Monitoring and incentives under multiple-bank lending: The role of collusive threats

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Online appendix

Proof of Proposition 8. The merged bank maximizes joint profits, but offer individualized contracts (m_i, s_i) and (m_j, s_j) to loan officers i and j, respectively. Therefore, we can solve the optimal contract for each loan officer separately. The optimal contract (m_i, s_i) between loan officer i and the merged entity solves

$$\max_{\{m_i, s_i\}} \pi(m_i, m_j)pr(2 - s_i - s_j) - 2,$$

subject to (8), (11), (12).

Denote by subscript 'mb' the threshold values of firm quality and the equilibrium variables. In a symmetric equilibrium, as in Proposition 5, when both no-collusion constraints (8) and (9) bind, the optimal contracts coincide with those under two bank lending. Therefore, we have $z^0_{mb} = z^0_2$, $m_{mb}(z) = m_2(z)$ and $s_{mb}(z) = s_2(z)$. This is because the rent-jamming effect has the same strength under both lending structures. By contrast, when none of the no-collusion constraints binds, the equilibrium contracts under merged banks differ from those under two-bank lending (strategic banks). The reason is simple. When the no-collusion constraints are slack, under both lending modes, we can ignore both the no-collusion and feasibility constraints, and use the binding participation constraints to write

$$\pi(m_i, m_j)prs_i = \frac{1}{2}cm_i^2, \quad \text{and} \quad \pi(m_i, m_j)prs_j = \frac{1}{2}cm_j^2.$$

When banks choose the contracts independently, substituting the above expressions into the objective functions, the payoffs of banks i and j respectively reduce to:

$$B_i(m_i, m_j) \equiv \pi(m_i, m_j)pr - \frac{1}{2}cm_i^2 - 1, \quad \text{and} \quad B_j(m_i, m_j) \equiv \pi(m_i, m_j)pr - \frac{1}{2}cm_j^2 - 1.$$

On the other hand, the objective function of the merged entity [under binding participation constraints] becomes

$$B(m_i, m_j) \equiv 2\pi(m_i, m_j)pr - \frac{1}{2}cm_i^2 - \frac{1}{2}cm_j^2 - 2.$$

Note that, under strategic banks, each bank maximizes the net surplus of the bank-monitor relationship, i.e., bank's expected revenue minus the sum of monitoring cost and the opportunity cost of capital. By contrast, under merged banks, they maximize the joint expected revenue minus the aggregate monitoring and opportunity costs. Clearly under merged banks, the aggregate surplus is higher due to the absence of the free-riding problem, and hence, monitoring efforts, shares and the aggregate monitoring intensity are higher than those under strategic banks. Thus, under merged banks, in the absence of collusion incentive problem, we have

$$m_{mb}(z) = \frac{2pr}{c + 2pr}, \; \pi_{mb}(z) = 1 - \left(\frac{2pr}{c + 2pr}\right)^2, \; \text{and} \; s_{mb}(z) = \frac{c}{2(c + 2pr)}.$$

Each of the above expressions are strictly higher than that under two-bank lending. It is also the case that $\bar{z}_{mb} < \bar{z}_2$, i.e., incentives for over-monitoring are lower under merged banks.

Proof of Proposition 9. When the banks choose contracts independently but the loan officers behave cooperatively, bank i chooses (m_i, s_i) to solve

$$\max_{\{m_i, s_i\}} \pi(m_i, m_j)pr(1-s_i)-1,$$
$$\text{subject to (10), (11), (12).}$$

We analyze a symmetric equilibrium, i.e., $m_i = m_j$ and $s_i = s_j$. When the no-collusion constraint (10) is slack, the equilibrium contracts coincide with those under two-bank lending. When (10) binds, we have $s_i = s_j = 1 - z/pr$. It follows from the binding participation constraints that

$$m_i = m_j = m_{cp}(z) = \frac{4(pr-z)}{c+2(pr-z)}.$$

We must have $m_{cp}(z) < 1$ which is equivalent to $z > pr - c/2 \equiv z^0_{cp} = z^0 < z^0_2$. The threshold value of borrower quality $\bar{z}_{cp}$ solves

$$\frac{c}{2(2c+pr)} = 1 - \frac{z}{pr},$$

whereas $\bar{z}_2$ solves

$$\frac{c}{2(2c+pr)} = 2\left(1 - \frac{z}{pr}\right).$$

Thus, $\bar{z}_{cp} < \bar{z}_2$, i.e., two-bank lending under cooperative loan officers ameliorates the collusion incentive problem relative to two-bank lending with competing monitors. As a result, there is a gain in efficiency under cooperative loan officers. It is immediate to show that $m_{cp}(z) < m_2(z)$ and $\pi_{cp}(z) < \pi(z)$ given that $c > 0$. Clearly, $s_{cp}(z) = 1 - z/pr < 2(1 - z/pr) = s_2(z)$.

Proof of Proposition 10. The firm's expected utility under separate financing is given by $F_S(z) = 2F(z) = 2[py - z - m(z)(pr - z)]$, whereas that under two-bank lending is given by $F_2(z) = 2[py - z - \pi_2(z)(pr - z)]$, where $\pi_2(z)$ is the aggregate monitoring intensity under two-bank lending, which is given by:

$$\pi_2(z) = 1 - (1 - m_2(z))^2 = \begin{cases} \pi^0_2(z) \equiv \dfrac{16c(pr-z)}{(c+4(pr-z))^2} & \text{for } z^0_2 < z \le \bar{z}_2, \\ \pi^*_2 \equiv \dfrac{pr(2c+pr)}{(c+pr)^2} & \text{for } \bar{z}_2 < z \le pr. \end{cases}$$

Note first that two-bank lending is feasible only if $z > z^0_2 = pr - \frac{c}{4}$ which is equivalent to $m_2(z) < 1$.

Because $F_S(z) - F_2(z) = 2(pr - z)(\pi_2(z) - m(z))$ and $pr - z \ge 0$, the firm prefers separate financing if and only if $\pi_2(z) \ge m(z)$. Recall also that $\bar{z} < z^0_2$, and hence, $m(z) = m^*$ for all $z \in [z^0_2, pr]$, where m^* is the non-delegation level of monitoring under single-bank lending. Because $\pi_2(z^0_2) = 1 > pr/c = m^* = m(z^0_2)$ and $\pi_2(z)$ is strictly decreasing on $[z^0_2, pr]$, we must compare $m(\bar{z}_2) = m^*$ with $\pi_2(\bar{z}_2) = \pi^*_2$. When $c \le 2(2+\sqrt{5})$ and $1 + \frac{c}{2} \le pr \le c$, or $c > 2(2+\sqrt{5})$ and $\frac{1}{2}(\sqrt{5}-1)c \le pr \le c$, we have $m^* > \pi^*_2$ which guarantees the existence and uniqueness of $\theta^F \in (z^0_2, pr)$ such that $\pi_2(z) \ge m(z)$ if and only if $z \le \theta^F$. Therefore, $F_S(z) \ge F_2(z)$ if and only if $z \le \theta^F$. On the other hand, if $c > 2(2+\sqrt{5})$ and $1 + \frac{c}{2} \le pr < \frac{1}{2}(\sqrt{5}-1)c$, then $m^* < \pi^*_2$, and hence, $\pi_2(z) > m(z)$, i.e., $F_S(z) > F_2(z)$ for all $z \in (z^0_2, pr]$.

Proof of Proposition 11. We skip many details of the proof as solving a bank's maximization problem is very similar to the other maximization problems we have analyzed in the previous sections. We have already shown how the minimum share constraint (18) is derived in the bribery subgame. Note first that each bank receives repayments only if no loan officer colludes with the borrower, conditional on the fact that both have succeeded in monitoring. In that event, the gross expected profit of bank i is given by:

$$p^2 r(1-s_i) + \frac{1}{2}p(1-p)y(1-s_i) + \frac{1}{2}(1-p)py.$$

When two good projects are implemented, bank i receives r with probability p^2, out of which the loan officer gets a share s_i, thus earning $p^2 r s_i$ in expectation. On the other hand, each bank receives $\frac{1}{2}y$ when one project succeeds and the other fails. However, the assumption of maximal incentive implies that loan officer i receives incentive pay only when the project financed by bank i succeeds. By contrast, when only the project financed by bank j is successful, bank i indeed receives $y/2$, but it does not share the repayment with loan officer i. Defining $\bar{r} \equiv pr + \frac{1}{2}(1-p)y$, the expected payoffs of bank i and loan officer i are therefore respectively given by:

$$B_i = m_i m_j \left(p\bar{r}(1-s_i) + \frac{1}{2}p(1-p)y \right) - 1 \quad \text{and} \quad M_i = m_i m_j p\bar{r}s_i + m_i(1-m_i)b(s_i) - \frac{1}{2}cm_i^2.$$

Thus, bank i chooses (m_i, s_i) to maximize (16) subject to (17) and (18). Let μ_P^i and μ_N be the Lagrange multipliers associated with the participation constraint, (17) and minimum share constraint, (18), respectively. Observe that $pr - z \geq 0$ is equivalent to $v \geq p(y-r)$. However, we assume that $v \geq v_J^{min} \equiv \frac{1}{2}py$. Given that $y < 2r$, we have $p(y-r) < v_J^{min}$. We later show that the maximum value v can take (so as to ensure that the equilibrium monitoring is less than 1) is $v_J^0 \equiv \frac{c}{2} + p^2(y-r)$. Thus, it would be useful to remember that

$$p(y-r) < v_J^{min} < v_J^0.$$

We next turn to solving the maximization exercise. We first argue that the participation constraint, (17) must bind at the optimum. Suppose not, i.e., $\mu_P^i = 0$, under which the first-order conditions with respect to m_i and s_i are respectively given by:

$$m_j \left(p\bar{r}(1-s_i) + \frac{1}{2}p(1-p)y \right) = 0, \quad \text{and} \quad p\bar{r}(\mu_N - m_i m_j) = 0.$$

If $\mu_N = 0$, i.e., the minimum share constraint, (18) is slack, then the second of the above two conditions yields $m_i = m_j = 0$, which is not possible because no bank would break even. Now, consider the case when the minimum share constraint binds at the optimum. Substituting for m_j from the second equation into the first one, we obtain

$$\frac{\mu_N}{m_i} \cdot \left(p\bar{r}(1-s_i) + \frac{1}{2}p(1-p)y \right) = 0,$$

which gives $\mu_N = 0$ (since m_i and s_i are both less than 1). This leads to $m_i = m_j = 0$, and hence, the participation constraint must bind. The binding participation constraint yields

$$s_i = s_i(m_i, m_j) = \frac{cm_i - 2(1-m_j)\beta v}{2m_j\bar{r} + (1-m_j)(1-\beta)py} \quad \text{with} \quad \frac{\partial s_i}{\partial m_i} = \frac{c}{2m_j\bar{r} + (1-m_j)(1-\beta)py} \geq 0. \tag{A.1}$$

Substituting the above, the objective function in (16) reduces to

$$m_i m_j \underbrace{\left(p\bar{r} + \frac{1}{2}p(1-p)y \right)}_{p\hat{r} \equiv pr + (1-p)y} + m_i(1-m_i)b(s_i(m_i, m_j)) - \frac{1}{2}cm_i^2 - 1,$$

which bank i maximizes subject to (18).

First, consider the case when the minimum share constraint does not bind. The first-order conditions of the maximization problems of banks i and j are respectively given by:

$$m_j p\bar{r} - m_j p\hat{r} s_i(m_i, m_j) - m_i m_j p\hat{r} \cdot \frac{\partial s_i}{\partial m_i}(m_i, m_j) = 0,$$
$$m_i p\bar{r} - m_i p\hat{r} s_i(m_i, m_j) - m_i m_j p\hat{r} \cdot \frac{\partial s_i}{\partial m_j}(m_i, m_j) = 0.$$

Solving the above two first-order conditions, we find these have a unique symmetric solution (because imposing symmetry, i.e., $m_i = m_j = m_J^*(v)$, both first order conditions become linear in $m_J^*(v)$). Thus we find

$$m_i = m_j = m_J^*(v, \beta) = \frac{2\beta v\bar{r} + (1-\beta)py\hat{r}}{2\beta v\bar{r} + (1-\beta)py\hat{r} + 2\bar{r}(c - p\hat{r})} \quad \text{with} \quad \frac{\partial m_J^*(v, \beta)}{\partial v} \geq 0.$$

Then, from (A.1) we obtain $s_J^*(v, \beta) \equiv s(m_J^*(v, \beta), m_J^*(v, \beta))$, which is given by:

$$s_J^*(v, \beta) = \frac{2\beta v\bar{r}(2p\hat{r} - c) + (1-\beta)cpy\hat{r}}{2p\bar{r}(2\beta v\bar{r} + (1-\beta)cy)} \quad \text{with} \quad \frac{\partial s_J^*(v, \beta)}{\partial v} \leq 0.$$

Because the minimum share constraint, (18) must be slack at the optimum, we have

$$\underbrace{p\bar{r}s_J^*(v, \beta)}_{S(v,\beta)} > \underbrace{v - p^2(y - r)}_{H(v)} \quad \Longleftrightarrow \quad v < \bar{v}_J.$$

Because $S(v, \beta)$ is decreasing in v and $H(v)$ is strictly increasing in v, $\bar{v}_J$ is unique.

Note also that $S(v, \beta)$ is strictly decreasing in β with $\max_\beta S(v, \beta) = S(v, 0) = \frac{p\hat{r}}{2}$ and $\min_\beta S(v, \beta) = S(v, 1) = p\hat{r} - \frac{c}{2}$. It is easy to show that

$$p(y - r) < \bar{v}_J < v_J^0,$$

as, by Assumption 2, $S(v, 0) < \max H(v) = H(v_J^0) = \frac{c}{2}$ and $S(v, 1) > \min H(v) = H(p(y - r)) = p(1 - p)(y - r)$. However, it is not possible to guarantee that $v_J^{min} \leq \bar{v}_J$. If $v_J^{min} > \bar{v}_J$, then the minimum share constraint, (18) is never slack at the optimum.[1]

Next, consider the case when the minimum share constraint, (18) binds. We consider the symmetric solution which is given by:

$$2p\bar{r}s = 2[v - p^2(y - r)] \quad \Longleftrightarrow \quad s_J^0(v) = \frac{v - p^2(y - r)}{p\bar{r}}.$$

Thus, the binding participation constraint, (19) reduces to

$$m_j p\bar{r}s_J^0(v) + (1 - m_j)b(s_J^0(v)) = \frac{1}{2}cm_i.$$

Differentiating the above with respect to m_j, we obtain

$$\text{sign}[m_i'(m_j)] = \text{sign}[p\bar{r}s_J^0(v) - b(s_J^0(v))] = \text{sign}[h(\beta)],$$

where

$$h(\beta) \equiv (2r - y)(v - p^2(y - r)) - \beta[py(y - r) + v(2r - y)].$$

Note that $h'(\beta) < 0$, $h(0) = (2r - y)(v - p^2(y - r)) \geq 0$ and $h(1) = -p(y - r)(2pr + (1 - p)y) < 0$. Therefore, there is a unique $\bar{\beta} \in (0, 1)$ such that $m_i'(m_j) > 0$ if and only $\beta < \bar{\beta}$. The symmetric equilibrium monitoring effort is given by:

$$m_J^0(v, \beta) = \frac{2b(s_J^0(v))}{c + 2(b(s_J^0(v)) - p\bar{r}s_J^0(v))} = \frac{2\beta v\bar{r} + (1-\beta)y(v - p^2(y - r))}{2\beta v\bar{r} + (1-\beta)y(v - p^2(y - r)) + \bar{r}(c - 2(v - p^2(y - r)))}.$$

[1] A sufficient condition for $v_J^{min} \leq \bar{v}_J$ is $c \leq py$. On the other hand, if $c > py$, then $v_J^{min} \leq \bar{v}_J$ if and only if β is low.

Note that $m_J^0(v,\beta)\le 1$ is equivalent to $v\le v_J^0\equiv \frac{c}{2}+p^2(y-r)$. Moreover, both $m_J^0(v,\beta)$ and $s_J^0(v)$ are increasing in v.

Finally, we have to verify that the equilibrium expected profit of each bank is non-negative. Because the expressions for the optimal monitoring and repayment share are cumbersome, it is difficult to have an analytical proof that the banks earn non-negative profits over the parameter range $[v_J^{min}, v_J^0]$. We therefore show that there is a lower-bound on v beyond which banks have positive profits. Further, we provide an example for which bank profits are positive. Each bank's symmetric equilibrium expected profits are given by:

$$B_J(v,\beta)=\begin{cases} B_J^*(v,\beta)\equiv (m_J^*(v,\beta))^2(p\hat{r}-p\bar{r}s_J^*(v,\beta))-1 & \text{for } v<\bar{v}_J,\\ B_J^0(v,\beta)\equiv (m_J^0(v,\beta))^2(p\bar{r}(1-s_J^0(v))+\dfrac{1}{2}p(1-p)y)-1=(m_J^0(v,\beta))^2(py-v)-1 & \text{for } v\ge \bar{v}_J.\end{cases}$$

Because $m_J^*(v,\beta)$ is increasing and $s_J^*(v,\beta)$ decreasing in v, we have $B_J^*(v,\beta)$ increasing in v. On the other hand, because $m_J^0(v,\beta)$ and $s_J^0(v)$ are both increasing in v, $B_J^*(v,\beta)$ is first increasing and then decreasing (concave or S-shaped) on $[v_J^{min}, v_J^0]$. Therefore, $B_J(v,\beta)$ is first increasing and then decreasing on $[v_J^{min}, v_J^0]$. Notice that

$$B_J(v_J^0,\beta)=B_J^0(v_J^0,\beta)=p\hat{r}-\frac{c}{2}-1,$$

which is positive under Assumption 2. So, by continuity, $B_J(v,\beta)\ge 0$ for v close to v_J^0.

Figure 9 depicts $B_J(v,\beta)$. As we have discussed before, there are two possible cases: (i) $v_J^{min}<\bar{v}_J$, which is the case in Figure 9, and (ii) $\bar{v}_J\le v_J^{min}$. When $v_J^{min}<\bar{v}_J$ in which case, $B_J^*(v,\beta)\ge B_J^0(v,\beta)$ for all $v\in[v_J^{min},\bar{v}_J]$ because $B_J^*(v,\beta)$ is the equilibrium utility of each bank when the minimum share constraint is slack. On the other hand, when $\bar{v}_J\le v_J^{min}$, the minimum share constraint binds for all $v\in[v_J^{min}, v_J^0]$, and hence, $B_J(v,\beta)\equiv B_J^0(v,\beta)$ for all $v\in[v_J^{min}, v_J^0]$. Therefore, taking both the aforementioned cases into account, a sufficient condition for non-negative $B_J(v,\beta)$ is

$$B_J^0(v_J^{min},\beta)=\frac{p^3y^3(2pr+y-(2-\beta)py)}{2[c(2pr+(1-p)y)+p^2(\beta y^2-(2r-y)(2p(r-y)+y)]^2}-1\ge 0.$$

Given that $B_J^0(v,\beta)$ depends on many parameters in a non-linear way, it is very difficult to verify the above inequality. If $B_J^0(v_J^{min},\beta)\ge 0$, then clearly each bank earns positive profits for all $v\in[v_J^{min}, v_J^0]$. By contrast, if $B_J^0(v_J^{min},\beta)<0$ (as drawn in Figure 9), then because $B_J^0(v_J^0,\beta)\ge 0$, by Intermediate value theorem there is a $\tilde{v}\in(v_J^{min}, v_J^0)$ such that $B_J^0(\tilde{v},\beta)=0$. Thus, we would consider only values of v which are higher than $\tilde{v}$ in order to guarantee non-negative profits for each bank. For example, for parameter values [which we use in Example 1] $\beta=0.4$, $p=0.5$, $c=5.5$, $r=10$ and $y=10.25$, we have $B_J(v_J^{min},0.4)=1.034$.

Proof of Proposition 12. Write the equilibrium monitoring under separate financing, which is the same as that under single-bank lending, as a function of v instead of z:

$$m_S(v)=\begin{cases} m^*(v)\equiv \dfrac{pr}{c} & \text{for } v^{min}\le v<\bar{v},\\ m^0(v)\equiv \dfrac{2(v-p(y-r))}{c} & \text{for } \bar{v}\le v<v^0,\\ \bar{m}(v)\equiv 1 & \text{for } v^0\le v\le py-1,\end{cases}$$

where $v^{min}\equiv p(y-r)$, $\bar{v}\equiv py-\frac{1}{2}pr$ and $v^0\equiv \frac{c}{2}+p(y-r)$. Clearly, $m^*(v)$ is constant with respect to v and $m^0(v)$ is linearly increasing in v with a slope $2/c$. Recall that $v^{min}<\frac{1}{2}py=v_J^{min}$ because $y<2r$. Also, $v^0>v_J^0$ as $p<1$. Therefore, we compare the equilibrium monitoring under separate and joint financing only for values of v in $[v_J^{min}, v_J^0]\equiv[\frac{1}{2}py, \frac{c}{2}+p^2(y-r)]$. The thresholds of v associated with separate and joint financing are ordered in a way that is depicted in the left panel of Figure 7. Also, it is easy to show that $m_J^*(v)$, the portion of $m_J(v)$

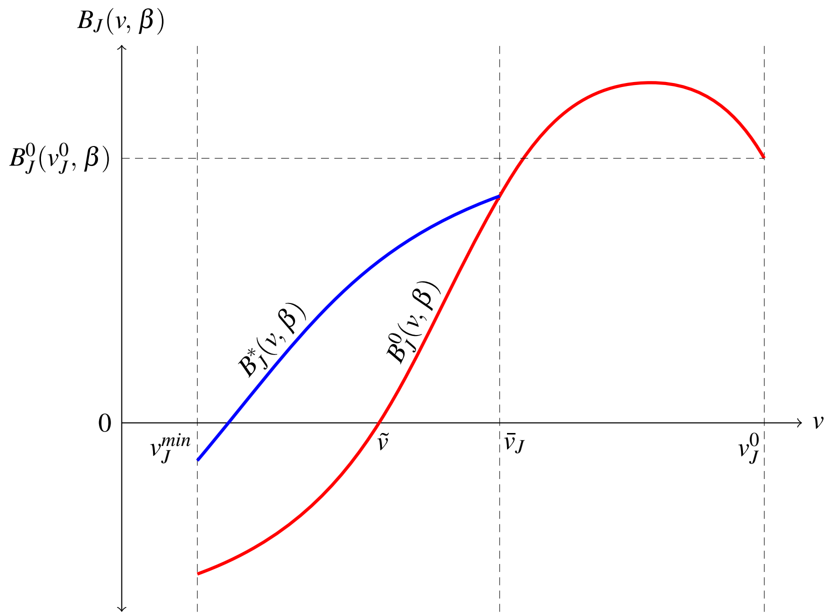


Figure 9: *The equilibrium profit of each bank under joint financing is the upper envelope of $B_J^*(v, \beta)$ and $B_J^0(v, \beta)$ when $v_J^{min} < \bar{v}_J$. $B_J^0(v, \beta) \geq 0$, which happens for $v \geq \tilde{v}$, guarantees non-negative profits for each bank. Note that if we had $\tilde{v} \leq v_J^{min}$ which is equivalent to $B_J^0(v_J^{min}, \beta) \geq 0$, then banks would have consumed non-negative profits for all $v \in [v_J^{min}, v_J^0]$.*

when the minimum share constraint is slack, is increasing and concave in v, whereas $m_J^0(v)$ is increasing and convex in v.

Note that for v large, i.e., close to v_J^0, $m_J(v)$ is greater than $m_S(v)$, because (a) $m_J(v_J^0) = m_J^0(v_J^0) = 1$, and (b) $m_S(v_J^0) = m^0(v_J^0) < m^0(v^0) = 1$ given that $m^0(v)$ is strictly increasing in v and $v_J^0 < v^0$. Then, all we have to show is $m_J(v)$ intersects $m(v)$ at most once. The main difficulty lies in the fact that $m^0(v)$ is a linear function while $m_J^0(v)$ is convex in v, and hence, in principle it is possible that these two functions cross each other twice. We would first establish that $\bar{v}_J \leq \bar{v}$ for any $\beta \in [0, 1]$. Therefore, in order to guarantee single-crossing between $m_S(v)$ and $m_J(v)$, it suffices to show that $m_J^0(v)$ is steeper than $m^0(v)$ for all $v \geq \bar{v}_J$ as $m_J(v) = \max\{m_J^*(v), m_J^0(v)\}$ and $m_S(v) = \max\{m^*(v), m^0(v)\}$. Clearly, $m_J^*(v)$ is steeper than $m^*(v)$ because $m_J^*(v)$ is strictly increasing in v, but $m^*(v)$ is a constant function.

We first show that $\bar{v}_J \leq \bar{v}$ for any $\beta \in [0, 1]$. Recall that $\bar{v}_J$ is defined as $p\bar{r}s_J^*(\bar{v}_J) = \bar{v}_J - p^2(y-r)$. Because $s_J^*(v)$ is decreasing in β, $\bar{v}_J$ decreases with β, and hence, $\bar{v}_J$ takes the maximum value [with respect to β] when $\beta = 0$ which is given by:

$$\bar{v}_J(0) = \frac{p\hat{r}}{2} + p^2(y-r) = \frac{1}{2}p(y + p(y-r)).$$

So, $\bar{v}_J(0) < \bar{v}$ is equivalent to $r < y$. Thus, $\bar{v}_J < \bar{v}$ for all $\beta \in [0, 1]$. Next, we show that $m_J^0(v)$ is steeper than $m^0(v)$ for all $v \geq \bar{v}_J$ whenever β is small. The slope of $m_J^0(v)$ at $\beta = 1$ is given by:

$$\frac{2}{c + p^2(y-r)},$$

which is less than $2/c$, the slope of $m^0(v)$.[2] On the other hand, the slope of $m_J^0(v)$ at $\beta = 0$, which is given by:

$$H(v) \equiv \frac{2cy(2pr+(1-p)y)}{c((2pr+(1-p)y)-2p(v-p^2(y-r))}$$

is greater than $2/c$. To show this, we proceed as follows. Because, $m_J^0(v)$ is convex in v for all $\beta \in [0, 1]$, the above expression is increasing in v, and hence, $\min_v H(v) = H(py/2)$. Now,

$$H(py/2) \geq \frac{2}{c} \quad \Longleftrightarrow \quad h(y) \equiv \frac{c^2 y(2pr+(1-p)y)}{[c(2pr+(1-p)y)-p^2(2r-y)(2p(r-y)+y)]^2} \geq 1.$$

Because $h(y)$ is decreasing in y, we have $\min_y h(y) = h(2r) = 1$. This proves that $dm^0(v)/dv \geq 2/c$ at $\beta = 0$. Because $dm^0(v)/dv \leq 2/c$ at $\beta = 1$, by the Intermediate value theorem, there is $\hat{\beta}$ such that $dm^0(v)/dv = 2/c$ at $\beta = \hat{\beta}$. If $\hat{\beta}$ is unique, set $\tilde{\beta} = \hat{\beta}$; otherwise, set $\tilde{\beta} = \min\{\hat{\beta}\}$. This completes the proof.

Proof of Proposition 13. We will analyze the equilibrium strategies of bank i in two situations—(a) loan officer j is strategic, and (b) he is honest. We start by observing that (C, C) cannot be an equilibrium since if (a) loan officer j is not honest, and neither bank implements the collusion-free contracts, then they would not receive any repayment, and would lose their entire investments, whereas (b) if loan officer j is honest then this possibility does not emerge because he never colludes. Next, consider any $z > \bar{z}_2$. For these values of firm quality z, neither (8) nor (9) binds, and hence, the banks do not worry about the incentive problems emerging from collusion possibilities. Thus, (NC, NC) is an equilibrium for all $z > \bar{z}_2$, and hence, we restrict attention to firm quality $z \leq \bar{z}_2$ to analyze whether the banks have incentives not to implement the collusion-free contracts. We proceed via several lemmata.

Lemma 2. *There is a unique threshold of firm quality $\tilde{\theta} < \bar{z}_2$, which depends on whether loan officer j is honest or not, such that for any $z \geq \tilde{\theta}$ there is an (NC, NC) equilibrium.*

Proof. First consider the case when loan officer j is strategic, and hence, bank j must impose the no-collusion constraint. Consider an (NC, NC) equilibrium and we examine if bank i can profitably deviate by not implementing the collusion-free contract, i.e., a contract that does not adhere to the no-collusion constraint (8). Recall that collusion possibility emerges if loan officer i is successful in monitoring, whereas loan officer j is not. Thus, if the equilibrium involves loan officer i colluding with the borrower, then this happens with probability $m_i(1-m_j)$. The Nash bargaining solution for the optimal bribe, for a given distribution $(\beta, 1-\beta)$ of bargaining power between loan officer i and the firm, is given by:

$$b_i^*(z) = \max\{0, 2\beta(pr-z)+(1-\beta)prs_i\}.$$

Thus, the participation constraint of loan officer i is given by:

$$\{m_j+(1-\beta)m_i(1-m_j)\}prs_i + 2\beta m_i(1-m_j)(pr-z) - \frac{1}{2}cm_i^2 \geq 0. \tag{A.2}$$

Given that bank i does not implement the collusion-free contract, it would optimally set $s_i = m_i = 0$, which is equivalent to not employing a loan officer.[3] Thus, bank i solves

$$\max_{m_i \in [0, 1]} m_j pr - 1,$$

[2] The expression of $dm^0(v)/dv$ at any $\beta \in [0, 1]$ is cumbersome and hence, is omitted.

[3] Bank i can also deviate to any $m_i > 0$ as long as it satisfies (A.2) with $s_i = 0$. This situation is equivalent to the bank offering no salary to loan officer i who is free to exert a positive level of monitoring effort, and consume a bribe if he succeeds in monitoring and the other loan officer does not.

subject to (A.2). Recall that in the (NC, NC) equilibrium both banks monitor at the level

$$m_2(z) = \frac{8(pr-z)}{c+4(pr-z)}$$

for $z \leq \bar{z}_2$ [cf. Proposition 5]. Thus the deviation payoff for bank i is computed by substituting $m_i = 0$ and $m_j = m_2(z)$, which is given by:

$$\tilde{B}_i(z) = \frac{8pr(pr-z)}{c+4(pr-z)} - 1.$$

Recall from Proposition 5 that the payoff of bank i in the (NC, NC) equilibrium is given by:

$$B_i(z) \equiv B_2(z) = \frac{16c(2z-pr)(pr-z)}{(c+4(pr-z))^2} - 1.$$

Consequently, bank i does not deviate from (NC, NC) if and only if

$$B_i(z) \geq \tilde{B}_i(z) \quad \iff \quad z \geq \frac{pr(3c+4pr)}{4(c+pr)} \equiv \theta^d.$$

To summarize, for all $z \geq \theta^d$, the equilibrium outcomes in Proposition 5 continue to constitute an equilibrium. In other words, the collusion-proofness principle holds for all $z \geq \theta^d$ in the sense that there is one equilibrium where imposing the no-collusion constraints is without loss of generality.

Next, consider the case when loan officer j is honest, and hence, bank j does not require to impose the no-collusion constraint, i.e., it implements the collusion-free contract at no cost. The proof in this case is very similar to the previous one. In the (NC, NC) equilibrium, we have $m_i = m_j = m_2(z) = pr/(c+pr)$ (the argument mimics that of Proposition 5(a) for the case when $z \geq \bar{z}_2$), and hence, $\pi_2(z) = pr(2c+pr)/(c+pr)^2$ and $prs_i = 2(pr-z)$. Thus, by implementing the collusion-free contract, bank i obtains

$$B_i(z) = \frac{pr(2c+pr)(2z-pr)}{(c+pr)^2} - 1,$$

whereas by deviating to $m_i = s_i = 0$, bank i has a payoff of

$$\tilde{B}_i(z) = \frac{p^2r^2}{c+pr} - 1.$$

Thus, bank i does not deviate from (NC, NC) if and only if

$$B_i(z) \geq \tilde{B}_i(z) \quad \iff \quad z \geq \frac{pr(3c+2pr)}{2(2c+pr)} \equiv \theta^h.$$

It is immediate to show that $\theta^h < \theta^d < \bar{z}_2$, i.e., the range of values of firm quality over which (NC, NC) is an equilibrium is larger when loan officer j is honest. Depending on the context, we have either $\tilde{\theta} = \theta^d$ (when loan officer j is strategic), or $\tilde{\theta} = \theta^h$ (when loan officer j is honest). $\square$

We next establish that there exists a cutoff of firm quality, $\hat{\theta}$, such that for any $z \leq \hat{\theta}$, there exists a (C, NC) equilibrium. Furthermore, we show that there are values of z for which multiple equilibria exist.

Lemma 3. *Let $\tilde{\theta} = \theta^d$ when loan officer j is strategic, and $\tilde{\theta} = \theta^h$ when loan officer j is honest. There is a unique threshold of firm quality $\hat{\theta}$ with $\tilde{\theta} < \hat{\theta} \leq \bar{z}_2$ such that for any $z \leq \hat{\theta}$ there is a (C, NC) equilibrium.*

Proof. Suppose first that loan officer j is not honest. From Lemma 2 it follows that there cannot be an equilibrium of type (NC, NC) for any $z \leq \theta^d$, and hence, (C, NC) is the only possible equilibrium for these values of z. Because in a (C, NC) equilibrium, we have $s_i = m_i = 0$ and $\pi(m_i, m_j) = m_j$, bank j solves

$$\max_{(m_j, s_j)\in[0,1]^2} m_j pr(1-s_j) - 1, \tag{A.3}$$

$$\text{subject to } m_j prs_j - \frac{1}{2}cm_j^2 \geq 0,$$

$$prs_j \geq 2(pr-z). \tag{A.4}$$

The above program is similar to the one of single-bank lending with the only modification that the no-collusion constraint is (A.4) instead of (2). The optimal monitoring effort of loan officer j is given by:

$$m_j^C(z) = \begin{cases} 1 & \text{for } z_j^{min} \leq z \leq z_2^0, \\ \frac{4(pr-z)}{c} & \text{for } z_2^0 < z \leq \bar{z}_j, \\ \frac{pr}{c} & \text{for } \bar{z}_j < z \leq \bar{z}_2, \end{cases}$$

where $z_j^{min} = (pr+1)/2$ such that $B_j(m_j^C(z)) \geq 0$ for all $z \geq z_j^{min}$, $z_2^0 = pr - \frac{c}{4}$ and $\bar{z}_j = 3pr/4 < \bar{z}_2$. Bank i's expected payoff at $s_i = 0$ is given by:

$$B_i^C(z) \equiv m_j^C(z)pr - 1 = \begin{cases} pr-1 & \text{for } z_j^{min} \leq z \leq z_2^0, \\ \frac{4pr(pr-z)}{c} - 1 & \text{for } z_2^0 < z \leq \bar{z}_j, \\ \frac{p^2r^2}{c} - 1 & \text{for } \bar{z}_j < z \leq \bar{z}_2. \end{cases}$$

In order to establish that such an equilibrium exists, we have to verify that, given $m_j^C(z)$, bank i has no incentive to deviate to a collusion-free contract. We first compute bank i's payoff from such a deviation when it solves the maximization problem (13) taking into account that $m_j = m_j^C(z)$. When the no-collusion constraint of loan officer i binds, bank i would choose $\tilde{m}_i(z) \equiv m_i(m_j^C(z))$, where $m_i(m_j)$ is given by (14), i.e.,

$$cm_i^2 = 4(pr-z)\pi(m_i, m_j).$$

On the other hand, when the no-collusion constraint of loan officer i does not bind, bank i would choose $\bar{m}_i(z) \equiv m_i(m_j^C(z))$ where $m_i(m_j)$ solves (6), i.e.,

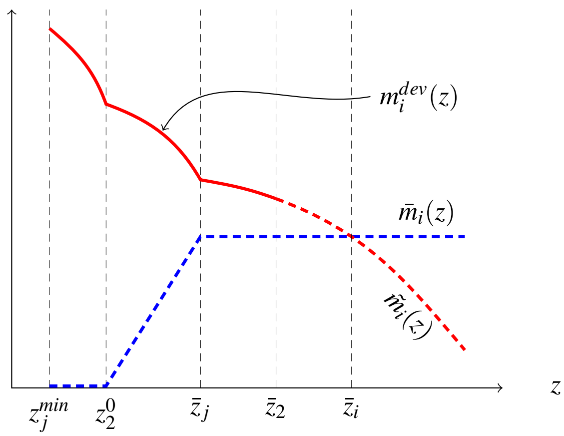
$$cm_i = pr(1-m_j).$$

Note that $\tilde{m}_i(z)$ is strictly decreasing in z because $m_j^C(z)$ is decreasing in z, and $m_i'(m_j) > 0$ (i.e., m_i and m_j are strategic complements) when the best reply is given by (14). By contrast, $\bar{m}_i(z)$ is non-decreasing in z because $m_j^C(z)$ is decreasing in z, and $m_i'(m_j) < 0$ (i.e., m_i and m_j are strategic substitutes) when the best reply of bank i is given by (6). The deviation strategies $\tilde{m}_i(z)$ and $\bar{m}_i(z)$ are depicted in the left panel of Figure 10, and they intersect each other only once at $z = \bar{z}_i$, which is given by:

$$\bar{z}_i = \frac{pr}{4}\left(3 + \frac{c^2}{2c^2 - 2cpr + p^2r^2}\right).$$

It is easy to show that $\bar{z}_2 < \bar{z}_i$ under Assumption 1. Hence, the monitoring level set by bank i in the above deviation strategy is given by $m_i^{dev}(z) \equiv \max\{\bar{m}_i(z), \tilde{m}_i(z)\} = \tilde{m}_i(z)$, which is the solid curve in the left panel of Figure 10.[4] Also, because the no-collusion constraint of loan officer i binds at this deviation strategy, we have $s_i(z) = 2(1 - \frac{z}{pr})$.

[4]Note that if $\tilde{m}_i(z) > \bar{m}_i(z)$, then the optimal monitoring level is given by $\tilde{m}_i(z)$ because for any lower level of monitoring the no-collusion constraint would be violated (as it binds at $\tilde{m}_i(z)$ and there is a monotonic relationship between m_i and s_i).



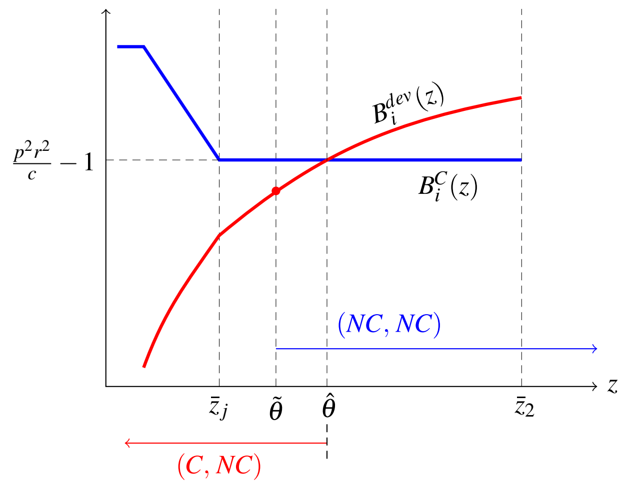


Figure 10: *Equilibria when loan officer j is not honest. In the left panel, the solid curve represents $m_i^{dev}(z)$. The right panel depicts $B_i^C(z)$ and $B_i^{dev}(z)$. Both banks offer collusion-free contracts when firm quality is high, i.e., $z \geq \tilde{\theta}$. On the other hand, at least one bank does not implement the collusion-free contract at any $z \leq \hat{\theta}$. There are two equilibria—namely, (NC, NC) and (C, NC) for all $z \in [\tilde{\theta}, \hat{\theta}]$.*

We next turn to compare bank i's expected profit in the (C, NC) equilibrium, $B_i^C(z)$ with $B_i^{dev}(z)$, its payoff from the deviation strategy $m_i^{dev}(z)$, which is given by:

$$B_i^{dev}(z) = \pi(\tilde{m}_i(z), m_j^C(z)) \underbrace{(2z - pr)}_{pr(1-s_i(z))} - 1.$$

In order to show the existence of $\hat{\theta}$, however, it suffices to concentrate only on the values of $z \in [\bar{z}_j, \bar{z}_2]$. The reason is the following. Note that $B_i^{dev}(z)$ is defined over three disjoint intervals of z as $m_j^C(z)$ is a piecewise function. It can be shown that $B_i^{dev}(z)$ is increasing on $[z_j^{min}, \bar{z}_2]$.[5] On the other hand, $B_i^C(z)$ is non-increasing in z with $\min_z B_i^C(z) = p^2r^2/c - 1$ which is a constant function for all $z \in [\bar{z}_j, \bar{z}_2]$. Both the functions $B_i^{dev}(z)$ and $B_i^C(z)$ are drawn in the right panel of Figure 10. We shall establish that there is a unique $\hat{\theta} \in (\bar{z}_j, \bar{z}_2]$ with the desired property. First, note that $B_i^{dev}(\bar{z}_j) < B_i^C(\bar{z}_j) = \frac{p^2r^2}{c} - 1$ because

$$\text{sign}\left[B_i^C(\bar{z}_j) - B_i^{dev}(\bar{z}_j)\right] = \text{sign}\left[c^2 + 2cpr - p^2r^2 - (c - pr)\sqrt{5c^2 - 2cpr + p^2r^2}\right],$$

which is strictly positive under Assumption 1. Thus, the fact that $B_i^{dev}(z)$ is strictly increasing on $[\bar{z}_j, \bar{z}_2]$ guarantees the existence of a unique $\hat{\theta} \in (\bar{z}_j, \bar{z}_2]$. For this we have to compare $B_i^{dev}(\bar{z}_2)$ with $B_i^C(\bar{z}_2) = p^2r^2/c - 1$. It turns out that $B_i^{dev}(\bar{z}_2)$ can be higher or lower than $p^2r^2/c - 1$.[6] If we have, $B_i^{dev}(\bar{z}_2) > B_i^C(\bar{z}_2)$, the situation that is depicted in the right panel of Figure 10, then there is a unique $\hat{\theta} \in (\bar{z}_j, \bar{z}_2)$ such that $B_i^{dev}(\hat{\theta}) = B_i^C(\hat{\theta})$. So, bank i does not deviate to a collusion-free contract if and only if $z \leq \hat{\theta}$. If, on the other hand, we have $B_i^{dev}(\bar{z}_2) \leq B_i^C(\bar{z}_2)$, then bank i cannot profitably deviate to a collusion-free contract for any $z \leq \bar{z}_2$. In this case, set $\hat{\theta} = \bar{z}_2$. So, we have a unique $\hat{\theta} \in (\bar{z}_j, \bar{z}_2]$ so that (C, NC) is an equilibrium for all $z \in [z_j^{min}, \hat{\theta}]$.

Recall from Lemma 2 that (NC, NC) is an equilibrium for all $z \geq \tilde{\theta} = \theta^d$. So, there are multiple equilibria on $[\tilde{\theta}, \hat{\theta}]$ if and only if $B_i^{dev}(\tilde{\theta}) < B_i^C(\tilde{\theta}) = p^2r^2/c - 1$, which is always the case under Assumption 1.

[5] Bank i's deviation payoff, $B_i^{dev}(z) = 2z - pr - 1$ on $[z_j^{min}, z_2^0]$, which is a strictly increasing function. This is because $m_j^C(z) = 1$, and hence, $\pi(m_i(m_j^C(z)), m_j^C(z)) = 1$ for all $z \in [z_j^{min}, z_2^0]$. On the other hand, it is piecewise concave and increasing on $(z_2^0, \bar{z}_2]$. In fact $B_i^{dev}(z)$ for $m_j^C(z) = pr/c$ is maximized at $z = \bar{z}_i$ which is strictly greater than $\bar{z}_2$.

[6] There is a unique $\bar{\gamma} \in (1 + c/2, c)$ such that $B_i^{dev}(\bar{z}_2) > p^2r^2/c - 1$ for $c > 6.26$ and $1 + c/2 \leq pr < \bar{\gamma}$. On the other hand, if (a) $2 < c \leq 6.26$ and $1 + c/2 \leq pr \leq c$ or (b) $c > 6.26$ and $\bar{\gamma} < pr \leq c$, then $B_i^{dev}(\bar{z}_2) \leq p^2r^2/c - 1$.

When loan officer j is honest, bank j would solve the maximization problem (A.3) without the no-collusion constraint (A.4). The optimal monitoring set by bank j is given by:

$$m_j^C(z) = \frac{pr}{c} \quad \text{for all } z \leq \bar{z}_2.$$

The rest of the proof is similar to the case when loan officer j is strategic. It is worth mentioning that $\hat{\theta}$ remains the same even if loan officer j is honest instead of being strategic. However, Lemma 2 asserts that $\theta^h < \theta^d$. Thus, there are two main differences between the cases when loan officer j strategic and he is honest. When monitor j is honest, (i) (NC, NC) is an equilibrium over a larger range of values of z, and (ii) consequently, the range of values of z, over which both equilibria exist, shrinks. $\square$

Combining the preceding two lemmata, we have Proposition 13.