analyzes the interaction between product and labor markets in a model that assigns worker talent to heterogeneous firms. Greater product market competition, as measured by demand elasticity, results in a reallocation of more talented managers from smaller to larger firms, and hence, an increase in the value of managerial efforts in such firms. Consequently, firms strengthen managerial incentives, and the resulting wage distribution becomes more right-skewed.

Our approach is novel because we analyze a new mechanism through which product market competition affects executive pay-performance sensitivity. In particular, we study how incumbent firms adjust their managerial contracts optimally when new firms are about to enter the market. As mentioned earlier, a model of sequential quantity competition is appropriate to analyze the effect of increased competition following a regulatory reform. In line with the empirical evidence, we find a positive relationship—as competition rises, incumbents find it optimal to strengthen executive pay-performance sensitivity in order to reduce managerial slack. Furthermore, we also contribute to the literature by noting that the time to build production capacity in an industry is a key factor in studying how competition affects managerial incentives. In particular, it allows us to relate our model to the earlier literature that finds a negative association between competition and managerial incentives. Our analysis builds on previous literature and conforms to empirical findings.

Our paper is also related to a well-known strand of literature in which incentive contracts are assumed to be linear combination of profit and revenue (e.g. Vickers, 1985; Fershtman and Judd, 1987; Sklivas, 1987). In these models, managers choose output (or price, depending on whether firms compete in quantities or prices) to maximize the incentive scheme. Wang and Wang (2009) extend this framework to sequential managerial delegation and obtain results similar to Daughety (1990), i.e., a more equal distribution on leaders and followers results in higher industry output, lower price, and higher welfare. The main difference of our approach is that, in our case, managers receive state-contingent contracts and choose cost-reducing R&D efforts instead of quantity or price. As a consequence, firms' cost parameters become endogenous, which results in ex-post asymmetry.

3. The model

3.1. Specifications

The economy consists of two classes of risk neutral agents, $n+m$ ex-ante identical firms who compete in quantities in a market for a homogeneous good, and $n+m$ ex-ante identical managers. The firms are divided in two groups—namely, a subset I of $n \geq 1$ incumbents and a subset J of $m \geq 0$ entrants, with $I \cap J = \varnothing$. Our main objective is to analyze the effect of increased competition, i.e., an increase in $|J| = m$, on the optimal managerial contracts in the firms that belong to I. Until Section 4.6, where we analyze cross-sectional variation in the number of incumbents, we consider I as a fixed collection of incumbent firms. A typical incumbent firm is denoted by i, and a typical entrant, by j. Often for convenience we will denote a generic firm (incumbent or entrant) by $k \in I \cup J$ with $|I \cup J| = n+m$.

Let q_k denote the production of firm k. The inverse market demand is given by $P = 1 - Q$, where Q denotes the aggregate industry output, and P the market price. Each firm k operates on a constant-returns-to-scale production technology with marginal cost $c_k \in \{0, c\}$ where $0 < c < 1$. Initially, all firms have the inefficient technology, i.e., $c_k = c$ for all k. Each firm hires a manager whose principal task is to exert non-verifiable R&D effort in order to mark down the marginal cost to 0. The probability that the marginal cost is reduced is given by e_k, which is

[5] See Legros and Newman (2014) for an excellent survey of the extant literature.

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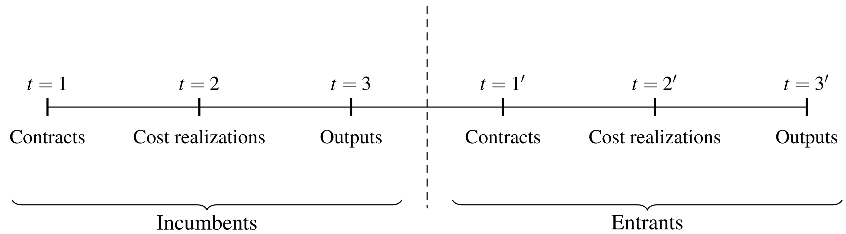

Fig. 1. The timing of events in the sequential quantity-setting oligopoly.

the effort exerted by the manager of firm k. Each firm k offers its manager a take-it-or-leave-it contract $(w_k(0),\ w_k(c))$ which is contingent on the realized marginal cost $c_k \in \{0,\ c\}$. Contracts are subject to limited liability of the managers. Managerial contracts are publicly observable, but the realized marginal costs remain private information of the firms. Every manager has the same effort cost function $\psi(e) = e^2/2$, and her outside option is normalized to 0.

3.2. Timing of events

The timing of events, which is divided into two phases, is described in Fig. 1. At date 1, the incumbents hire a manager apiece by offering publicly observable contingent contracts. At $t = 2$, the manager at each incumbent firm exerts non-verifiable effort, and the marginal cost of each incumbent is privately realized. At $t = 3$, the incumbents simultaneously set quantities. After observing the aggregate quantities set by the incumbents, the entrants repeat the timing at dates $t = 1', 2', 3'$. Finally, after date $t = 3'$, the market price is set, and the profits of all firms (incumbents and entrants) are realized.

3.3. Managerial contract and effort

Each manager k chooses her effort e_k optimally, given the contracts $w_k(0)$ and $w_k(c)$ at firm k. Because the realizations of marginal costs are independent, managerial contracts at each firm k are independent of the realizations of marginal costs at the rival firms. The optimal effort at firm k is given by:

$$\begin{aligned} e_k &= \underset{\hat{e}_k}{\operatorname{argmax}} \left\{ \hat{e}_k w_k(0) + (1-\hat{e}_k) w_k(c) - \frac{1}{2}\hat{e}_k^2 \right\} \\ &= w_k \equiv w_k(0) - w_k(c). \end{aligned} \tag{IC}$$

The above is the *incentive compatibility constraint* of the manager at firm k in which w_k represents the incentive component of the managerial contract. Therefore, we will refer to a higher (lower) value of w_k, or equivalently, of e_k as 'stronger (weaker) managerial incentives'. We assume limited liability (non-negative income for the manager at each state of nature), i.e.,

$$w_k(c) \geq 0, \quad \text{and} \quad w_k(0) \geq 0. \tag{LL}$$

Finally, the expected utility of the manager at each firm k must be at least as high as her outside option 0, i.e., the *participation constraint* of the manager is given by:

$$u_k \equiv e_k w_k(0) + (1-e_k) w_k(c) - \frac{1}{2} e_k^2 \geq 0. \tag{PC}$$

3.4. Quantity competition

We follow Daughety (1990), which is a generalization of the standard notion of Stackelberg competition, to model market competition in the present context. After managers have exerted effort, each incumbent i learns its marginal cost c_i privately. Then, the incumbent firms (the "leaders") choose quantities $(q_1, \ldots, q_n)$ simultaneously to maximize expected profit. After observing the aggregate incumbent quantity, $Q_I \equiv \sum_{i\in I} q_i$, the entrants choose managerial contracts simultaneously, taking Q_I as given. Following the choice of managerial effort, e_j, each entrant firm j learns its marginal cost c_j privately. Finally, the entrant firms (the "followers") choose quantities $(q_1, \ldots, q_m)$ in a Cournot fashion to maximize expected profit. We assume that in equilibrium all m entrants decide to enter, i.e., regardless of their own and the incumbents' cost realizations, each entrant finds it optimal to produce a positive output in equilibrium. This rules out the possibility that the incumbents may deter entry. The incumbents are also assumed to produce a positive output in equilibrium regardless of their realized marginal cost. This implies a restriction of the parameter space—namely, an upper bound on c. This is an innocuous but conservative assumption as the incentives to attain a low marginal cost would have been stronger otherwise. We solve for the equilibrium by backward induction, and show that it is unique, and symmetric for incumbents and entrants.

4. Managerial incentives in sequential oligopoly

4.1. Choice of quantities and managerial efforts by the entrants

Let $Q_J = \sum_{j\in J} q_j$ be the aggregate entrant output, and $q_{-j} \equiv Q_J - q_j = \sum_{k\in J\setminus\{j\}} q_k$, the aggregate output of the rival entrants. Further, let the managerial effort and bonus vectors be denoted by $(e_i,\ e_j)$ and $(w_i,\ w_j)$, respectively for $i \in I$ and $j \in J$. At the quantity setting stage, $t = 3'$, each entrant j takes Q_I and q_{-j} as given to solve

$$\max_{q_j}\ q_j(1 - Q_I - q_j - \mathbb{E}q_{-j} - c_j).$$

The subgame played by the entrants at the quantity setting stage, $t = 3'$, is simply a Cournot game among m firms with a residual demand $P = 1 - Q_I - \sum_{j\in J} q_j$. The quantity of each rival entrant is a random variable because its realized marginal cost is unknown to entrant firm j. The expected cost of firm j is $\mathbb{E}c_j = c(1-e_j)$, where e_j is the incentive compatible level of managerial effort chosen at date $t = 2'$. Because the managerial contracts of all entrant firms are publicly observable, every firm j knows the expected cost of every rival firm. Further, let $e_{-j} \equiv \sum_{k\in J\setminus\{j\}} e_k$. The quantity and expected profit of each entrant firm in the subgame perfect equilibrium are respectively given by:

$$q_j(c_j, e_j, e_{-j}, Q_I) = \frac{2(1-Q_I)-(m+1)c_j+(m-1)c(1+e_j)-2ce_{-j}}{2(m+1)},$$

$$\pi_j(c_j, e_j, e_{-j}, Q_I) = \left\{\frac{2(1-Q_I)-(m+1)c_j+(m-1)c(1+e_j)-2ce_{-j}}{2(m+1)}\right\}^2.$$

Note that $\pi_j(c_j, e_j, e_{-j}, Q_I)$ is the expected market profit of each entrant firm j conditional on its realized cost, c_j. It depends on e_j even when conditioning on c_j because the effort exerted by the manager at firm j pins down the beliefs of the rival entrants about c_j. These beliefs affect the rivals' output decisions in the same way as e_{-j} affects those of firm j, so the effort exerted by the manager at firm j is profitable beyond its cost realization. If the realized marginal costs were publicly observable, the product market profits would not depend on managerial efforts; instead, they would depend on the observed numbers of high- and low-cost firms (cf. Golan et al., 2015), and managerial effort would not be profitable beyond the value of cost reduction.

The optimal contracting problem at $t = 1'$ at each entrant firm j is solved in two stages (e.g. Grossman and Hart, 1983). First, firm j minimizes the expected incentive costs in order to implement a given level of effort subject to the constraints described in Section 3.3, i.e.,

$$C_j(e_j) = \min_{\{w_j(0),\, w_j(c)\}} e_j w_j(0) + (1-e_j)w_j(c), \tag{Min$_j$}$$

subject to (IC), (LL) and (PC).

The value function, called the 'incentive cost function', of the above minimization problem is given by:

$$C_j(e_j) = C(e_j) = e_j^2 \quad \text{for all } j \in J.$$

In the second stage, firm j chooses the effort level e_j in order to maximize the expected profits

$$\Pi_j(e_j, e_{-j}, Q_I) \equiv e_j\pi_j(0, e_j, e_{-j}, Q_I) + (1-e_j)\pi_j(c, e_j, e_{-j}, Q_I)$$

net of its incentive costs $C(e_j)$, i.e.,

$$\max_{e_j} \Pi_j(e_j, e_{-j}, Q_I) - C(e_j). \tag{Max$_j$}$$

Let the equilibrium managerial effort in the entrant firms be denoted by $e_J(Q_I, m)$, which is derived from the first-order condition of the maximization problem (Max$_j$). It is analyzed in the following lemma.

Lemma 1. *Given the aggregate output Q_I of the incumbent firms, the equilibrium managerial effort in the entrant firms is unique, symmetric, and is given by:*

$$e_J(Q_I, m) = \frac{c[8m(1-Q_I)+c(m^2-6m+1)]}{2[4(m+1)^2+c^2(m-1)^2]} \quad \textit{for all } j \in J. \tag{EE}$$

The higher the aggregate output of the incumbents, Q_I, the lower is the managerial effort in each entrant firm. This is because when the aggregate output of the incumbents expands, the entrants face a shrunken residual demand, and hence, it is optimal for each of them to offer weaker incentives to its manager, which elicit lower effort.

4.2. Quantity choice of the incumbents

To set output levels at date $t = 3$, the incumbents solve the following profit maximization:

$$\max_{q_i} \pi_i^q(q_i, Q_J) \equiv q_i(1-q_i-\mathbb{E}q_{-i}-Q_J-c_i). \tag{Max$_q$}$$

In setting quantities, the incumbents take into account the best response of the entrant firms and anticipate their managerial efforts. Let $q_j(c_j, e, Q_I)$ denote the quantity of an entrant firm j in the subgame perfect equilibrium for a common level of effort e (among the entrants), i.e., with $e_j = e$ for all $j \in J$. Then, the expected aggregate output of the entrants is given by:

$$Q_J(Q_I, m) = \sum_{j\in J}\mathbb{E}q_j(c_j, e_J(Q_I, m), Q_I) = \kappa(m)(1-C_\kappa Q_I),$$

where

$$\kappa(m) \equiv \frac{(4+c^2)m(m+1)}{4(m+1)^2+c^2(m-1)^2}, \quad \text{and}$$

$$C_\kappa \equiv 1-\frac{c(8+c^2)}{2(4+c^2)} \in (0,\ 1).$$

It is easily verified that $\kappa'(m) > 0$. Hence, the aggregate best response $Q_J(Q_I, m)$ is linear in the aggregate incumbent quantity Q_I, and it shifts upward as m grows. Importantly, $\partial^2 Q_J(Q_I, m)/\partial m\partial Q_I = -C_\kappa\kappa'(m) < 0$. This means that the incumbent output softens the impact of firm entry on the market price, or, equivalently, that more entrants make incumbent output more effective in deterring entrant output.[6] Therefore, (Max$_q$) takes the following form:

$$\max_{q_i} q_i(1-q_i-\mathbb{E}q_{-i}-Q_J(q_i+\mathbb{E}q_{-i}, m)-c_i)$$
$$\iff \max_{q_i} q_i(A(m)-B(m)(q_i+\mathbb{E}q_{-i})-c_i), \tag{1}$$

where $A(m) \equiv 1-C_\kappa\kappa(m)$ and $B(m) \equiv 1-\kappa(m)$. From the incumbents' perspective, entry of new firms implies two countervailing effects. On the one hand, more firms imply a lower market price, i.e., $A(m) < 1$. However, as the aggregate incumbent output diminishes the optimal effort and output of the entrants, it also implies that the price is less responsive to the incumbents output, i.e., $B(m) < 1$. This gives them more leeway; they can increase output without reducing the equilibrium price too much. For reasons that will become clear below, it is convenient to consider these effects in a different but equivalent way. Note that the solution to (1) is equivalent to the solution of the following 'normalized' problem:

$$\max_{q_i} q_i(a(m)-(q_i+\mathbb{E}q_{-i})-\theta(m)c_i),$$

where

$$a(m) \equiv \frac{A(m)}{B(m)} = \frac{1-C_\kappa\kappa(m)}{1-\kappa(m)},$$

$$\theta(m) \equiv \frac{1}{B(m)} = \frac{1}{1-\kappa(m)} \quad \text{with} \quad a'(m),\ \theta'(m) > 0.$$

That is, from the perspective of each incumbent i, the entry of new firms is equivalent to an increase in the market size, $a(m) > 1$, and the size of cost reduction, $\theta(m)c > c$. This means that, even though entrants reduce the market price, the market size increases from the incumbents perspective as the price is less responsive to their output. Which also equates to a higher size of cost reduction.

We depict the equivalence mentioned above graphically in Fig. 2 by means of the marginal revenue (derived from the residual demand faced by i) and marginal cost curves. The black downward-sloping line is the marginal revenue function derived

[6] This effect is similar to the one in Daughety (1990), except that, in our model, there is an extra strategic device to achieve this product-deterrence effect—namely, increasing the strength of managerial incentives to reduce marginal costs.

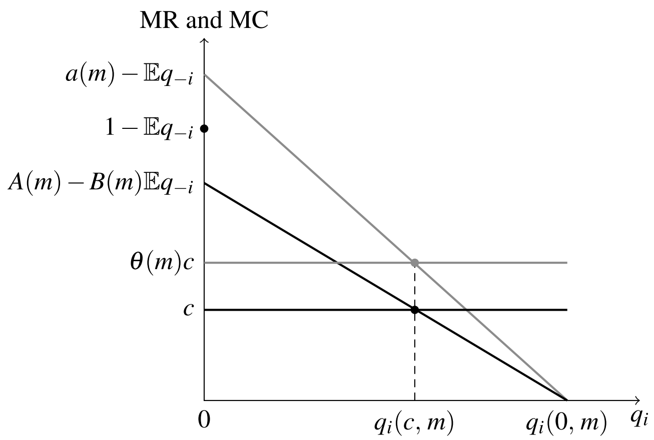

Fig. 2. The optimal output of a representative incumbent firm for a given number of entrants under the actual (black line) and normalized (gray line) marginal revenue and cost functions.

from the residual demand $[A(m) - B(m)\mathbb{E}q_{-i}] - B(m)q_i$ of incumbent i for $m > 0$. This marginal revenue function has a slope equal to $-2B(m)$. The maximum price is represented by the point $A(m) - B(m)\mathbb{E}q_{-i}$, and the market size is represented by $a(m) - \mathbb{E}q_{-i}$. Hence, the horizontal intercept of the marginal revenue function is given by $[a(m) - \mathbb{E}q_{-i}]/2$. If there were no entrants, we would have $A(0) = B(0) = 1$. Following the entry of at least one firm, we have $A(m) < 1$ and $B(m) < 1$. The black horizontal line is the marginal cost of a high-cost incumbent i. The equilibrium quantity $q_i(c, m)$ is determined by the intersection of the marginal revenue and marginal cost of the high-cost incumbent i for a given number of entrants m. The normalized marginal revenue function that is derived from the normalized residual demand $a(m) - \mathbb{E}q_{-i} - q_i$ with $a(m) > 1$, and the normalized marginal cost curve, $\theta(m)c$, are shown by the gray lines. The normalized marginal revenue curve is steeper than the actual marginal revenue curve because it has a slope equal to -2. These two normalized functions intersect at the same equilibrium output level $q_i(c, m)$ of each high-cost incumbent. For each low-cost incumbent, the equilibrium quantity is given by $q_i(0, m) = [a(m) - \mathbb{E}q_{-i}]/2$ because for such a firm i, $c_i = \theta(m)c_i = 0$.

Let $e_{-i} = \sum_{k \in I \setminus \{i\}} e_k$ be the aggregate managerial efforts of the rival incumbents. The equilibrium output and profit of incumbents are described in the following lemma.

Lemma 2. *Given the number of entrants, m, the privately realized marginal costs* $\{c_1, \ldots, c_n\}$*, and the managerial efforts* $\{e_1, \ldots, e_n\}$ *of the incumbent firms, the equilibrium quantity and profit of each incumbent firm are respectively given by:*

$$q_i(c_i, e_i, e_{-i}, m) = \frac{2a(m) - (n+1)\theta(m)c_i + (n-1)\theta(m)c(1+e_i) - 2\theta(m)ce_{-i}}{2(n+1)},$$

$$\pi_i(c_i, e_i, e_{-i}, m) = \frac{1}{\theta(m)} \times \left\{ \frac{2a(m) - (n+1)\theta(m)c_i + (n-1)\theta(m)c(1+e_i) - 2\theta(m)ce_{-i}}{2(n+1)} \right\}^2.$$

Although the equilibrium quantity and profit of each entrant j depend on the aggregate incumbent quantity Q_I, those of each incumbent firm i do not depend on the entrant quantity because the incumbents act as Stackelberg leaders in the product market. But they do depend on the number of entrants via the market size $a(m)$ and the size of cost reduction $\theta(m)c$ for the incumbent firms.

4.3. Equilibrium managerial efforts and incentives in the incumbent firms

In the contracting stage at date 1, each incumbent firm i solves a maximization problem similar to (Max$_j$) (replace j by i everywhere, and drop Q_I from the profit function). Define by $\Delta\pi_i(e_i, e_{-i}, m) \equiv \pi_i(0, e_i, e_{-i}, m) - \pi_i(c, e_i, e_{-i}, m)$ the expected value of cost reduction of each incumbent firm i. The first-order condition for the contracting problem of each incumbent i is given by:

$$\frac{\partial \Pi_i(e_i, e_{-i}, m)}{\partial e_i} \equiv \Delta\pi_i(e_i, e_{-i}, m) + \left[e_i \frac{\partial \pi_i(0, e_i, e_{-i}, m)}{\partial e_i} + (1-e_i)\frac{\partial \pi_i(c, e_i, e_{-i}, m)}{\partial e_i} \right] = 2e_i. \tag{FOC$_i$}$$

At the optimal managerial effort, the marginal benefit of effort is equalized with the marginal incentive cost. The left-hand-side of (FOC$_i$) is the marginal benefit of effort which comprises of two terms—namely, the *expected value of cost reduction*, $\Delta\pi_i(e_i, e_{-i}, m)$, and the *expected marginal profitability of effort*, $\mathbb{E}[\partial\pi_i(c_i, e_i, e_{-i}, m)/\partial e_i]$. On the right-hand-side of the above equation is the marginal incentive cost, $C'(e_i)$. Let the equilibrium managerial effort and incentives of incumbents be denoted by $e_I(m)$ and $w_I(m)$, respectively, which are determined from (IC) and (FOC$_i$). Note also that the manager's utility, i.e., the net level of compensation of the manager in each incumbent firm is given by:

$$u_I(m) \equiv e_I(m)w_I(m) - \frac{1}{2}(e_I(m))^2 = \frac{1}{2}(w_I(m))^2. \tag{2}$$

The following proposition describes the equilibrium managerial effort, incentives, and the level of executive compensation in the incumbent firms.

Proposition 1. *The equilibrium managerial effort and incentives of the incumbent firms are unique, symmetric, and given by:*

$$e_I(m) = w_I(m) = \frac{c[8a(m)n + \theta(m)c(n^2 - 6n + 1)]}{2[4(n+1)^2 + \theta(m)c^2(n-1)^2]} \in (0, 1). \tag{EI}$$

The equilibrium utility accrued to each manager at the incumbent firms is given by $u_I(m)$*, as in* (2)*. Moreover, for fixed* $n \geq 1$ *and* $m \geq 0$*, there exists* $\hat{c} \in (0, 1)$ *such that every firm (incumbent or entrant) produces a positive output in equilibrium regardless of its realized cost, provided that* $c \in (0, \hat{c})$*.*

Note that the first-order condition (FOC$_i$) defines implicitly the best reply in effort at firm i as a function of the aggregate effort at the rival incumbent firms, e_{-i}, which is linear and downward sloping (see proof of Result 1-(a) in Appendix A for more details). Managerial efforts and incentives are strategic substitutes. As a result, the symmetric equilibrium effort $e_I(m)$ is the unique equilibrium outcome. Now, in order to determine the equilibrium managerial effort, we evaluate the first-order condition (FOC$_i$) at a common effort level e. The marginal benefit of effort, denoted by $MB(e, m)$, is strictly decreasing in e as shown by the downward sloping line in Fig. 3. The upward sloping line, labeled $C'(e)$, is the marginal incentive cost as a function of e. The intersection of $MB(e, m)$ and $C'(e)$ yields the unique equilibrium managerial effort $e_I(m)$.

To find the upper bound $\hat{c}$ on the high marginal cost, note that the firm that produces the least in equilibrium is a high-cost entrant in a market in which all incumbents are low-cost. Let $q_i(c_i, e, m)$ denote the equilibrium output of an incumbent firm i at marginal cost c_i and a common effort level e (among the incumbents), which is obtained from Lemma 2. Let $\hat{Q}_I \equiv$

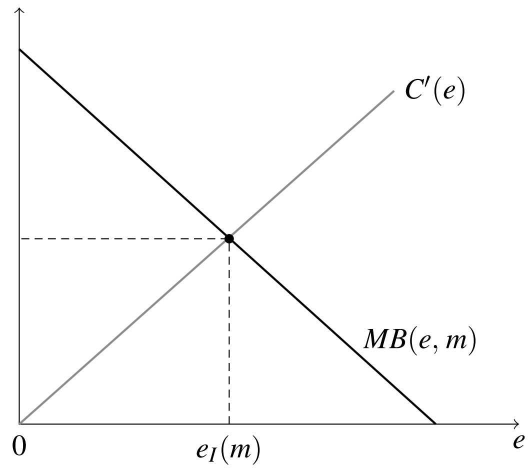

Fig. 3. The equilibrium managerial effort in the incumbent firms.

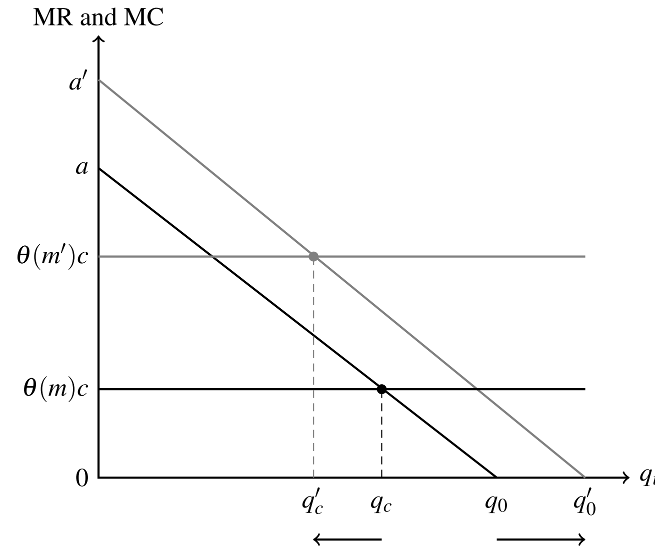

Fig. 4. Effect of an increase in the number of entrants from m to m' on the equilibrium outputs of low- and high-cost incumbents. Let $a \equiv a(m) - \mathbb{E}q_{-i}$, $a' \equiv a(m') - \mathbb{E}q_{-i}$, $q_{c_i} \equiv q_i(c_i, e, m)$ and $q'_{c_i} \equiv q_i(c_i, e, m')$ for $c_i \in \{0, c\}$. Following an increase in m, q_0 increases to q_0', but q_c decreases to q_c'.

$\sum_{i\in I} q_i(0, e_I(m), m)$ be the aggregate incumbents output at equilibrium when all of them are low-cost. The upper bound $\hat{c}$ is implicitly defined by $q_j(c, e_J(Q_I, m), Q_I) = 0$. For more details, see the proof of Proposition 1 in Appendix B.

4.4. Competition and managerial incentives in the incumbent firms

Our objective is to analyze how increased competition due to the entry of new firms into the market affects the provision of managerial incentives at the incumbent firms. The following proposition states our main result.

Proposition 2. *Let $m' > m \geq 0$. Given any number of incumbents $n \geq 1$, entry of new firms induces each incumbent firm to elicit higher managerial effort, i.e., $e_I(m') > e_I(m)$, by providing stronger incentives, i.e., $w_I(m') > w_I(m)$, and higher compensation, i.e., $u_I(m') > u_I(m)$.*

The above proposition implies two sorts of effects of competition on managerial incentives. The first one is an *extensive margin* effect. The equilibrium managerial effort, incentives, and compensations, are lower in the incumbent firms in the absence of any entrant firm. Even the entry of only one firm which sets quantity as a Stackelberg follower induces the incumbents to elicit higher managerial effort by offering stronger incentives and compensation. This is a consequence of the fact that both $e_I(m)$ and $w_I(m)$ are strictly increasing in m. The second is an *intensive margin* effect. As the competitive pressure intensifies, each incumbent firm elicits higher managerial effort and offers stronger incentives and compensations.

The effect of an increase in the number of entrants on the equilibrium output of both low- and high-cost incumbents is shown in Fig. 4. Entry of new firms induces the low-cost incumbents to produce more because both their market size and size of cost reduction increase. As $a(m)$ and $\theta(m)$ are both increasing functions of m, entry benefits the low-cost incumbents implying that $q_i(0, e, m)$ is strictly increasing in m. The same is not obtained for high-cost incumbents. The direction of the change in $q_i(c, e, m)$ following an increase in the number of entrants is *a priori* ambiguous because both $a(m)$ and $\theta(m)$ are increasing in m. From Fig. 4, it is immediate to see that $q_i(c, e, m)$ is decreasing in m if and only if $a'(m) - \theta'(m)c < 0$, which turns out to be the case, i.e., the loss to the high-cost incumbents due to an increase in the size of cost reduction outweighs the gain from an increase in market size.[7]

In order to see why entry of new firms induces the incumbents to elicit higher managerial effort, we analyze how an increased number of entrants affects the expected value of cost reduction and the expected marginal profitability of effort of incumbents, i.e., the two terms in the left-hand-side of (FOC$_i$) evaluated at a common effort level e of the incumbent firms. The expected value of cost reduction is given by:

$$\Delta\pi_i(e, m) = \frac{c\,[4a(m) + \theta(m)(n-3)c - 2\theta(m)(n-1)ce]}{4(n+1)}.$$

Note that if all incumbents increase their effort level, e, the value of the cost reduction diminishes, since it is more likely for all incumbents to lower their marginal cost.[8] Importantly, this effect is amplified if there are more entrants. Hence, it can easily be shown that the 'value-of-cost-reduction effect' is positive, i.e., it is strengthened by the entry of new firms, if the common effort level is sufficiently low:

$$\frac{d\Delta\pi_i(e, m)}{dm} > 0 \iff e < F(c) \equiv \frac{2(1 - C_\kappa)}{(n-1)c} + \frac{n-3}{2(n-1)}.$$

The expected marginal profit of effort is given by:

$$MPE_i(e, m) \equiv \mathbb{E}\left[\frac{\partial\pi_i(c_i, e, m)}{\partial e_i}\right] = \frac{c(n-1)\,[a(m) - \theta(m)c + \theta(m)ce]}{(n+1)^2}.$$

Note that the expected marginal profit of effort, $MPE_i(e, m)$, is increasing in the common effort level, e. That is, if all the firms believe that all of them are more likely to reduce the cost, then it is

7 Note that

$$a'(m) - \theta'(m)c = -\frac{c^3\kappa'(m)}{(4+c^2)[1-\kappa(m)]^2} < 0$$

because $\kappa'(m) > 0$. See Appendix B for details.

8 Note that this is not the case if a single firm increases its managerial effort. In such case, such firm would see its value of cost reduction rise while that of the other firms would diminish.

more profitable to exert effort. Intuitively, this effect is dampened by the number of incumbents n; however, it is amplified by the number of entrants, m. One can easily show that the 'marginal-profitability-of-effort-effect' is positive, i.e., it is strengthened by the entry of new firms, if the common effort level is sufficiently high:

$$\frac{dMPE_i(e,\ m)}{dm} > 0 \quad \Longleftrightarrow \quad e > G(c) \equiv \frac{c^2}{2(4+c^2)}.$$

Overall, a higher number of entrants, m, steepens the expected value of a cost reduction and the marginal profit of effort. Recall that the marginal benefit of effort is the sum of the two effects, i.e.,

$$MB(e,\ m) = \Delta\pi_i(e,\ m) + MPE_i(e,\ m).$$

Therefore, a sufficient condition for the equilibrium effort $e_I(m)$ to be increasing in the number of entrants is for both effects to be positive:

$$G(c) < e_I(m) < F(c) \quad \Longrightarrow \frac{\partial MB_i(e_I(m),\ m)}{\partial m} > 0$$
$$\Longleftrightarrow \quad \frac{de_I(m)}{dm} > 0.$$

One way of proving Proposition 2 is to show that the inequality $G(c) < e_I(m) < F(c)$ holds for every $c \in (0, \hat{c})$, which is, indeed, the case.[9] Nonetheless, showing these inequalities directly is not the most suitable way of proving Proposition 2. The difficulty lies in that all three, $G(c)$, $e_I(m)$, and $F(c)$, are increasing functions of c. Because the upper bound $\hat{c}$ does not have a closed form solution (see Appendix B), it is simpler to verify that the inequality holds numerically, by doing an extensive search in the parameter space.[10] Therefore, the proof of Proposition 2 consists in showing directly that $e_I(m)$ is an increasing function of m. Equivalently, one can show that it is an increasing function of $\kappa(m)$.

4.5. Equilibrium firm value and market profits

In this section, we focus on the effect that firm entry has on the equilibrium "firm value" and market profits of incumbents. The firm value is given by a firm's expected product market profits net of incentive costs. In equilibrium, the expected firm value corresponds to the value function of problem (Max$_j$). Let $V_i(m)$ and $\Pi_i(m)$ denote the expected firm value and expected market profits of incumbent $i \in I$ in equilibrium, respectively. Then,

$$V_i(m) \equiv \Pi_i(m) - C_i(e_i),$$

where $C_i(e_i) = e_i^2$ is the incentive cost function, defined in (Min$_j$), and e_i denotes the equilibrium effort of each incumbent firm i. The expected equilibrium profits are given by:

$$\Pi_i(m) = e_I(m)\pi_i(0, e_I(m), m) + (1 - e_I(m))\pi_i(c, e_I(m), m),$$

where $\pi_i(c_i, e_I(m), m)$ denotes the incumbent expected market profits, conditional on having realized cost c_i, on the equilibrium path (see Lemma 2). Recognizing that the firm value is the value function of a maximization problem, which depends itself on other value functions (the equilibrium profit functions) yields the following result.

Proposition 3. *The equilibrium firm value of the incumbent firms is decreasing in the number of entrants.*

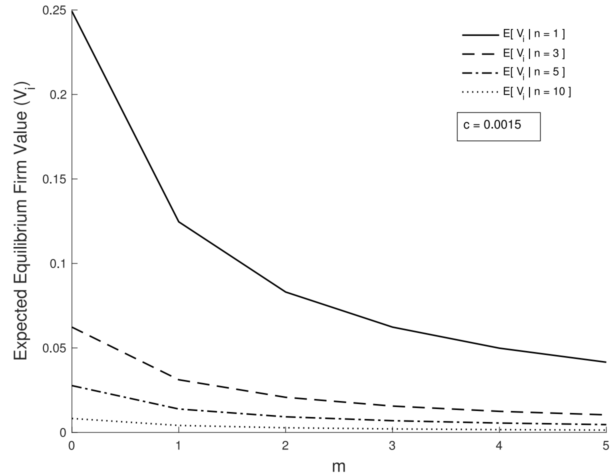

Fig. 5. Equilibrium expected incumbent firm value.

The intuition behind Proposition 3 is straightforward. The objective function in (Max$_j$) depends on m only through the equilibrium expected profit functions at realized marginal costs 0 and c, $\pi_i(0, e_I(m), m)$ and $\pi_i(c, e_I(m), m)$, respectively. Hence, it follows from the Envelope theorem that, if both of the expected profit functions are decreasing in m, then $V_i(m)$ is also decreasing in m. This is indeed the case because the objective function of the profit maximization problem, $\pi_i^q(q_i, Q_J)$ in (Max$_q$), only depends on m through Q_J. Since $Q_J = Q_J(Q_I,\ m)$ is strictly increasing in m, due to $\kappa'(m) > 0$, by the Envelope theorem, $\partial\pi_i^q(q_i,\ Q_J)/\partial Q_J < 0$ implies $\partial\pi_i(c_i,\ e_I(m),\ m)/\partial m < 0$ for $c_i \in \{0, c\}$.[11] Fig. 5 shows graphically the result in Proposition 3.

4.6. Cross-sectional variation in the number of incumbents

Until now, we have maintained the number of incumbents fixed. To emphasize the significance of Proposition 2, we analyze how the equilibrium managerial effort varies with the number of incumbents, n, for a fixed number of entrants. This corresponds to comparing the managerial contracts offered at firms in two markets that face the same amount of competitive pressure (same number of entrants), but one of which is initially more competitive than the other (has more incumbents). Let $e_I(n,\ m) \equiv e_I(m)$, and $w_I(n,\ m) \equiv w_I(m)$, as defined in (EI), and $u_I(n,\ m) \equiv u_I(m)$, as defined in (2). (In this section we take the liberty of using notation defined previously, but change m for n to highlight the comparative statics in n.)

Proposition 4. *Let $n' > n \geq 1$. Given any fixed number of entrants $m \geq 0$, incumbents in more competitive markets, i.e., in ones with more incumbents, elicit lower managerial effort, i.e., $e_I(n',\ m) < e_I(n,\ m)$, by providing weaker incentives, i.e., $w_I(n',\ m) < w_I(n,\ m)$, and lower compensation, i.e., $u_I(n',\ m) < u_I(n,\ m)$.*

The marginal benefit of effort, the left-hand-side of (FOC$_i$), differs in two ways in markets that are initially more competitive. The first one is through the standard 'output channel'. More incumbents means greater aggregate production by rivals, which implies that each firm optimally reduces its output at any realization of marginal cost as quantities are strategic substitutes. The expected value of cost reduction is lower as the effect of the number of incumbents on the optimal output level does not

[9] It is easy to check that $F(c) > G(c)$ if and only if $n + 1 > 0$.

[10] Despite the fact that we prove this claim numerically, the proof of Proposition 2 is fully analytical. For more details on this and the other claims below that we show numerically, see Appendix C.

[11] We owe this observation to an anonymous referee.

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depend on the realized cost. That is, because $\partial q_i(0, e, n)/\partial n = \partial q_i(c, e, n)/\partial n < 0$, one obtains

$$\frac{d\Delta\pi_i(e, n)}{dn} = \frac{2}{\theta(m)}[q_i(0, e, n) - q_i(c, e, n)] \cdot \frac{\partial q_i(c_i, e, n)}{\partial n} < 0.$$

Notably, this 'value-of-cost-reduction effect' would work under the same logic if the realizations of marginal costs would have been public knowledge.

Due to the presence of privately realized marginal costs, a higher number of incumbents also changes the marginal benefit of managerial effort through the 'marginal-profitability-of-effort' effect. By eliciting a higher managerial effort, each incumbent i induces its rivals to believe that it has attained a low marginal cost, and hence, the aggregate rival quantity is lower in expectation. This raises the expected market price, and hence, the expected profits of firm i, i.e., $\partial\pi_i(c_i, e, n)/\partial e_i > 0$ at any realization of marginal cost. The marginal profitability of effort is greater if rivals believe that a given firm has attained cost reduction. In a more concentrated market (less firms) it is easier to influence rivals by affecting their beliefs, and hence, the marginal profitability of effort is increasing in the number of incumbents. In a market with many firms, by contrast, it is harder that more rivals are so influenced (as there are more firms). Thus, the marginal profitability of effort is decreasing in the number of incumbents in competitive markets. Formally,

$$\frac{d\mathbb{E}[\partial\pi_i(c_i, e, n)/\partial e_i]}{dn} = -\frac{c(n-3)[a(m) - \theta(m)c(1-e)]}{(n+1)^3},$$

which is strictly positive (negative) for $n < (>) 3$ as $c < \hat{c} < a(m)/\theta(m)$. (To see why this last inequality holds, note that the equilibrium output of a high-cost incumbent would be negative otherwise. See the proof of Proposition 1 in Appendix B for more details.). Thus, the effect of an increase in n on the marginal profitability of effort may be positive or negative depending on the number of incumbents. Nonetheless, in either case, the aggregate effect of a higher number of incumbents on the marginal benefit of effort turns out to be always negative, i.e., $MB(e, n)$ in Fig. 3 shifts down as n increases with $C'(e)$ remaining unaltered, and hence, $e_I(n, m)$ is decreasing in n. To see this formally, it suffices to note that

$$\begin{aligned}\frac{d\,MB(e, n)}{dn} &= \frac{d\Delta\pi_i(e, n)}{dn} + \frac{d\mathbb{E}[\partial\pi_i(c_i, e, n)/\partial e_i]}{dn} \\ &= -\frac{2c(n-1)[a(m) - \theta(m)c(1-e)]}{(n+1)^3} < 0.\end{aligned}$$

The crucial difference between varying the number of entrants and incumbents, is that the entry of new firms affects the incumbents' output decision by altering the effective market size and the size of cost reduction. If there are more incumbents to start with, this alters directly the number of firms incumbents are competing against, and leaves the market size and the size of cost reduction unaffected. The juxtaposition of Propositions 2 and 4 conveys the main message of our paper. The fact that incumbents find it optimal to elicit higher managerial effort by offering steeper incentive contracts when they foresee the entry of new firms to the market, is due to incumbents being able to affect the entrants' output decisions by committing to an output level before they start producing.

4.7. Managerial incentives in simultaneous oligopoly

The objective of this section is to analyze the effect of entry on managerial efforts and incentives in the incumbent firms when the m entrant firms are allowed set quantities simultaneously along with the n incumbents. The simultaneous setting is nothing but a Cournot market with $n + m$ symmetric firms and privately realized marginal costs $(c_1, \ldots, c_n, c_1, \ldots, c_m)$. The equilibrium managerial effort in each firm (incumbent or entrant) can be obtained directly from the expression (EI) as follows. As the entrants are treated equally as the incumbents, remove the entrants by setting $m = 0$, and replace the number of incumbents, n, by $n+m$. In this case, $a(m) = \theta(m) = 1$.

Let the symmetric equilibrium managerial effort and incentives in each firm (incumbent or entrant) be denoted by $e^{sim}(n + m)$ and $w^{sim}(n + m)$, respectively, and note that a manager's equilibrium utility is given by:

$$\begin{aligned}u^{sim}(n+m) &\equiv e^{sim}(n+m)w^{sim}(n+m) - \frac{1}{2}(e^{sim}(n+m))^2 \\ &= \frac{1}{2}(w^{sim}(n+m))^2.\end{aligned}$$

The effect of an entrant on an incumbent's optimal managerial effort and contract in this setting is analogous to considering a market that has one more incumbent (in this setting, entrants and incumbents are symmetric). Hence, we obtain the following corollary directly from Proposition 4.

Corollary 1. *Let $m' > m \geq 0$. In a simultaneous quantity-setting oligopoly in which m entrants set quantities and managerial contracts along $n \geq 1$ incumbents, entry of new firms implies that each incumbent elicits lower managerial effort, i.e., $e^{sim}(n + m') < e^{sim}(n + m)$, by providing weaker incentives to its manager, i.e., $w^{sim}(n + m') < w^{sim}(n + m)$, and lower compensation, i.e., $u^{sim}(n + m') < u^{sim}(n + m)$.*

The result in Corollary 1 is not new in the literature (see Martin, 1993; Hermalin, 1994; Golan et al., 2015). The intuition behind it goes in the same line as the one underlying Proposition 4. The entrants affect the marginal benefit of effort of the incumbents through the 'value-of-cost-reduction' and 'marginal-profitability-of-effort' effects. As noted in Section 4.6 above, in this case, entry implies a lower expected value of cost reduction for the incumbents, and also a lower expected marginal profit of effort as long as the market is already sufficiently competitive or, equivalently, the number of entrants is sufficiently high, i.e., as long as $n + m > 3$. Notably, as highlighted in the extant literature, the result in Corollary 1 does not depend on marginal costs being privately realized. On the contrary, we show that the negative effect of competition on managerial incentives in this setting is reinforced with privately realized marginal costs if the market is sufficiently competitive.

5. Testable implications

5.1. Nature of industry competition and time to build production capacity

A key insight of our stylized model is the juxtaposition of Proposition 2 with Corollary 1. If entrants set quantities as Stackelberg followers, incumbent firms offer stronger managerial incentives as the number of entrants grows, whereas the opposite is obtained if they set quantities simultaneously, along with the incumbents.

Allen, Deneckere, Faith, and Kovenock (2000) examine the role of capacity pre-commitment as an instrument to deter production in a Bertrand–Edgeworth model of price competition. In particular, they analyze a three-stage game where an incumbent firm chooses its capacity first. Having observed the incumbent's capacity level, the entrant then chooses capacity. Finally at stage 3, the firms simultaneously set prices. The authors show that the outcome of this game coincides with that of a Stackelberg quantity competition. The crux of Allen et al.'s (2000) analysis is that production capacity cannot be adjusted instantaneously in

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the post-entry game, i.e., for a potential entrant, capacity requires *time to build*. This is in contrast with the Bertrand–Edgeworth model analyzed by Kreps and Scheinkman (1983), in which firms are able to adjust capacity instantaneously prior to engaging in simultaneous price competition. In this sense, the outcome of Kreps and Scheinkman (1983), which coincides with that of Cournot competition, corresponds to industries in which capacity requires *no* time build. This disparity in the time required to build production capacity leads to the following implication.

Implication 1. *(i) In an industry where production capacity requires 'time to build', the incumbent firms offer higher managerial compensation and stronger incentive pay following an increase in the market competition induced by the entry of new firms. (ii) By contrast, if the production capacity can be adjusted instantaneously, entry of new firms implies that incumbents would provide lower compensation and weaker incentives to their managers following entry.*

The Stackelberg outcome is more plausible in an industry in which sequential capacity choices are followed by simultaneous price competition. In such industries, such as the airline or banking industries, the sluggishness of capacity adjustment gives rise to an output-deterrence effect due to capacity pre-commitment. By contrast, in industries in which production capacities can be built almost instantaneously, such as services and technology, building capacity does not have a pre-commitment value. Our results imply that this industry-specific feature is key when analyzing the effect of market product competition on executive compensation.

It is worth emphasizing that Implication 1 applies both at the *extensive* and the *intensive margins*. When entrants set quantity as Stackelberg followers, (i) firm entry increases incumbents managerial effort, i.e., at the extensive margin, and (ii) a higher number of entrants increases each incumbent's managerial effort by a larger magnitude, i.e., at the intensive margin. Fig. 6 depicts the juxtaposition of Proposition 2 with Corollary 1. From (EI) it follows that $e_I(n, 0) = e^{sim}(n + 0)$. In the absence of any entrant firm ($m = 0$), the equilibrium efforts coincide because it makes no difference whether entrants set managerial contracts and quantities after or along with the incumbents. Because $e_I(n, m)$ is strictly increasing in m, and $e^{sim}(n + m)$ is strictly decreasing in m, the equilibrium managerial incentives are not only higher when time is required to build capacity, but also their differences magnify as the number of entrants grows. Therefore, even a monopolist incumbent ($n = 1$) would respond more aggressively to an increase in the threat of competition under time-to-build-capacity, whereas she would provide weaker managerial incentives if the time to build capacity were negligible.

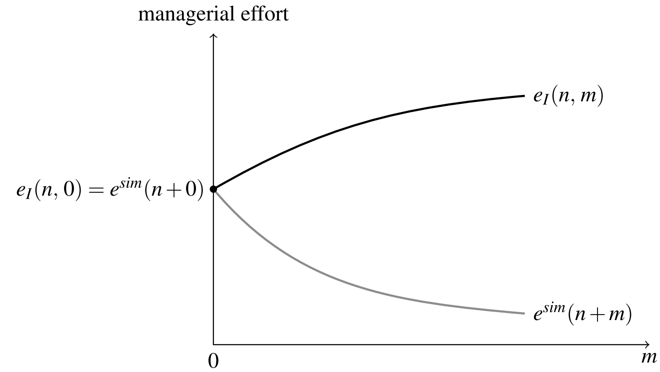

Fig. 6. Equilibrium managerial effort as a function of the number of entrants m under simultaneous and sequential quantity-setting oligopolies for a given number of incumbents n.

5.2. Equilibrium social welfare

In this section, we focus on the welfare analysis. We simplify the analysis by analyzing the equilibrium welfare numerically. We use a granular grid of the model's parameters to validate Implication 2 below (see Appendix C). The total welfare in the industry consists of three components: (i) consumer surplus (CS), (ii) total producer surplus of incumbents (PS_I), and (iii) total producer surplus of entrants (PS_J). We sketch how to compute each of these components in turn. We provide full details in Appendix D.

The consumer surplus is directly obtained from the demand function. Conditional on the total industry output, $Q = Q_I + Q_J$, the consumer surplus can be readily computed as $CS = Q^2/2$. Hence, the expected consumer surplus is given by $\mathbb{E}CS = 0.5 * \mathbb{E}Q^2$. To compute the producer surplus of the incumbents and the entrants, define the producer surplus of a generic firm $k \in K$, as the sum of its firm value and the utility of its manager. Since managerial wages are simply a transfer between a firm and its manager, the producer surplus of a firm is equal to its market profits net of its manager's effort cost. That is, the expected producer surplus of firm $k \in K$, denoted by PS_k is given by:

$$PS_k(m) = \Pi_k(m) - \psi(e_k),$$

where $\psi(e) = e^2/2$ is the managerial effort cost function, and e_k denotes the equilibrium effort of firm $k \in K$. Note that the producer surplus differs from the firm value in that, instead of accounting for the *incentive* costs of effort provision, it accounts for the effort costs from an *efficiency* point of view. The total producer surplus of incumbents is given by $PS_I = \sum_{i \in I} PS_i(m)$, and the total producer surplus of entrants is given by $PS_J = \sum_{j \in J} PS_j(m)$. The total welfare of the industry is defined by:

$$W = CS + PS_I + PS_J.$$

As a measure of social welfare, we use the expected total welfare at equilibrium, $\mathbb{E}W$. As in the previous subsections, define the analog welfare measures for the case in which entrants produce along the incumbents simultaneously, by W^{sim}, CS^{sim}, PS_I^{sim}, and PS_J^{sim}.

Implication 2. *Entry of new firms affects the consumer surplus, the producer surplus and the social welfare in the following ways.*

(i) *The expected consumer surplus both under sequential (Stackelberg) and simultaneous (Cournot) competition, $\mathbb{E}\,CS$ and $\mathbb{E}\,CS^{sim}$ are increasing in the number of entrants. Moreover, the expected consumer surplus under sequential competition is higher than that under simultaneous competition, i.e., $\mathbb{E}\,CS > \mathbb{E}\,CS^{sim}$ for every $m \geq 1$ and $n \geq 1$;*

(ii) *The aggregate expected producer surplus both under sequential and simultaneous competition, $\mathbb{E}(PS_I + PS_J)$ and $\mathbb{E}(PS_I^{sim} + PS_J^{sim})$ are decreasing in the number of entrants. Moreover, the aggregate expected producer surplus under sequential competition is lower than that under simultaneous competition, i.e., $\mathbb{E}(PS_I + PS_J) < \mathbb{E}(PS_I^{sim} + PS_J^{sim})$ for every $m \geq 1$ and $n \geq 1$;*

(iii) *The expected social welfare both under sequential and simultaneous competition, $\mathbb{E}\,W$ and $\mathbb{E}\,W^{sim}$ are increasing in the number of entrants. Moreover, the expected social welfare under sequential competition is higher than that under simultaneous competition, i.e., $\mathbb{E}\,W > \mathbb{E}\,W^{sim}$ for every $m \geq 1$ and $n \geq 1$.*

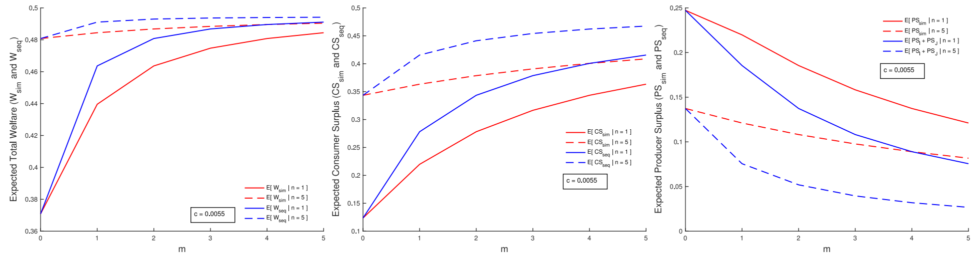

Fig. 7. Social welfare (left), consumer surplus (center), and producer surplus (right) in the simultaneous and sequential oligopolies.

According to Implication 2, regardless of the nature of competition, i.e., whether entrants produce after or along with the incumbents, social welfare increases with firm entry. Moreover, for any combination of parameter values, it also obtains that social welfare is higher when the incumbents are able to set quantities before the entrants. This can be seen in Fig. 7 (left panel), which shows how social welfare is higher under sequential competition (blue curves) than under simultaneous competition (red curves). In the central and right panels of Fig. 7, we plot the consumer surplus and the aggregate producer surplus. Notably, while the consumer surplus is always increasing in the number of entrants, the producer surplus is decreasing. Qualitatively, consumer surplus behaves in a similar fashion as social welfare with respect to entry. However, the producer surplus is higher in the simultaneous case than with sequential competition. All of these findings point to the conclusion that the effect of increased competition via firm entry is fierce when incumbents behave as Stackelberg leaders. It is worth noting that the above finding that social welfare is higher under Stackelberg quantity competition is similar to what Daughety (1990), and Wang and Wang (2009) find, except that in our case the production technology is endogenized through managerial incentive contracts.

6. Extensions

6.1. Effect of hierarchical entry on managerial incentives

Entry of firms seldom takes place simultaneously. The Airline Deregulation Act of 1978 stipulated a transition period of three years over which several small carriers entered the U.S. airline industry sequentially. Even in the absence of entry barriers some firms are quicker than others to learn about market conditions. Prescott and Visscher (1977) argue that "some entrants become aware of a profitable market before others or require longer periods of time in which to *tool up* [our italics]". In what follows, we analyze an entry game where firms enter sequentially. In particular, following the quantity choice of the incumbents, firms enter in a hierarchical fashion (as in Boyer and Moreaux, 1986). For simplicity, we consider only two entrants, i.e., $J = \{1, 2\}$. There are two consecutive periods of entry, entrant 1 enters in period 1, and entrant 2 enters the market in period 2. We show that hierarchical competition reinforces the effect of entry on managerial effort relative to the case when the post-entry quantity competition is simultaneous.

Note that when the two entrants compete simultaneously by setting quantities, the symmetric equilibrium managerial effort of the incumbent firms are given by $e_I(2)$ which is obtained by substituting $m = 2$ in the expression (EI) in Proposition 1. Denote by $e_I^h(2)$ the symmetric equilibrium effort elicited by the incumbents under hierarchical entry of firms 1 and 2.

Proposition 5. *Incumbents elicit greater managerial effort under hierarchical entry than that under simultaneous entry, i.e.,* $e_I^h(2) \geq e_I(2)$ *for any number of incumbents,* $n \geq 1$.

Recall that the key determining factors of managerial efforts are the sizes of the market and cost reduction for the incumbent firms. The key to proving Proposition 5 is that, under hierarchical entry, both the market size and the size of the cost reduction for incumbents are higher relative to simultaneous entry. Hierarchical entry implies more intense competition because each predecessor produces more aggressively in order to deter the production of subsequent entrants. As a result, the incumbents, being the first movers, provide stronger incentives (relative to the case of simultaneous entry) in order to reduce managerial slack.

6.2. Effect of price competition on managerial incentives

In this section, we analyze the effect of price competition on managerial incentives. Consider the setting described in Section 3 with the only difference that firms set prices instead of quantities. Competition is à la Bertrand, i.e., all firms produce a single homogeneous good, and there is no capacity constraint. The timing of the game is analogous to the one described in Fig. 1. Incumbents post prices at date $t = 3$, which the entrants observe, and then entrants set prices at date $t' = 3$.

Let p_k be the price set by firm $k \in I \cup J$, and let $P_I = \min\{p_i : i \in I\}$ be the lowest price among those set by the incumbents. Consider the sub-games played by the entrants and the incumbents once their respective marginal costs have been privately realized. Standard arguments show that these sub-games do not have equilibria in pure strategies.[12] We analyze symmetric equilibria with atom-less mixed strategies in the price-setting stages. Also, to simplify the analysis, we restrict attention to the symmetric equilibrium in the choice of managerial effort.

Proposition 6. *Under price competition,*

(a) *the symmetric equilibrium managerial effort of the entrants is given by* $e_J^B(m, P_I)$ *which solves*

$$\frac{e_J}{(1-e_J)^{m-1}} = \frac{P_I(1-P_I)}{2}. \tag{3}$$

12 First, note that the only case in which an entrant can obtain positive profits is if $P_I > 0$. Also, see that $P_I \leq c$ in equilibrium (since incumbents would obtain negative profits otherwise). To note that there is no equilibrium in pure strategies, see that (i) setting $p_j > 0$ is not an equilibrium since any other entrant could undercut this price and obtain all the demand, and (ii) setting $p_j = 0$ is also not an equilibrium since there exists the possibility that j is the only entrant with a low cost, in which case it would be profitable to increase the price marginally. Analogous arguments apply to the sub-game in which incumbents set prices.

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(b) *Similarly, the symmetric equilibrium effort elicited by the incumbents is given by* $e_I^B(m)$ *which solves*

$$\frac{e_I}{(1-e_I)^{n-1}} = \frac{c(1-c)\left(1-e_J^B(m,\,c)\right)^m}{2}. \tag{4}$$

(c) *Both* $e_J^B(m,\,P_I)$ *and* $e_I^B(m)$*, are decreasing in the number of entrants m, i.e.,* $e_J^B(m',\,P_I) < e_J^B(m,\,P_I)$ *and* $e_I^B(m') < e_I^B(m)$ *for* $m' > m \geq 0$.

Proposition 6 establishes that the equilibrium effort of incumbents is decreasing in the number of entrants m in a price-setting environment. The intuition behind this result lies in the expected equilibrium profits of a low-cost firm who sets its price according to an equilibrium in mixed strategies. Two observations that follow from the above proposition are worth noting.

- First, the expected profits in a Bertrand game with privately realized costs are the same as in the game with publicly observed ones.[13] In our case, this implies that the expected profits of a low-cost entrant are given by:

  $$\pi_j(0,\,e_{-j},\,P_I) = P_I(1-P_I)(1-e)^{m-1}, \tag{5}$$

  where $e_k = e$ for every $k \in J \setminus \{j\}$. Note that the expected profits in (5) correspond to the case with publicly observable costs. In this case, entrant j obtains non-negative profits if and only if it is the only entrant that attains cost reduction by setting price equal to P_I and serving the entire market demand (we assume that entrants have priority over incumbents if they set the same price since they can always undercut any positive price set by an incumbent marginally). From (5), one can easily see why the equilibrium effort of the entrants is decreasing in m. The likelihood of being the only entrant who attains cost reduction is decreasing in the number of entrants, which diminishes the marginal profitability of managerial effort.
- Second, the equilibrium managerial effort elicited by the incumbent firms is decreasing in the number of firms when both incumbents and entrants set prices simultaneously. From (4), it is immediate to see that the left-hand-side is strictly increasing in both e_I and n, whereas the right-hand-side is constant with respect to n for a given value of m. Thus, as n increases, the left-hand-side of (4) shifts up, which implies that e_I^B decreases with n.

The expected profits of a low-cost incumbent are given by:

$$\pi_i(0,\,e_{-i}) = c(1-c)(1-e_J^B(m,\,c))^m(1-e)^{n-1}, \tag{6}$$

where $e_k = e$ for every $k \in I \setminus \{i\}$. The expected profit of a low-cost incumbent is also the same as that with known marginal costs. In this case, an incumbent obtains non-negative profits if and only if it is the only firm in the industry which succeeds in attaining cost reduction. Therefore, the only channel through which the number of entrants affects the incumbents expected profits is the probability that all entrants fail in reducing marginal cost, which is given by $(1-e_J^B(m,\,c))^m$. One can easily see from (3) that this probability is decreasing in m. Even though each entrant is more likely to fail to attain the cost reduction individually as m increases, there are more of them, so the probability that all of them fail to attain it decreases in m. This, in turn, drives the profits of every incumbent to zero, which is sufficient to counteract the fact that the entrants themselves provide less effort when there are more of them. Therefore, in a price-setting environment, fierce competition among the entrants themselves causes them to offer weaker managerial incentives, and in turn makes it profitable for the incumbents to weaken managerial incentives.

[13] For a formal proof of this statement, see the proof of Proposition 6.

7. Conclusion

Motivated by empirical evidence, in this paper we investigate how firms adjust executive compensation packages following deregulation policies that intensify product market competition by allowing the entry of new firms. Using a standard incentive contracting model under quantity-setting oligopoly, we show that incumbent firms find it optimal to elicit higher managerial effort by offering stronger incentive contracts when they foresee entry of new firms into the product market. Our model allows us to tease out in detail the channels through which product market competition affects managerial incentives in a setting with firm entry. In our model, the key features that link the number of entrants with an incumbent's contracting problem are the market size and the size of cost reduction, both of which affect the marginal benefit of effort, through the expected value of cost reduction, and the expected marginal profitability of effort. By showing that firm entry increases both the market size and the size of cost reduction for incumbents, and analyzing, in turn, how these two affect the expected value of cost reduction, and the expected marginal profitability of effort, we show that incumbents find it optimal to offer stronger managerial incentives when new firms enter the market. Furthermore, we also show that the magnitude in which incumbents strengthen managerial incentives is increasing in the number of entrants—a greater competitive pressure triggers a starker reaction by the incumbents.

Beyond conforming to the empirical regularities, our model also sheds light on how the nature of competition in product market affects managerial incentives. Namely, we explore the connection between the time to build production capacity in an industry and the effect that product market competition has on managerial incentives. We find that firm entry increases the pay-performance sensitivity of managerial contracts in markets in which production capacity takes time to build. In other words, the key driver of our result is that entrants act as Stackelberg followers in the product market by taking the aggregate output of incumbents as given. In the opposite case in which production capacity may be obtained instantaneously, i.e., entrants are symmetric to incumbents and set contracts and output simultaneously along with them, the association is negative—incumbents find it optimal to offer weaker managerial incentives as more firms enter the market.

Appendix A. The base model

The subgames played by the entrants and the incumbents have the same underlying structure. In this section, we analyze a more general version of the simultaneous quantity competition model with a fixed number N of firms, called the *base model*, which yields most of the results in Section 4. Let K be the set of firms, with $|K| = N \geq 1$, and index firms with k. Let $P = A - BQ$ be the inverse market demand with $A,\,B > 0$. The marginal cost of a representative firm k is given by c_k, with $c_k \in \{0,\,c\}$ with $c \in (0,\,\bar{c})$. The upper bound $\bar{c}$ is such that all firms produce a positive output in equilibrium regardless of their realized marginal costs. We will prove that such bound exists. In what follows we describe the main results of the base model.

Result 1. *Consider the base model.*

(a) *The equilibrium effort is symmetric and unique across all firms. It is given by:*

$$e(N) = \frac{c[8AN + c(N^2 - 6N + 1)]}{2[4B(N+1)^2 + c^2(N-1)^2]}. \tag{EC}$$

Moreover, the second order condition associated with the individual firm maximization problem in a Bayesian Cournot equilibrium is satisfied for every firm if all of them produce a positive quantity in equilibrium.

(b) *If A is independent of c, then there is $\bar{c} \in (0, A)$ such that, if $c \in (0, \bar{c})$, every firm produces a strictly positive quantity of output and elicits strictly positive level of managerial effort in a symmetric equilibrium, regardless of its realized marginal cost.*

(c) *If $c \in (0, \bar{c})$, the equilibrium effort in (EC) is decreasing in the number of firms, i.e., for every $N \geq 1$, we have $e'(N) < 0$.*

Proof. Let q_k denote the production of any firm $k \in I \cup J$.

(a) Once all the contracts are observed and marginal costs are privately realized, each firm k solves

$$\max_{q_k} \; q_k[A - B(q_k + \mathbb{E}q_{-k}) - c_k].$$

The first-order condition of the above maximization problem is given by:

$$\begin{aligned} & A - 2Bq_k - B\mathbb{E}q_{-k} - c_k = 0 \\ \Longleftrightarrow \; & 2Bq_k = A - c_k - B\mathbb{E}q_{-k} \\ \Longleftrightarrow \; & q_k(c_k, \mathbb{E}q_{-k}) = \frac{A - c_k}{2B} - \frac{1}{2}\mathbb{E}q_{-k}. \end{aligned} \tag{7}$$

Taking expectation in the above equation, we get

$$\mathbb{E}q_k = \frac{A - \mathbb{E}c_k}{2B} - \frac{1}{2}\mathbb{E}q_{-k} = \frac{A - c(1 - e_k)}{2B} - \frac{1}{2}\mathbb{E}q_{-k}. \tag{8}$$

Summing the above over k we get

$$\begin{aligned} & \sum_{k=1}^{N}\mathbb{E}q_k = \frac{N(A - c)}{2B} + \frac{c}{2B}\sum_{k=1}^{N} e_k - \frac{N-1}{2}\sum_{k=1}^{N}\mathbb{E}q_k \\ \Longleftrightarrow \; & \sum_{k=1}^{N}\mathbb{E}q_k = \frac{1}{B(N+1)}\left[N(A - c) + c\sum_{k=1}^{N} e_k\right]. \end{aligned} \tag{9}$$

On the other hand, (8) can be written as

$$\begin{aligned} & \mathbb{E}q_k = \frac{A - c(1 - e_k)}{2B} - \frac{1}{2}\left(\sum_{l=1}^{N}\mathbb{E}q_l - \mathbb{E}q_k\right) \\ \Longleftrightarrow \; & \frac{1}{2}\mathbb{E}q_k = \frac{A - c(1 - e_k)}{2B} - \frac{1}{2B(N+1)}\cdot\left[N(A - c) + c\sum_{k=1}^{N} e_k\right] \\ \Longleftrightarrow \; & \mathbb{E}q_k = \frac{A - c + c(Ne_k - e_{-k})}{B(N+1)}, \end{aligned} \tag{10}$$

where $e_{-k} = \sum_{l \in K \setminus \{k\}} e_l$. Thus, using the fact that $\mathbb{E}q_{-k} = \sum_{l=1}^{N}\mathbb{E}q_l - \mathbb{E}q_k$, and substituting for $\sum_{l=1}^{N}\mathbb{E}q_l$ and $\mathbb{E}q_k$ from (9) and (10), from (7) we obtain the quantity and profit of each firm in the Bayesian Cournot equilibrium, which are respectively given by:

$$q_k(c_k, e_k, e_{-k}) = \frac{2A - (N+1)c_k + (N-1)c(1 + e_k) - 2ce_{-k}}{2B(N+1)} \tag{11}$$

$$\pi_k(c_k, e_k, e_{-k}) = B\left(\frac{2A - (N+1)c_k + (N-1)c(1 + e_k) - 2ce_{-k}}{2B(N+1)}\right)^2. \tag{12}$$

At date 1, each firm k chooses the optimal managerial incentives to solve

$$\max_{e_k} \; e_k\pi_k(0, e_k, e_{-k}) + (1 - e_k)\pi_k(c, e_k, e_{-k}) - e_k^2. \tag{13}$$

The expected value of cost reduction, $\Delta\pi_k(e_k, e_{-k}) := \pi_k(0, e_k, e_{-k}) - \pi_k(c, e_k, e_{-k})$ of firm k is given by:

$$\Delta\pi_k(e_k, e_{-k}) = \frac{c[4A + (N-3)c + 2(N-1)ce_k - 4ce_{-k}]}{4B(N+1)}. \tag{14}$$

Also, note that

$$\begin{aligned} & e_k \cdot \frac{\partial\pi_k(0, e_k, e_{-k})}{\partial e_k} + (1 - e_k)\cdot\frac{\partial\pi_k(c, e_k, e_{-k})}{\partial e_k} \\ & = \frac{(N-1)c[A - c + c(Ne_k - e_{-k})]}{B(N+1)^2}. \end{aligned} \tag{15}$$

Using the expressions (14) and (15), the first-order condition of the maximization problem in (13) is given by:

$$\begin{aligned} & \Delta\pi_k(e_k, e_{-k}) + e_k\cdot\frac{\partial\pi_k(0, e_k, e_{-k})}{\partial e_k} \\ & \quad + (1 - e_k)\cdot\frac{\partial\pi_k(c, e_k, e_{-k})}{\partial e_k} = 2e_k \\ \Longleftrightarrow \; & \frac{c[8AN + (N^2 - 6N + 1)c + 2(N-1)(3N+1)ce_k - 8Nce_{-k}]}{4B(N+1)^2} \\ & = 2e_k. \end{aligned} \tag{FOC$'_k$}$$

Condition (FOC$'_k$) defines the best response (in effort) $e_k(e_{-k})$ of the manager at firm k, which is given by:

$$\begin{aligned} e_k(e_{-k}) = & \underbrace{\frac{c[8AN + c(N^2 - 6N + 1)]}{2[4B(N+1)^2 - c^2(N-1)(3N+1)]}}_{\alpha \equiv \alpha(N, A, B)} \\ & - \underbrace{\left(\frac{4c^2N}{4B(N+1)^2 - c^2(N-1)(3N+1)}\right)}_{\beta \equiv \beta(N, A, B)} e_{-k} \end{aligned} \tag{BR$'_k$}$$

The best response is linear and downward sloping. Let $e_K = \sum_{k \in K} e_k$. Summing over all k, in equilibrium: $e_K = N\alpha - \beta\sum_k e_{-k}$. Thus,

$$e_K = \frac{N\alpha}{1 + \beta(N-1)},$$

where we use $\sum_k e_{-k} = (N-1)e_K$. As $e_K = e_{-k} + e_k$, the equilibrium effort is given by:

$$e_k = \frac{\alpha}{1 + \beta(N-1)}.$$

Replacing the values for the constants α and β yields the equilibrium effort given in (EC). Because effort choices are strategic substitutes with linear best response functions, there exists a unique symmetric equilibrium.

Next, we show that the second order condition is satisfied for every firm if all of them produce a positive output in equilibrium. Note that the second-order condition of firm k's maximization problem (13) is given by:

$$\begin{aligned} & 2\left(\frac{\partial\Delta\pi_k}{\partial e_k}\right) + e_k\cdot\frac{\partial^2\Delta\pi_k}{\partial e_k^2} + \frac{\partial^2\pi_k(c, \cdot)}{\partial e_k^2} - 2 \leq 0 \\ \Longleftrightarrow \; & \frac{c^2(N-1)}{B(N+1)} + \frac{c^2(N-1)^2}{2B(N+1)^2} - 2 \leq 0. \end{aligned} \tag{SOC$_k$}$$

Note that (SOC$_k$) is strict for $N = 1$, and it is equivalent to

$$\frac{1}{B} \leq \frac{4(N+1)^2}{c^2(N-1)(3N+1)} \quad \text{for} \quad N \geq 2. \tag{SOC$'_k$}$$

Let $q_k(c_k) \equiv q_k(c_k, e_k, e_{-k})$ with $e_k = e(N)$ for every $k \in I \cup J$. Note that $q_k(0) - q_k(c) = c/2B$, so $q_k(c) > 0$ for all k

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implies

$$\frac{1}{B} < \frac{2}{cN} \cdot \sum_k q_k(0). \tag{16}$$

The upper bound on $1/B$ in (16) is lower than the one in (SOC$'_k$) as, by construction, $\sum_k q_k(0) < 1$ (otherwise the equilibrium price would be negative), and $4(N+1)^2/(N-1)(3N+1) > 1$ for each $N > 0$.

(b) We prove the existence of $\bar{c} \in (0, A)$ such that $c \in (0, \bar{c})$ implies $q_k(c_k) > 0$ in equilibrium for every $k \in K$ and $c_k \in \{0, c\}$. Fix $N \geq 1$. Write $e(N, c) \equiv e(N)$. From (11), see that the symmetric equilibrium production of a high-cost firm is lower than that of a low-cost firm and satisfies:

$$q_k(c) = \frac{2(A-c) - (N-1)ce(N, c)}{2B(N+1)} > 0$$

$$\iff f(N, c) \equiv \frac{2(A-c)}{c(N-1)} - e(N, c) > 0. \tag{17}$$

Note that

$$\lim_{c \to 0} f(N, c) = \infty,$$

$$f(N, A) = 0 - \frac{A^2(N+1)^2}{2[4B(N+1)^2 + A^2(N-1)^2]} < 0.$$

Therefore, by Intermediate Value Theorem, there is $c_0 \in (0, A)$ such that $f(N, c_0) = 0$. If c_0 is unique, then take $\bar{c} = c_0$. Otherwise, take $\bar{c} = \min\{c_0\}$. Next, we prove that $e(N, c) > 0$ for $c \in (0, \bar{c})$, which is equivalent to the following:

$$8AN + c(N^2 - 6N + 1) > 0. \tag{18}$$

Given that $A > c$, we have

$$8AN + c(N^2 - 6N + 1) > 8cN + c(N^2 - 6N + 1) = c(N+1)^2 > 0,$$

which proves (18) for all $N > 0$.

(c) Fix $N \geq 1$. Differentiating (EC) with respect to N we obtain

$$e'(N) = -\frac{2c(N^2-1)[8B(A-c) + c^2(2A-c)]}{[4B(N+1)^2 + c^2(N-1)^2]^2}.$$

The above expression is negative for $A > c$ and $N \geq 1$. It is immediate to see that $e(N)$ is strictly increasing in A and strictly decreasing in B for all $N \geq 1$.

This completes the proof. □

Appendix B. Proofs

Most of the following proofs follow directly from the analysis of the base model, see Result 1 in Appendix A.

B.1. Proof of Lemma 1

The proof directly follows from Result 1-(a) with $A = 1 - Q_I$, $B = 1$, and $N = m$. □

B.2. Proof of Lemma 2

The proof directly follows from the proof of Result 1 with $A = A(m)$, $B = B(m)$, $a(m) \equiv A(m)/B(m)$, $\theta(m) \equiv 1/B(m)$, and $N = n$; see Eqs. (11) and (12). □

B.3. Proof of Proposition 1

The maximization problem of each incumbent i is given by:

$$\max_{q_i} \; q_i(1 - q_i - \mathbb{E}q_{-i} - Q_J(q_i + \mathbb{E}q_{-i}) - c_i)$$

$$\iff \max_{q_i} \; q_i[\underbrace{(1 - C_\kappa \kappa(m))}_{A(m)} - \underbrace{(1-\kappa(m))}_{B(m)}(q_i + \mathbb{E}q_{-i}) - c_i]$$

Therefore, setting $a \equiv a(m) = A(m)/B(m)$, $\theta \equiv \theta(m) = 1/B(m)$ and $N = n$ it follows from Result 1-(a) that

$$e_I(m) = \frac{c[8a(m)n + \theta(m)c(n^2 - 6n + 1)]}{2[4(n+1)^2 + \theta(m)c^2(n-1)^2]}.$$

Recall that the subgame played by the entrants is equivalent to the base model with $B = 1$, $a = A/B = 1 - Q_I$ and $\theta = 1/B = 1$. We cannot apply the bound in Result 1-(b) directly as a depends on c (Q_I is an equilibrium object that depends on the model's parameters). Obtain the equilibrium output of a high-cost entrant by replacing $a = 1 - Q_I$ and $\theta = 1$ in (11):

$$q_j(c, Q_I) = \frac{2(1 - Q_I - c) - c(m-1)e_J(m, Q_I)}{2(m+1)}, \tag{19}$$

where $e_J(m, Q_I)$ is the optimal effort of the entrants given in (EE). Because low-cost entrants produce more than high-cost ones in equilibrium, the interior solution condition is equivalent to $q_j(c, Q_I^*) > 0$, where Q_I^* is the total output of the incumbents in the symmetric equilibrium. Note that $q_j(c, Q_I)$ is decreasing in Q_I as

$$\frac{\partial q_j(c, Q_I)}{\partial Q_I} < 0$$

$$\iff 2 - \frac{4c^2m(m-1)}{4(m+1)^2 + c^2(m-1)^2} > 0$$

$$\iff m^2(4 - c^2) + m(8 - c^2) + 4 + c^2 > 0.$$

Hence, a high-cost entrant produces the least when all incumbents have low costs. Let $q_i(0)$ be the optimal output of a low-cost incumbent, so the interior solution for each entrant j requires $q_j\left(c, \sum_{i \in I} q_i(0)\right) > 0$. By (19), this is equivalent to

$$\sum_i q_i(0) < 1 - c - \frac{c(m-1) \cdot e_J\left(m, \sum_i q_i(0)\right)}{2}. \tag{20}$$

From (11), the equilibrium output of a low-cost incumbent is given by:

$$q_i(0) = \frac{2a(m) + \theta(m)c(n-1)(1 - e_I(m))}{2(n+1)}. \tag{21}$$

Note that $a(m) \to 1$, $\theta(m) \to m+1$ and $e_I(m) \to 0$ as $c \to 0$, and hence, we have

$$\lim_{c \to 0} \sum_i q_i(0) = \frac{n}{n+1} < 1$$

$$= \lim_{c \to 0} 1 - c - \frac{c(m-1) \cdot e_J\left(m, \sum_i q_i(0)\right)}{2}.$$

Therefore, from (20), there exists $\bar{c}_J > 0$ such that every entrant produces a positive output in equilibrium, provided $c \in (0, \bar{c}_J)$. Furthermore, $\bar{c}_J < 1 - Q_I^*$ as (20) also implies

$$c < c + \frac{c(m-1) \cdot e_J\left(\sum_i q_i(0)\right)}{2} < 1 - \sum_i q_i(0) \leq 1 - Q_I^*.$$

Finally, we characterize the interior solution condition of the incumbents. From (11), the interior solution condition for incumbents, $q_i(c) > 0$, is equivalent to

$$e_I(m) < \frac{2(A(m) - c)}{c(n-1)} = \frac{2(a(m) - \theta(m)c)}{\theta(m)c(n-1)}. \tag{22}$$

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Because $e_I(m) \to 0$ as $c \to 0$, there is $\bar{c}_I > 0$ such that all incumbents produce a positive output in equilibrium provided that $c \in (0, \bar{c}_I)$, although the right-hand-side of (22) tends to ∞. Moreover, $\bar{c}_I < A(m) = a(m)/\theta(m)$ because (22) does not hold if $c > a(m)/\theta(m)$. Define $\hat{c} = \min\{\bar{c}_J, \bar{c}_I\}$ to obtain the appropriate bound. By Result 1-(a), every firm's second order condition of the optimal contracting problem is satisfied if $c \in (0, \hat{c})$. □

B.4. Proof of Proposition 2

We first establish that $\kappa(m)$ is strictly increasing in m. Note that

$$\kappa'(m) = \frac{(4+c^2)[(4+c^2)(m+1)^2 - 4c^2m^2]}{(4(m+1)^2 + c^2(m-1)^2)^2}.$$

The numerator of the above expression is strictly positive if and only if

$$\underbrace{\frac{4+c^2}{4c^2}}_{h(c)} > \left(\frac{m}{m+1}\right)^2.$$

Note that $h(c)$ is strictly decreasing on $[0, 1]$ with $\min\{h(c)\} = h(1) = 5/4 > 1$. The right-hand-side of the above inequality is always strictly less than 1 for $m \geq 1$. Hence, $\kappa'(m) > 0$. Next,

$$A(m) = 1 - C_\kappa \kappa(m) \implies A'(m) = -C_\kappa \kappa'(m),$$
$$B(m) = 1 - \kappa(m) \implies B'(m) = -\kappa'(m).$$

Because

$$e_I(m) = \frac{c[8nA(m) + c(n^2 - 6n + 1)]}{2[4(n+1)^2 B(m) + c^2(n-1)^2]},$$

we have

$$e_I'(m) = \frac{8\kappa'(m)[(n+1)^2 e_I(m) - C_\kappa cn]}{2[4(n+1)^2 B(m) + c^2(n-1)^2]},$$

Thus, $e_I'(m) > 0$ if and only if

$$e_I(m) > \frac{ncC_\kappa}{(n+1)^2}$$
$$\iff \frac{8nA(m) + c(n^2 - 6n + 1)}{2[4(n+1)^2 B(m) + c^2(n-1)^2]} > \frac{nC_\kappa}{(n+1)^2}. \quad (23)$$

We prove the following condition:

$$\frac{8nA(m) + c(n^2 - 6n + 1)}{2[4(n+1)^2 B(m) + c^2(n-1)^2]} > \frac{n}{(n+1)^2}$$
$$\iff \underbrace{A(m) - B(m)}_{\kappa(m)(1-C_\kappa)} > \frac{c[2cn(n-1)^2 - (n+1)^2(n^2 - 6n + 1)]}{8n(n+1)^2}. \quad (24)$$

which implies (23) because $C_\kappa < 1$. Note that $A(m) - B(m)$ is a strictly increasing function of m as $A'(m) - B'(m) = \kappa'(m)(1 - C_\kappa) > 0$. So, in order to prove condition (24), it suffices to show that the inequality holds for $m = 1$. Note that

$$A(1) - B(1) = \kappa(1)(1 - C_\kappa) = \frac{4+c^2}{8} \cdot \frac{c(8+c^2)}{2(4+c^2)} = \frac{c(8+c^2)}{16}.$$

Hence, for $m = 1$, (24) boils down to:

$$8 + c^2 > \underbrace{\frac{2[2cn(n-1)^2 - (n+1)^2(n^2 - 6n + 1)]}{n(n+1)^2}}_{H(n;\, c)} \quad (25)$$

It is easy to show that $H(n; c)$ is strictly decreasing in n for all $c \in (0, 1)$. Thus, $H(n; c)$ achieves a maximum at $n = 1$, which is equal to $H(1; c) = 8 < 8 + c^2$, and hence, the proposition. □

B.5. Proof of Proposition 3

The proof is in the text. It follows directly from applying the envelope theorem to the profit maximization problem, and then to the contracting problem. The key is to first note that Q_J is increasing in m, then that the profit functions $\pi_i(c_i, e, m)$ are decreasing in Q_J and only depend on m through Q_J (by the Envelope theorem). Hence, $\pi_i(c_i, e, m)$ is decreasing in m for $c_i \in \{0, c\}$. Lastly, note that by the Envelope theorem, the firm value $V_i(m)$ only depends on m through the profit functions. Since $V_i(m)$ is increasing in both profit functions, it is decreasing in the number of entrants m. □

B.6. Proof of Proposition 4

The proof directly follows from Result 1-(c) with $A = A(m)$, $B = B(m)$, and $N = n$. □

B.7. Proof of Corollary 1

The corollary follows directly from Proposition 4. Because $e_I(n, m)$ is strictly decreasing in n by Proposition 4, and $e^{sim}(n + m) = e(n+m, 0)$, $e^{sim}(n+m') < e^{sim}(n+m)$ for every $m' > m$. □

B.8. Proof of Proposition 5

A direct but more mechanical way to prove Proposition 2 is to show that $e_I(m)$ is strictly increasing in $\kappa(m)$. Note that

$$e_I(m) = \frac{c[8n(1 - C_\kappa \kappa(m)) + c(n^2 - 6n + 1)]}{2[4(n+1)^2(1 - \kappa(m)) + c^2(n-1)^2]} \equiv \hat{e}_I(\kappa(m)).$$

It is easy to show that

$$\text{sign}[\hat{e}_I'(\kappa)] = \text{sign}$$
$$\times \underbrace{[2c(n+1)^2(8n + c(n^2 - 6n + 1)) - 4ncC_\kappa(4(n+1)^2 + c^2(n-1)^2)]}_{h(n,\, c)}.$$

For all $n > 0$ and $c \in (0, 1)$, $h(n, c) > 0$, and hence, $\hat{e}_I(\kappa)$ is strictly increasing in κ.

To show that the managerial effort elicited by the incumbents is higher under hierarchical entry than that under simultaneous entry the only thing we require to show is that, under hierarchical entry, $\kappa(2)$ is higher than that under simultaneous entry. First, consider the case of simultaneous entry. Note that

$$\kappa(2) = \frac{6(4+c^2)}{36+c^2}.$$

Next, consider hierarchical entry. The last mover, entrant 2 solves

$$\max_{q_2} q_2 (1 - Q_I - q_1 - q_2 - c_2),$$

where Q_I is the aggregate incumbent output, and q_1 is the production of entrant 1. The optimal output and profit of entrant 2 are respectively given by:

$$q_2(c_2) = \frac{1 - (Q_I + q_1) - c_2}{2},$$

$$\pi_2(c_2) = \frac{(1 - (Q_I + q_1) - c_2)^2}{4}.$$

The optimal managerial effort of entrant 2 is given by:

$$e_2 = \frac{\pi_2(0) - \pi_2(c)}{2} = \frac{c(2 - 2(Q_I + q_1) - c)}{8}.$$

Thus, the expected output of entrant 2 is given by:

$$\mathbb{E}q_2(Q_I + q_1) = e_2 q_2(0) + (1 - e_2) q_2(c)$$
$$= \underbrace{\frac{c^2(2-c) + 8(1-c)}{16}}_{G_2(c)} - \underbrace{\frac{4+c^2}{8}}_{K_2(c)}(Q_I + q_1).$$

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In previous stage of entry, entrant 1 solves

$$\max_{q_1} q_1 \left(1 - Q_I - q_1 - \mathbb{E}q_2(Q_I + q_1) - c_1\right).$$

Following the same procedure as in the case of entrant 2, we obtain

$$\mathbb{E}q_1(Q_I) = \underbrace{\frac{32 - 32c + c^3(12 - 2c + c^2)}{4(4-c^2)^2}}_{G_1(c)} - \underbrace{\frac{4 + c^2}{8 - 2c^2}}_{K_1(c)} \cdot Q_I.$$

Using the recursive formulation, we thus get

$$\begin{aligned} Q_J(Q_I) = \mathbb{E}q_1 + \mathbb{E}q_2 &= G_1(c) - K_1(c)Q_I + G_2(c) \\ &\quad - K_2(c)\underbrace{(Q_I + G_1(c) - K_1(c)Q_I)}_{Q_I + \mathbb{E}q_1} \\ &= G_2(c) + G_1(c)(1 - K_2(c)) \\ &\quad - (K_2(c) + K_1(c)(1 - K_2(c)))Q_I, \end{aligned}$$

i.e., the aggregate best reply of the entrants is linear in Q_I. Each incumbent i thus solves

$$\begin{aligned} &\max_{q_i} q_i \left(1 - (q_i + \mathbb{E}q_{-i}) - Q_J(q_i + \mathbb{E}q_{-i}) - c_i\right) \\ \iff &\max_{q_i} q_i \left(a^h(2) - (q_i + \mathbb{E}q_{-i}) - \theta^h(2)c_i\right), \end{aligned}$$

where

$$\begin{aligned} a^h(2) &= \frac{1 - [G_2(c) + G_1(c)(1 - K_2(c))]}{1 - [K_2(c) + K_1(c)(1 - K_2(c))]} \\ &= \frac{4}{4 - c^2} + c\left(\frac{12}{4 - 3c^2} - \frac{c}{4 + 2c}\right), \end{aligned}$$

$$\theta^h(2) = \frac{1}{1 - [K_2(c) + K_1(c)(1 - K_2(c))]} = \frac{16}{4 - 3c^2}$$

are respectively the market size and the size of cost reduction of the incumbents under hierarchical entry. Note that

$$\theta^h(2) = \frac{16}{4 - 3c^2} = \frac{1}{1 - \kappa^h(2)} \quad \iff \quad \kappa^h(2) = \frac{12 + 3c^2}{16}.$$

It is immediate to see that $\kappa^h(2) > \kappa(2)$for any $c \in (0, 1)$, and hence, the proposition follows. $\square$

B.9. Proof of Proposition 6

To derive the equilibrium managerial efforts, we proceed by backward induction. Consider the problem of an entrant firm. Let $c_j \in \{0, c\}$ be its realized cost, which is private information. Entrants share the following public beliefs about their marginal costs, $\Pr.\{c_j = 0\} = e_j$. Let $P_I \in [0, c]$ be the minimum price chosen by the incumbents. In the price-setting stage of entrants, consider a symmetric mixed strategy equilibrium, where $p_j(c_j = c) = c$ and $p_j(c_j = 0) \sim F_J[\underline{p}_J, P_I]$ for every j. That is, high-cost entrants set a price equal to their marginal cost and obtain zero profits, and low-cost entrants randomize their price according to distribution F_J, which has support on $[\underline{p}_J, P_I]$, where $\underline{p}_J \geq 0$. We focus on the equilibrium in which the cdf F_J is a smooth function, i.e., the distribution has no atoms. As explained in the text, there is no equilibrium in pure strategies. Also, note that the low-cost incumbents would obtain zero profits by setting their prices above P_I.

To derive the equilibrium mixed strategy F_J, we exploit the fact that an entrant must be indifferent between setting any price in the support of F_J when all other entrants are playing the equilibrium mixed strategy. Let $p_j \in [\underline{p}_J, P_I]$, and set $e_k = e$ for every $k \in J \setminus \{j\}$. Under this strategy profile, a low-cost entrant obtains positive profits if and only if its price is the lowest among all the prices set by the rival entrants. The probability of this event is given by:

$$\Pr.\{p_j \leq p_k \text{ for all } k \in J \setminus \{j\}\} = \left(1 - eF_J(p_j)\right)^{m-1}. \tag{26}$$

Hence, for a low-cost entrant, the expected profits of setting price p_j are given by:

$$\mathbb{E}\left(\pi_j \mid c_j = 0,\ e_{-j}, P_I\right) = p_j(1 - p_j)\left(1 - eF_J(p_j)\right)^{m-1}. \tag{27}$$

The indifference condition implies that (27) is a constant function of p_j for the equilibrium strategy F_J, i.e., there exists a constant K_J such that $K_J = \mathbb{E}\left(\pi_j \mid c_j = 0,\ e_{-j},\ P_I\right)$ for every $p_j \in [\underline{p}_J, P_I]$. Using this indifference condition, one obtains

$$F_J(p_j) = \frac{1}{e}\left[1 - \left(\frac{K_J}{p_j(1 - p_j)}\right)^{\frac{1}{m-1}}\right]. \tag{28}$$

To find the value of K_J, note that $F(P_I) = 1$, which results in $K_J = P_I(1 - P_I)(1 - e)^{m-1}$. Plugging this value of K_J in (28) results in the cdf of the equilibrium mixed strategy, which can easily be shown to be a smooth and increasing function in p_j. Similarly, one may find the value of $\underline{p}_J$ by using the fact that $F(\underline{p}_J) = 0$. It is easily shown that $\underline{p}_J \in (0, P_I)$.

The value of K_J gives the expected profits of a low-cost entrant, prior to its own cost realization. That is, if $e_k = e$ for every $k \in J \setminus \{j\}$, a low-cost entrant has expected profits given by:

$$\pi_j(0,\ e_{-j},\ P_I) = P_I(1 - P_I)(1 - e)^{m-1}. \tag{29}$$

Two key remarks follow from expression (29). First, the expected equilibrium profits do not depend on an entrant's own managerial effort. As opposed to the quantity-setting game, managerial effort has no value beyond the true cost realization, i.e., the marginal profitability of effort is null in a price-setting environment. Second, the expected equilibrium profits are the same as if marginal costs were public information. Note that, if marginal costs were known, (i) entrants with high-marginal cost would have zero profits as well, and (ii) the only case in which a low-cost entrant can have a positive profit is that for every other entrant to have high marginal cost (in which case the entrant would set the price at $p_j = P_I$ and capture the entire market demand).

Therefore, at the contracting stage, the problem of entrant j is given by:

$$\max_{e_j} e_j\pi_j(0,\ e_{-j},\ P_I) - e_j^2, \tag{30}$$

which yields the following best-response for effort choice among entrants:

$$e_j(e) = \frac{P_I(1 - P_I)(1 - e)^{m-1}}{2}, \tag{31}$$

which implies that effort choices are strategic substitutes, i.e., $e_j'(e) < 0$. Expression (31) yields the equilibrium effort of entrants in the symmetric equilibrium, $e_J^B(m, P_I)$, defined implicitly in (3).

Now, consider the problem of an incumbent firm. Using similar arguments as above, one can see that an incumbent firm realizes positive profits if it attains low marginal cost, sets a price lower than every other incumbent, and every entrant realizes high cost. Also, it can easily be seen that there is no equilibrium in pure strategies in the price-setting stage of the incumbents. Therefore, conditional on setting price p_i, an incumbent with low cost will have expected profits equal to

$$\mathbb{E}(\pi_i \mid c_i = 0,\ p_i) = (1 - e_J)^m \left(1 - eF_I(p_i)\right)^{n-1} p_i(1 - p_i), \tag{32}$$

where $(1-e_J)^m$ is the probability that all entrants have high cost, e is the symmetric managerial effort elicited by each rival incumbent, and F_I is the symmetric mixed strategy equilibrium in price choices among the incumbents. Note that conditional on the event that i has the lowest price among incumbents implies $p_i = P_I$, and hence, from (3) we obtain

$$(1-e_J)^m = \frac{2e_J(1-e_J)}{p_i(1-p_i)}. \tag{33}$$

Therefore, we can write

$$\mathbb{E}(\pi_i \mid c_i = 0,\ p_i) = 2e_J(p_i)(1-e_J(p_i))\,[1-eF_I(p_i)]^{n-1} \tag{34}$$

To have an equilibrium in mixed strategies, $\mathbb{E}(\pi_i \mid c_i = 0,\ p_i)$ must be constant for all p_i. Set $\mathbb{E}(\pi_i \mid c_i = 0,\ p_i) = K_I$ and solve for F_I to obtain

$$F_I(p_i) = \frac{1}{e}\left[1-\left(\frac{K_I}{2e_J(p_i)(1-e_J(p_i))}\right)^{n-1}\right] \tag{35}$$

The support of F_I is given by $[\underline{p}_I,\ c]$ with $\underline{p}_I > 0$. Find the value of K_I by solving $F_I(c) = 1$, which yields

$$K_I = 2e_J(c)(1-e_J(c))(1-e)^{n-1}. \tag{36}$$

Following the same steps as above, one can verify that the equilibrium mixed strategy F_I is well-defined, i.e., smooth and increasing, and that $\underline{p}_I \in (0,\ c)$.

As with entrants, the value of K_I gives the expected profits of low-cost incumbents, i.e., $\pi_i(0,\ e_{-i}) = K_I$ if $e_k = e$ for every $k \in I \setminus \{i\}$. Solving a problem analogous to (30), and using (3), one obtains the best-response for the effort choice of incumbents:

$$e_i(e) = \frac{c(1-c)(1-e_J(c))^m(1-e)^{n-1}}{2}. \tag{37}$$

Note that the effort choices among incumbents are also strategic substitutes. Using the best-response in (37) results in the equilibrium effort of incumbents, provided in (4).

Now, we prove that both equilibrium managerial efforts [of the entrants and the incumbents] are decreasing in m. First, from (3) note the equilibrium effort level of the entrants is decreasing in m: the left-hand side of (3) is increasing as a function of both e_J and m, hence e_J is decreasing as a function of m. Second, from (4), note that the left-hand side is increasing as a function of e_I. Therefore, the sign of the derivative of $e_I^B(m)$ with respect to m is equal to the sign of the derivative of $[1-e_J^B(m,\ c)]^m$ with respect to m. In what follows, we prove that $[1-e_J^B(m,\ c)]^m$ is decreasing in m, and conclude the proof.

From (3), note that $(1-e_J^B(m,\ c))^m$ is decreasing in m if and only if $e_J^B(m,\ c)(1-e_J^B(m,\ c))$ is also decreasing in m. Because $e_J^B(m,\ c)$ is decreasing in m, $e_J^B(m,\ c)(1-e_J^B(m,\ c))$ is decreasing in m if and only $e_J^B(m,\ c) \le 1/2$. We prove this statement by contradiction.

$$e_J^B(m,\ c) > \frac{1}{2} \implies \frac{1}{2^{m-1}} > (1-e_J^B(m,\ c))^{m-1}$$

$$\implies \frac{e_J^B(m,\ c)}{(1-e_J^B(m,\ c))^{m-1}} > 2^{m-2}$$

$$\implies \frac{c(1-c)}{2} > 2^{m-2} \quad \text{[follows from (3)]},$$

which is a contradiction since the maximum value of $c(1-c)/2$ is $1/8$ for $c \in [0,1]$, and the minimum value of 2^{m-1} is $1/4$ for $m \ge 0$. □

Appendix C. Numerical implementation

In order to show $G(c) < e_I(m) < F(c)$ for every $c \in (0, \hat{c})$, $n \ge 1$, $m \ge 1$, and the validity of Implication 2, in Section 5.2, we compute the model numerically. We define a grid over the parameter space $(c,\ n,\ m) \in \mathcal{C} \times \mathcal{N} \times \mathcal{M}$, where $\mathcal{C}$ is a grid of $(0,\ 1)$, $\mathcal{N} = \{1,\ 2,\ \dots,\ 50\}$, and $\mathcal{M} = \{0,\ 1,\dots,50\}$. For each $(n,\ m) \in \mathcal{N} \times \mathcal{M}$, we solve numerically for the upper bound of c, given by $\hat{c}(n,\ m)$. For each $(n,\ m) \in \mathcal{N} \times \mathcal{M}$, we then show the validity of the claims at every point $c \in \mathcal{C}(n,m) \equiv \left\{c = \hat{c}(n,\ m) * H/51 : H = 1,\dots,50\right\}$.

Appendix D. Social welfare

D.1. Expected consumer surplus

As explained in the text, the expected consumer surplus is given by $\mathbb{E}CS = 0.5 * \mathbb{E}Q^2$. Straightforward computations show that

$$\begin{aligned}\mathbb{E}Q^2 &= n\mathbb{E}q_i^2 + n(n-1)\,(\mathbb{E}q_i)^2 \\ &\quad + m\mathbb{E}q_j^2 + m(m-1)\left(\mathbb{E}q_j\right)^2 + 2nm\mathbb{E}q_i\mathbb{E}q_j,\end{aligned} \tag{38}$$

where $\mathbb{E}q_i$ and $\mathbb{E}q_j$ are the expected incumbent and entrant outputs in equilibrium, respectively.[14] Computing the expected output of an incumbent in equilibrium, $\mathbb{E}q_i$, is straightforward using expressions derived in the text. Namely,

$$\mathbb{E}q_i = e_I(m)q_i(0,e_I(m),m) + (1-e_I(m))q_i(c,e_I(m),m).$$

To compute the expected output of an entrant in equilibrium, one needs to compute the expectation over the incumbents aggregate output along the equilibrium path, Q_I. Along the equilibrium path, Q_I is a random variable determined by the realized number of incumbents that attain the cost reduction, denoted by L. Let $Q_I(L)$ be the aggregate incumbent output conditional on L incumbents attaining the cost reduction, i.e.,

$$Q_I(L) = Lq_i(0,e_I(m),m) + (n-L)q_i(c,e_I(m),m).$$

Note that L is a random variable with support $\{0,1,\dots,n\}$ and probability distribution

$$p_L(l) \equiv \mathbb{P}\left[L=l\right] = \binom{n}{l} e_I(m)^l(1-e_I(m))^{n-l}, \quad l \in \{0,1,\dots,n\}.$$

Let $q_j(c_j,e,L) = q_j(c_j,e,Q_I(L))$, and $e_J(L,m) = e_J(Q_I(L),m)$. Then, the expected output of an entrant in equilibrium is given by:

$$\begin{aligned}\mathbb{E}q_j &= \mathbb{E}\Big[\mathbb{E}\left[q_j \mid L\right]\Big] \\ &= \mathbb{E}\Big[e_J(L,m)q_j(c,e_J(L,m),L) + (1-e_J(L,m))q_j(0,e_J(L,m),L)\Big]\end{aligned}$$

14 To see how (38) is obtained, it suffices to note:

$$\begin{aligned}\mathbb{E}Q^2 &= \mathbb{E}\left(\sum_{i\in I} q_i + \sum_{j\in J} q_j\right)^2 \\ &= \mathbb{E}\left(\sum_i q_i^2 + 2\sum_{i\neq i'} q_iq_{i'} + \sum_j q_j^2 + 2\sum_{j\neq j'} q_jq_{j'} + 2\sum_{i,j} q_iq_j\right) \\ &= n\mathbb{E}q_i^2 + 2\binom{n}{2}(\mathbb{E}q_i)^2 + m\mathbb{E}q_j^2 \\ &\quad + 2\binom{m}{2}\left(\mathbb{E}q_j\right)^2 + 2nm\mathbb{E}q_i\mathbb{E}q_j.\end{aligned}$$

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$$= \sum_{l=0}^{n} p_L(l) \times \left[e_J(l,m) q_j(c, e_J(l,m), l) + (1 - e_J(l,m)) q_j(0, e_J(l,m), l) \right].$$

The expectations $\mathbb{E}q_i^2$ and $\mathbb{E}q_j^2$ can be computed analogously.

D.2. Expected producer surplus of incumbents

The expected producer surplus of an incumbent can be computed directly from its definition. Note that the expected market profits are given by $\Pi_i(m)$, which is provided in Section 4.5. Furthermore, at equilibrium, the effort cost of incumbents is given by $\psi(e_I(m))$.

D.3. Expected producer surplus of entrants

To compute the expected producer surplus of an entrant, one needs to take the expectation over the incumbents aggregate output along the equilibrium path, Q_I. Following the same reasoning as in Appendix D.1, the expected market profits of entrant j can be expressed as:

$$\Pi_j(m) = \sum_{l=0}^{n} p_L(l) \times \left[e_J(l,m) \pi_j(c, e_J(l,m), l) + (1 - e_J(l,m)) \pi_j(0, e_J(l,m), l) \right],$$

where $\pi_j(c_j, e, L)$ denotes the expected market profits of an entrant conditional on having realized marginal cost c_j, at a common effort level $e_j = e$ for all $j \in J$, when $Q_I = Q_I(L)$, see Section 4. Similarly, the effort cost of entrants along the equilibrium path depends on the incumbents aggregate output. Hence, the expected producer surplus of an entrant at equilibrium is given by:

$$PS_j(m) = \Pi_j(m) - \mathbb{E}\psi(e_J(L,m)),$$

where

$$\mathbb{E}\psi(e_J(L,m)) = \sum_{l=0}^{n} p_L(l) \left[\frac{e_J(L,m)^2}{2} \right].$$

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