Competition for Managers and the Rise in Skill Premium*

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February 6, 2024

Abstract

This paper examines the growing role of managerial occupations in influencing wage inequality and the skill-premium in the U.S. economy. Following the literature on managerial practices, we argue that managers not only increase firm productivity but also influence the wage dynamics of high-skill workers. Using the American Community Survey data from 1950 to 2019, we document a significant growth in managerial occupations from 4% to 20% and a corresponding rise in the wage premium for these roles from 39% to 100% with respect to low-skill workers. Additionally, our analysis, when augmented with the relative supply of managers, shows a positive correlation between the expansion of managerial roles and the wages of high-skill workers. We explain these facts by introducing a standard general equilibrium model where we highlight how firms' competition for managerial services affects both the compensation of managers and of high-skill workers. Our model successfully replicates the observed U.S. wage trends from 1950 to 2019 and suggests that the inclusion of managers can explain a significant portion of the observed increase in the skill-premium, reducing the emphasis on exogenous skill-biased technical change.

JEL classification: D43, D50, D24, E24, J21

*We would like to thank valuable comments from Issac Baley, Michele Boldrin, Matthias Hoelzlein, Victoria Nuguer, Jordi Paniagua, Ricardo Reyes-Heroles, Cesar Sosa Padilla, Mitchell Valdes, and the participants at the (Please mention the ITAM-ND conference and the Midwest Macro) ITAM's macro-readings for useful comments. All errors and omissions are our own responsibility.

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Keywords: Skill-premium, managers compensation, general equilibrium, labor force composition

1 Introduction

Managerial occupations represent a significant and expanding segment of the US labor force. At the same time, good managerial practices enhance production efficiency. Competition among firms to hire managers' services influences their remunerations depending on the technological contribution of a manager to efficiency and on the scarcity of these services in the market. Moreover, since the improvement in the efficiency of production also changes firms' demand for other factors of production, the general compensation of other types of labor, particularly high-skill workers, also change in a market economy. Given the extensive and expanding body of research on the rise of wage inequality in the US economy[1], this paper adds to the literature by analyzing how the increase in managerial compensation can be accommodated with an expansion of their services and how these contribute to the increase in the skill-premium.

Answering this question requires bridging two related literatures. On one hand, studies such as Ichniowski, Shaw, and Prennushi (1997), Syverson (2011),or Bloom, Sadun, and Van Reenen (2012) show that managerial practices influence firms productivity. This literature highlights that the proliferation of effective managerial practices and the increased delegation of CEO tasks to middle management positions are associated associated with heightened firm sales, value added, and employee compensation. On the other hand, economist have been documenting a raise in the skill premium, linked with growing wage inequality between low-skill and high-skill workers, even amid an increased relative supply of the latter. Typical explanations for this phenomenon rely on models that add various forms of skill-biased technical change that stimulate higher demand for high-skill labor, thereby widening the wage gap. Several authors have proposed different forces that may trigger this type of technological shift in favor of high-skill labor, for example, exogenous technological growth, capital-skill complementaries, or structural change (Tinbergen, 1974, Krusell, Ohanian, Ríos-Rull, and Violante, 2000, or Buera, Kaboski, Rogerson, and Vizcaino, 2021). In the present study, we partially endogenize this skill-biased technical change by allowing managers to enhance firms' productivity. We show that a simple model that accommodates this role for managers can account for most of the increase in the high-skill premium as observed in the data, while simultaneously generating a trajectory for managerial compensation in the US economy.

[1] A non-exhaustive list include Tinbergen (1974), Katz and Murphy (1992), Krusell, Ohanian, Ríos-Rull, and Violante (2000), Acemoglu and Autor (2011), Carneiro and Lee (2011), or Card, Cardoso, Heining, and Kline (2018) among others.

Using census data from the American Community Survey (ACS), we start by documenting an expansion of the relative importance of managerial occupations in the US labor force. Between 1950 and 2019 we observed an increase of occupations related to managers from 4% to about 20%. At the same time, the managers' premium, measured as the relative hourly wage of manager occupation relative to the wage of low-skill non-college educated workers increased from 39% to 100%. To inspect how the expansion in managerial positions also affected wages of high-skill workers we augment the typical Tinbergen (1974) regressions that project the log wage premium of high-skill workers (non-managers) against their relative supply and a time trend by also including the relative supply of managers. We find results consistent with the empirical literature that accounts the increase in the skill-premium to both supply factors (the relative abundance of high-skill workers) and demand factors (the skill-biased technical change captured by the time trend). However, when including the relative supply of managers in the regressions we find positive and significant coefficients and a diminished estimated role for the time trend. These results provide suggestive evidence consistent with a non-negligible role of managers at accounting for the observed patterns of the skill premium.

To study such relationship, we introduce a simple general equilibrium model where firms compete in a monopolistic environment and the skill-biased technological occurs endogenously. Our model consists of one representative household and two firms, each producing a distinct consumption good. These goods exhibit imperfect substitutability in the product market, characterized by monopolistic competition. The production process for each good relies on a standard constant-returns-to-scale technology, which utilizes both high- and low-skill workers as inputs. A unique aspect of our model is the introduction of managers as a third factor of production. Managers constitute a fixed proportion of the college-educated workforce, yet they play a distinct role within the production process compared to standard high-skill workers. Specifically, the presence of managers is intended to augment the productivity of high-skill workers relative to their low-skill counterparts. Consequently, within our model, the degree of skill-biased technological change in each firm is contingent upon the quantity of managerial resources available, and it exhibits an increasing relationship with the managerial stock. A novel aspect of our model is that, while firms remunerate "non-manager" high-skill workers at competitive wage rates, they actively engage in competition within the market for managers, vying for managerial services through bidding processes.

We analyze the sequential game of the model. In the first stage, firms engage in a bidding phase to lure managers. In the subsequent stage, they produce consumption goods using both high-skilled (non-manager) and low-skilled workers. In our model, managerial and non-manager high-skilled employment exhibits complementarity. We decompose the premium associated with higher education into two distinct components: the skill premium, which signifies the wages earned by non-manager high-skilled workers compared to low-skilled workers, and the managerial wage premium, representing the wages earned by managers compared to low-skilled workers. Our objective is to assess the impact of an increased supply of managers on both wage premiums. When the supply of managers within a firm rises, it leads to a higher relative demand for high-skilled workers. Consequently, the skill premium also increases in sync with the expansion of the managerial workforce.

The Nash equilibrium of the bidding game uniquely determines managerial compensation as a function of a firm's profit relative to what it would have achieved in the absence of managers. We identify three distinct channels through which an increase in managerial supply affects managerial compensation. First, there is a direct negative effect of a greater managerial supply, which tends to decrease the managerial wage premium. Second, there is a positive indirect effect—having more managers in one firm makes it more productive relative to the other, thus increasing the demand for managers. Consequently, higher managerial employment tends to boost the wage premium for managers. Third, there is a negative indirect effect. Because higher managerial employment enhances the productivity of the high-skilled non-managerial workforce compared to low-skilled workers, firms have a higher relative demand for such inputs, which, in turn, tends to increase the skill premium. As a higher skill premium implies lower profits for firms, the managerial wage premium, which is paid out of a firm's profits, tends to decrease. The negative indirect effect that we identify emerges from the feedback within the general equilibrium economy under consideration. The final impact of an increase in managerial supply on their compensation depends on the relative strength of the above countervailing effects. We demonstrate that, depending on the parameter values −specifically, the marginal product of managerial employment and the degree of product substitutability −the managerial wage premium may decrease, increase, or vary non-monotonically with managerial supply.

This demonstrated that, within the model, the skill premium consistently increases, while the managerial wage premium may decrease, increase, or exhibit a non-monotonic pattern in response to an increase in managerial supply. Consequently, the premium associated with higher education may decrease, increase, or follow a non-monotonic trajectory following an expansion in higher-education employment. This conclusion aligns with findings in the literature on endogenous skill-biased technological change, as seen in studies such as Kiley (1999) and Acemoğlu (2002). However, the innovation in our approach lies in the following aspect: We partition the labor force with college education into two distinct categories –namely, non-manager high-skill workers and managers– and treat them as separate entities within the production process based on the tasks they undertake. As a result, the complementarity between managers and non-managers helps elucidate the fluctuations observed in the premium associated with higher education over the past few decades.

With the model structure, we then perform a quantification of the mechanisms highlighted relying on a standard calibration strategy. To do this, we input the model with the observed sequence of relative supply of low-skill, high-skill, and managerial labor as observed in the US Census data. Additionally, we add an additional auxiliary exogenous skill-biased technical change free-variable that allow us to match exactly the observed path of the high-skill premium. In this exercise, we find that the model mimics the pattern of the managers' premium displayed in the US economy between 1950 and 2019. At the same time, albeit present, we find a diminished role of the exogenous skill-biased technical change. This indicates that the inclusion of managers in an, otherwise, standard model of income distribution in a economy can account for a large part of the observed increase in the skill-premium.

Related literature This paper contributes to the extensive literature addressing the rise in the skill-premium within the US economy. A central challenge in this literature is reconciling the simultaneous increase in the relative supply of high-skill workers with their rising wages. Tinbergen (1974) provides an early explanation for this phenomenon, attributing the widening gap in wages to an augmented demand for high-skill workers. He posits that this demand surge is a result of exogenous improvements in the production efficiency of these workers. Such findings have been confirmed in other influential studies, for instance Katz and Murphy (1992) or Acemoglu and Autor (2011) among others.

Finding the explanation of an exogenous skill-biased technical change incomplete, several explanations have emerged to endogenize the surge in demand for high-skill workers. Some authors have highlighted the significance of skill-biased structural change, where development and non-homothetic preferences induce households to consume more consume goods that intensively use high-skill labor (e.g. Buera, Kaboski, Rogerson, and Vizcaino, 2021; Jaimovich, Rebelo, Wong, and Zhang, 2020). Other authors have focused into the role of capital deepening, particularly in cases displaying capital-skill complementarities (e.g. Krusell, Ohanian, Ríos-Rull, and Violante, 2000; Burstein and Vogel, 2017; Acemoglu and Restrepo, 2022; Caunedo, Jaume, and Keller, 2023). In such instances, the accumulation of capital enhances the productivity and task-performing abilities of high-skill workers, further increasing their demand. In contrast to these perspectives, our paper takes a distinctive approach, centering on the complementarities that arise from incorporating additional managerial content in production units. This approach not only allows us to account for the rise in the skill-premium but also provides insights into the patterns of managerial compensation.

In light of the evolution of managerial compensation, particularly that of top-level executives like CEOs, over the past few decades, several theories have been proposed to elucidate the trends and fluctuations in executive pay. Three distinct perspectives can be related with the contributions advanced in this paper.[2]

The first group of theories attempts to establish a connection between CEO compensation and firm size. Following the 'span of control' theories Lucas (1978), Gabaix and Landier (2008) and Terviö (2008) analyze an assignment model that links managerial talent with firm size to elucidate the dynamics of top executive pay. These authors assume that more talented CEOs exhibit a comparative advantage in larger firms, making large firms more appealing to such CEOs through a positive assortative matching pattern. Consequently, as firms expand over time, the distribution of executive compensation becomes increasingly positively skewed. Building upon the assignment models, Bao, De Loecker, and Eeckhout (2022) explore the interplay between CEO pay, firm size, and the structure of the product market. In their analysis, they incorporate managerial talent into the production process, where managerial ability enhances a firm's total factor productivity (TFP). Bao et al. (2022) discover that, on average, market power accounts for 45.8% of the total variation in top executive compensation between 1994 and 2019. While the models examined by Gabaix and Landier (2008) and Terviö (2008) employ a reduced-form approach, Bao, De Loecker, and Eeckhout (2022) integrate managerial talent directly into the production function of firms. This approach provides a microfoundation for understanding how managerial talent influences firm productivity and profitability, consequently leading to the dependency of CEO compensation on firms' market power. In this regard, our modeling choice aligns with that Bao, De Loecker, and Eeckhout (2022). However, our specific focus centers on analyzing the relationship between the return to skill and the supply of managers, and not the influence

[2]See Edmans, Gabaix, and Jenter (2017) for an excellent survey of the literature.

of firm size or market power in determining the level of managerial compensation schemes.

The second cluster of theories focuses on how the incentive structure of CEO compensation influences its level. Enhancing the sensitivity of executive income to performance is a means of aligning managerial incentives with those of shareholders. In their analysis of a dataset encompassing top executives in large firms from 1936 to 2005, Frydman and Saks (2010) attribute the strong correlation between executive compensation levels and firm growth since the 1980s to the increased utilization of incentive-based compensation, such as stocks and stock options. While our model does not revolve around the agency problem between shareholders and managers, the pay structure that emerges from the bidding mechanism bears resemblance to managerial incentive theories. In our models, managers receive compensation derived from the firm's profits.

Finally, the rent extraction view posits that poor corporate governance amplifies managers' capacity to siphon corporate resources (Bertrand and Mullainathan, 2001; Bebchuk and Fried, 2004). This results in a substantial upsurge in CEO compensation. Consequently, executive pay tends to be elevated during periods of weak governance. The current model similarly leads to rent extraction by management. However, this occurs as a Bertrand-like outcome in the bidding equilibrium, without necessitating a reliance on the quality of the corporate environment.

The rest of the paper is organized as follows. Section 2 provides empirical motivation for the mechanisms highlighted in the model that is introduced in section 3. Next, in section 4, we use a calibrated version of the simple model to assess the quantitative validity of the role of managers at explaining the skill premium. We provide conclusions in section 5.

2 Empirical motivation

In this section, we outline the main trends of the wage distribution and occupational choice in the US labor market since the 1950s, with a particular focus on workers with a high school education or less compared to those with a college education or more. Consistent with previous studies[3], we observe an increase in the wage premium of college-educated workers despite an increase in its relative supply. This pattern is particularly pronounced when we restrict our sample to college-educated workers in managerial and related occupations.

[3] See, for example, Tinbergen (1974), Katz and Murphy (1992), Krusell, Ohanian, Ríos-Rull, and Violante (2000), Autor, Katz, and Kearney (2008), Carneiro and Lee (2011), or Hoffmann, Lee, and Lemieux (2020).

Moreover, we examine the cross-sectional variation in these trends and document that sectors with larger increases in the relative supply of managers correspond to sectors with faster growth in the wage premium for non-manager, college-educated workers.

2.1 Data

To document the facts presented we rely on the US Census samples and the American Community Survey (ACS).[4] The US Census data encompasses 1% samples of the US population for the years 1950, 1960, 1970, 1980, and 1990. For years following 2000, we use the ACS, an annual survey conducted by the Census Bureau. The ACS has similar questions to the Census and provides a 1% random sample of the entire U.S. population. To maintain consistency with the US Census, we include ACS data from 2000, 2010, and 2019 in our analysis.[5] One key advantage of the data provided by the U.S. Census/ACS is its substantially larger sample sizes compared to alternative datasets such as the Current Population Survey (CPS), typically used to measure the skill premium. This allows for a detailed analysis of the changes in wages and occupational employment across different sectors while controlling for fine-grained individual characteristics.

Each observation contains information on an individual's demographics, education, occupation, sector of activity, wage income, and work hours. We limit our sample to employed workers aged between 25 and 60 years old at the time of the survey. Individuals are categorized into five educational groups (less than high school, high school, some college, college, and more than college), four groups of potential experience (9 or less years, 10 to 19 years, 20 to 29 years, and more than 29 years), and by sex (male, or female). We also distinguish observations between managerial and non-managerial occupations using a harmonized coding scheme based on the Census Bureau's 2010 ACS as provided by IPUMS. Occupations associated with 'management, business, science, and arts', 'business operations specialists', and 'financial specialists' are included in our broad category of managerial roles.

The data is further restricted to full-time workers, who report more than 40 weeks of work over the previous year. Observations with hourly wages that are 50% below the federal minimum wage or with a weekly wage smaller than $50, adjusted for inflation using

[4]Data from both the US Census and the ACS can be retrieved from the IPUMS website (https://usa.ipums.org).

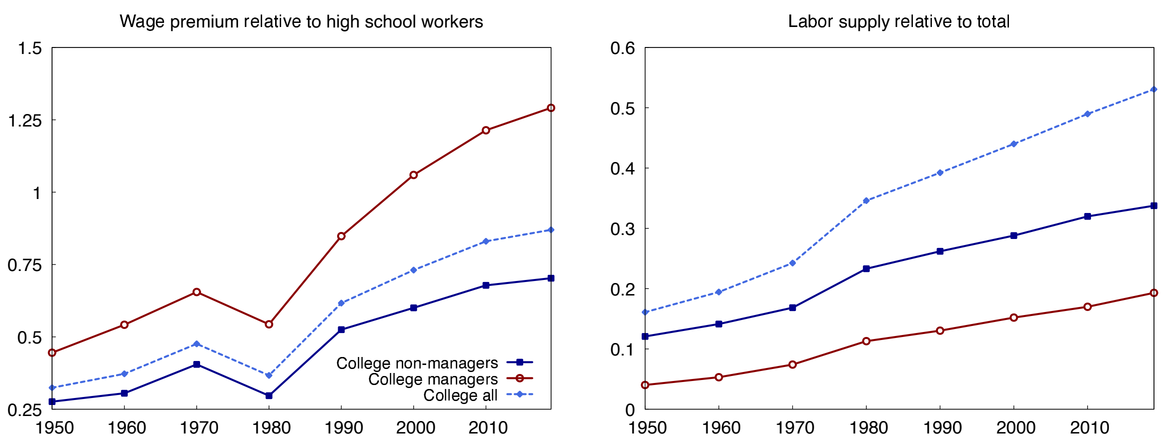
[5]The inclusion of the ACS survey of 2019 instead of the one from 2020 is related with the impact of the covid-19 epidemic on the collection and data quality that induced the Census Bureau to use experimental weights on the observations.

the Consumer Price Index (CPI) with 1982 as the base year, are also dropped. Workers are categorized into sectors of activity using the 1990 Census Bureau industrial classification scheme, which allows for sector harmonization across years. This enables us to divide our data into 12 sectors.[6] It is with this individualized data on hourly wages and total hours worked that we compute relative wages and relative labor supply across various educational and occupational groups in the subsequent analysis.

2.2 Managers intensity and the college education wage premium

As our objective is to examine trends in the skill premium and managerial compensation, we first illustrate wage dynamics and work hours over the past seven decades in the US economy. We first define 'skilled' and 'unskilled' workers as those with completed college education or higher, and those with incomplete college education or less, respectively. Within the 'skilled' category, we further distinguish between 'managerial' and 'non-managerial' occupations based on our classification. For each group, we compute total yearly hours as the product of reported weeks of work over the year with the typical number of hours worked in a week. Hourly wages are determined by dividing total annual labor income by the total hours worked.

Figure 1: Composition-adjusted wages and relative labor supply across education and occupations



Source: US Census samples and the American Community Survey.
Notes: The average wages and hours supply of different groups follows the methodology used in Acemoglu and Autor (2011) as explained in section 2.2. The wage premium relative to high school workers is defined as the ratio of the average wage of a particular group relative to the average wage of high-school workers normalized to zero (when $w_t^i/w_t^{\text{high}school} = 1$, the wage premium is 0). The labor supply relative to total just gives the ratio of the total hours worked in a group relative to total hours worked in the economy. All the occupations that are considered as managers can be found in the appendix A.1.2.

[6] The appendix A.1 lists the included sectors and provides additional details about the data cleaning.

To measure average wages within groups, we adopt the methodology used by Acemoglu and Autor (2011), computing composition-adjusted wages in the following manner. First, for each year, we project log real wages onto dummy variables that capture our previously defined groups, specifically, two sexes, five educational groups, four potential experience brackets, and two occupations. Second, we calculate mean wages for broader groups (managers, high, and low skill workers), holding constant the relative employment shares of our 40 labor groups across all sample years. This method ensures that changes in average wages are not the result of shifts within the narrow groups' composition of sex, education, experience, or occupation. Figure 1 plots the trend of composition-adjusted wages for college workers relative to high school workers from 1950 to 2019, as well as the relative supply of hours.[7]

The figure illustrates trends in the skill premium and relative labor supply similar to those documented in other studies.[8] Specifically, we observe an increase in the wage gap from 32% in 1950 to 83% in the 2010, and 87% in 2019. At the same time, the supply of college educated relative to the total supply of workers increased from 16% in 1950 to 49% and 53% in 2010 and 2019, respectively. This pattern becomes even more pronounced when we differentiate college-educated workers into managerial and non-managerial occupations. Between 1950 and 2019, the wage premium of managers soared from 45% to 129%, while the relative labor supply of college-educated managers rose from 4% to 19%.

Similar results emerge when we measure composition-adjusted wages within each broad sector in the sample. Table 1 shows the evolution of these measures between 1950 and 2019. We observe common general trends across sectors where both the wage premium and the relative supply increase for managers and non-managers. However, these increases vary considerably across sectors. For instance, the increment of managerial employment ranges from 5 percentage points (retail sector) to 34 percentage points (finance), while the premium increase ranges from 8 percentage points (agriculture) to 96 percentage points (manufacturing). A similar pattern is evident for college-educated non-managerial workers, albeit with less intensity. Notably in figure 2, we observe a clear cross-sectoral positive correlation between wages of college-educated non-manager' and both managers' wages or managers' relative

[7]The results depicted in figure 1 are truncated at 2019 instead of 2020 due to the utilization of experimental weights in that year, a response to the global pandemic's impact on the survey data collection. Using the year of 2021 (already available in the ACS dataset) instead of 2019 does not change any of the main results presented in this section (see appendix A.1.4 for more details).

[8]For example, Autor, Katz, and Kearney (2008), Acemoglu and Autor (2011), or Autor (2022) also document trends in the skill premium and relative labor supply with similar patterns and magnitudes to the ones presented in figure 6.

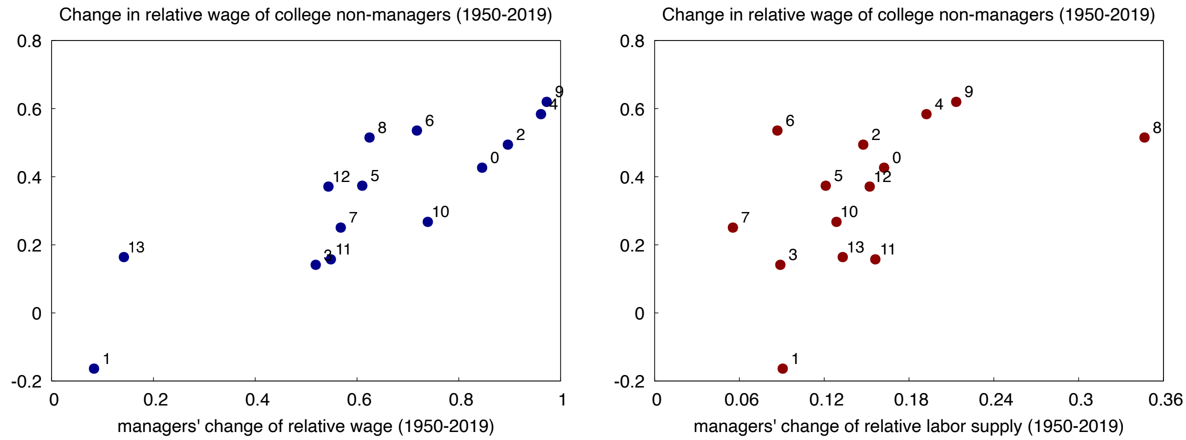
Table 1: Across-sectoral trends in composition-adjusted wages and relative labor supply (1950/2019)

		Managers				Non-managers			
		%wage gap		%labor supply		%wage gap		%labor supply	
	Sector	1950	2019	1950	2019	1950	2019	1950	2019
1	Agriculture and fisheries	71	79	3	11	78	62	1	17
2	Mining	41	130	3	18	31	80	9	19
3	Construction	32	84	3	12	16	30	5	8
4	Manufacturing	72	168	4	22	32	90	8	24
5	Transportation and comm.	51	112	2	13	21	58	5	22
7	Retail and wholesale trade	50	107	4	9	25	50	4	22
8	Finance and real estate	47	109	9	43	16	68	13	27
9	Business and repair serv.	63	160	5	26	31	93	9	35
10	Personal services	45	119	3	14	12	39	3	18
11	Entertainment	30	84	5	20	24	40	8	31
12	Professional serv.	68	122	4	18	39	76	56	54
13	Public administration	54	68	4	17	28	44	16	41
0	All sectors	45	129	4	19	28	70	12	34

Source: US Census samples and the American Community Survey.
Notes: The definitions and methodology associated with the measures of the wage gap and the labor supply correspond the to wage premium and the relative supply as explained in the footnote of figure 6. The listed sectors in the table correspond to the classification used in the American Community Survey as provided by IPUMS.

labor supply. This suggests a potential interaction between college-educated workers wages and the intensity of managers utilization and their respective compensation.

Figure 2: Correlation between wages and labor supply across sectors of activity



Source: US Census samples and the American Community Survey.
Notes: The definitions and methodology associated with the measures of the relative wage the relative labor supply are the same as the ones described in the footnote of figure 6. The listed sectors in the table correspond to the classification used in the American Community Survey as provided by IPUMS.

To better understand the relationship between the observed patterns of the college-educated non-manager premium, we adopt the standard analysis from Tinbergen (1974).[9] This analysis is inspired by a canonical model of the labor market, in which the evolution of the college premium is explained through the effect of skill-biased technical change (demand factors) and the availability of high-skill to low-skill labor (supply factors). The forces associated the the demand and supply factors are usually measured through the recourse of linear regressions where the log wage premium for college-educated workers is regressed against a time trend and their relative supply. In this paper, given our interest on role of managers, we augment the standard estimation equation by including the relative supply of managers. Our main regression specification estimates:

$$w_{jt}^{college} = \alpha + \beta_0 \times t + \beta_1 \times h_{jt}^{college} + \eta \times h_{jt}^{managers} + \gamma_j + \epsilon_{jt}, \tag{1}$$

where $w_{jt}^{college}$ is the logarithm of the ratio of the average wage of college-educated (non-manager) workers to high-school workers at time t in sector j, $h_{jt}^{college}$ the logarithm of of the relative supply of college (non-managers) hours, and $h_{jt}^{managers}$ the relative supply of mangers' hours. Additionally, we include a common time trend variable t, and a sectoral fixed effect

[9]This framework has been applied by many authors studying the college premium, for example, Katz and Murphy (1992), Card and Lemieux (2001), or Acemoglu and Autor (2011).

γ_j. The residual of the regression is captured in ϵ_{jt}.

Table 2: Regression models for the college non-manager wage premium between 1950 and 2019

Dependent variable: log relative wage college to high school	(I)	(II)	(III)
Regressors:			
relative employment of college workers	.079***	-.074***	-.075***
	(0.016)	(.023)	(.022)
relative employment of managers			.541**
			(.228)
time trend		.0051***	.0040***
		(.0006)	(.0008)
sectoral fixed effects	Yes	Yes	Yes
Observations	96	96	96
R-squared	0.59	0.78	0.80

Source: US Census samples and the American Community Survey.
Notes: The average wages and hours supply of different groups follows the methodology used in Acemoglu and Autor (2011) as explained in section 2.2. The wage premium relative to high school workers is defined as the ratio of the average wage of a particular group relative to the average wage of high-school workers normalized to zero (when $w_t^i/w_t^{\text{high}school} = 1$, the wage premium is 0). The labor supply relative to total just gives the ratio of the total hours worked in a group relative to total hours worked in the economy. This figure uses a definition of managers that includes the occupations listed in table 6 of the appendix.

Table 2 displays the estimation results, which align well with the canonical model. Specifically, when we incorporate a time trend into the regression, the coefficient on the relative supply of college hours turns negative. A direct interpretation of the coefficients suggests that the trend variable captures demand effects via skill-biased technical change (with a positively estimated sign), while the relative employment of college-educated workers captures supply effects (with a negatively estimated sign). Augmenting the regression to include a relative employment of managers independent variable, does not change the pattern of correlations for the time trend or for the relative employment of college educated workers. However, the estimated sign for this new variable is positive and significant, suggesting that a higher prevalence of managers in a sector may increase demand for college-educated workers implying higher wages. Consistent with this interpretation is the 20% decrease in the estimated coefficient for the time trend between regression (II) and (III): after controlling for the relative employment of managers, the time trend effect on the relative wage of college educated workers becomes less important.

2.3 Summary

In this section we utilized ACS data to document a significant increase in the wage premium of high-skill labor in the US over the last 70 years. This increase in the wage premium is especially pronounced in managerial occupations. Concurrently, the relative supply of both managers and high-skill labor has also increased. These data trends are consistent with standard neoclassical theory, provided we account for the role of skill-biased technological demand factors. This is corroborated by our high-skill wage regression results, which show a negative coefficient for the supply of high-skill labor (indicating a negative supply effect) and a positive coefficient for a time trend (indicating a positive skill-bias technical change effect). However, when we augment these regressions to include the effect of the relative supply of managers, we uncover a positive impact of managers on high-skill wages and a reduced importance of the time trend. We find this evidence suggestive of a role of managers at explaining part of the dynamics of the skill-premium in the US.

3 A simple model

Given the empirical results documented in the previous section, we propose a simple model that ilustrates how changes in the supply of different type of workers (managers, high-skill, and low-skill) contribute to wage inequality. In our framework, we portray managers as agents who can directly influence firm productivity and, consequently, profits. Under some conditions, competition among firms for the limited availability of managers implies that managers' salaries can increase along with their supply, thereby also elevating the income of high-skill workers.

We analyze a general equilibrium economy with a monopolistically competitive product market wherein firms employ managers, high-skill, and low-skill workers to produce imperfectly substitutable goods. The economy comprises two classes of agents—a continuum of identical households of measure 1, and two firms (or sectors). Households consume two goods that are imperfect substitutes and supply three types of workers—namely, managers, high-skill workers and low-skill workers who are in fixed supplies, M, H and L, respectively. Each firm produces one good by employing all three types of workers. Think of the workforce of the economy consisting of 'college graduates' and 'high school graduates'. The high school graduates are designated as 'low-skill workers', whereas the college graduates comprise 'managers' and 'high-skill workers'. Unlike high- and low-skill workers, whom the firms hire in a competitive labor market, managers are hired in firms through a bidding process.

Preferences and technology. A representative household derives utility from the consumption of the two goods that are imperfect substitutes. The utility function is given by

$$U(x_1, x_2) \equiv \left(x_1^{\frac{\sigma-1}{\sigma}} + x_2^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}, \tag{2}$$

with $\sigma > 1$ representing the elasticity of substitution between the two goods. In a monopolistically competitive product market, σ determines the market power of the firms—the higher the product substitutability, the lower is the market power firms enjoy.

Firms (potentially) differ in their production technology which are given by

$$y_i = f(h_i,\, l_i;\, z_i) \equiv A\left(\alpha(z_i h_i)^{\frac{\zeta-1}{\zeta}} + (1-\alpha) l_i^{\frac{\zeta-1}{\zeta}}\right)^{\frac{\zeta}{\zeta-1}}, \tag{3}$$

Parameters A, α and $\zeta \leq \sigma$ are technology parameters: A represents a Hicks-neutral technological change, $\alpha \in (0,\, 1)$ is the factor intensity associated with high-skill workers, and $\zeta > 1$ is the elasticity of input substitution between high- and low-skill workers. The parameter z_i represents the firm-specific skill-biased technological change (SBTC). Our framework endogenizes z_i. In particular, firm i's SBTC is given by

$$z_i = z(m_i) \equiv z_0 + m_i^{\gamma}, \quad z_0,\, \gamma > 0. \tag{4}$$

The firm-specific SBTC depends on some firm attribute, z_0 which is same across both firms (e.g., firm size, baseline input) and the number of managers employed in firm i, m_i. Firm i produces good i by using employing managers, high-skill workers (in quantity h_i) and low-skill workers (in quantity l_i).

The timing of events. The economy lasts for two subperiods, $t = 1,\, 2$. At $t = 1$, firms hire managers from the pool of M managers. We assume that managers are not hired in a competitive labor market, rather through a bidding process. Each firm i 'bids' for the managers by offering per-manager salary w_m^i to employ m_i managers in the firm. Once m_i managers are employed in firm $i = 1,\, 2$, the SBTC, $z(m_i)$ becomes common knowledge. At date 2, each firm i hires high- and low-skill workers at competitive wages $(w_h,\, w_l)$, and carry out the production of good i. At the same time, the representative household submits the demand for each good by taking its prices as given. Finally, firms set prices by maximizing profits.

3.1 Equilibrium

The general equilibrium of our economy is solved sequentially in the two subperiods. We first determine the salary offers for each manager by the firms 1 and 2, (r_1, r_2), and the allocation of managers across firms, (m_1, m_2) such that $m_1 + m_2 = M$. The equilibrium of the stage 2, given (m_1, m_2), is a standard Walrasian equilibrium of the economy that comprises a monopolistically competitive product market where goods 1 and 2 are traded at prices (p_1, p_2), and a competitive labor markets where firms hire high-skill and low-skill workers at wages (w_h, w_l). We normalize w_l, the low-skill wage, to 1, and write $w_h \equiv w$.

In stage 2, the representative household submits demands for both goods by maximizing utility in (2), taking the prices (p_1, p_2) as given, i.e.,

$$(x_1(p_1, p_2), x_2(p_1, p_2)) \equiv \underset{\{x_1, x_2\}}{\operatorname{argmax}} \{U(x_1, x_2) \mid \text{subject to } p_1 x_1 + p_2 x_2 = I\}, \tag{5}$$

where $I > 0$ is the household income. Firm profits and the aggregate worker (managerial, high- and low-skill) incomes accrue to the households, and hence, the household income, I is the sum of the profits of the two firms, and aggregate worker incomes.

Firms optimize in two stages. First, each firm i employs high- and low-skill workers by minimizing cost, taking w and the production technology in (3) as given, i.e., firm i solves

$$C_i(y_i) \equiv \min_{\{h_i, l_i\}} \{wh_i + l_i \mid \text{subject to } y_i = f(h_i\, l_i;\, m_i)\}. \tag{6}$$

Next, firms compete in the product market. The market-clearing condition for each good implies $x_i(p_1, p_2) = y_i(p_1, p_2)$ for $i = 1, 2$. Thus, each firm i sets price p_i to maximize profit, i.e.,

$$\tilde{\pi}_i \equiv \max_{p_i} p_i y_i(p_1, p_2) - C_i(y_i(p_1, p_2)). \tag{7}$$

We solve the model by backward induction.

3.2 Analysis of the equilibrium

Households. The economy has 2 goods, 1 and 2. A representative consumer has utility function $U(x_1, x_2)$. We assume The consumer, given the prices (p_1, p_2), maximizes the above utility function subject to the budget constraint

$$p_1 x_1 + p_2 x_2 = I,$$

where I is consumer's income. Optimality implies

$$\frac{x_1}{x_2} = \left(\frac{p_2}{p_1}\right)^{\sigma}.$$

Substituting the above into the budget constraint one can derive the following demand functions:

$$x_1 \equiv x_1(p_1, p_2, I) = \frac{I}{P p_1^{\sigma}}, \tag{8}$$

$$x_2 \equiv x_2(p_1, p_2, I) = \frac{I}{P p_2^{\sigma}}, \tag{9}$$

where $P \equiv (p_1^{1-\sigma} + p_2^{1-\sigma})^{\frac{1}{1-\sigma}}$ is the composite price index, giving the expenditure associated with one unit of total utility. Equations (8) and (9) show that each good's expenditure decreases with its own relative price with respect to the price index at a constant elasticity σ, and increases with income at a unit elasticity.

Firms. Each good is produced by a distinct firm. Firms 1 and 2 are heterogeneous in terms of production technology. In particular, let $c_i > 0$ be the constant marginal cost of firm $i = 1, 2$. We assume that the product market is monopolistically competitive. Each firm i solves the following maximization problem (by taking the composite price index, P as given):

$$\tilde{\pi}_i = \max_{p_i} (p_i - c_i) x_i(p_i, p_j, I) \tag{10}$$

for $i \neq j$. The first-order condition (Ramsey rule) is given by:

$$p_i \left(1 - \frac{1}{\varepsilon_i}\right) = c_i, \tag{11}$$

where ε_i is the price elasticity of good i. It follows from (8) and (9) that $\varepsilon_1 = \varepsilon_2 = \sigma$. Therefore, (11) implies

$$p_i = \frac{\sigma}{\sigma - 1} c_i \quad \text{for } i = 1,\, 2. \tag{12}$$

Because $\sigma > 1$, we have $p_i > c_i$, i.e., price of good i is a constant mark-up over its marginal cost of production. The Lerner index of each firm is given by $1/\sigma$, i.e., market power of the firms decreases with product substitutability.

Normalizing the low-skill wage, w_l to 1, and denoting by $w \equiv w_h/w_l$ the high-skill wage premium, lemma 3.1 summarizes firms' factor demands and cost functions.

Lemma 3.1. *The (conditional) factor demand and cost functions of firm $i = 1,\, 2$ are given by*

$$h_i(w,\, y_i) = \frac{1}{A}\left(\frac{\alpha}{w}\right)^{\zeta} z_i^{\zeta-1} c_i^{\zeta} y_i, \tag{13}$$

$$l_i(w,\, y_i) = \frac{1}{A}(1-\alpha)^{\zeta} c_i^{\zeta} y_i. \tag{14}$$

Firm i's cost function is linear in y_i, and the associated constant marginal cost is given by

$$c_i \equiv c(w,\, z_i) = \frac{1}{A}\left(\alpha^{\zeta} z_i^{\zeta-1} w^{1-\zeta} + (1-\alpha)^{\zeta}\right)^{\frac{1}{1-\zeta}} \tag{15}$$

with c_i increasing in w and decreasing in z_i.

Proof. In the appendix A.2. □

3.3 Determination of high-skill equilibrium premium and aggregate income

Next, we determine the high-skill wage premium, w and household income, I in the equilibrium of the second stage, that is, conditional on z_i for $i = 1, 2$. This equilibrium can be found by applying market clearing conditions in the factor markets:

$$L = l_1(w, y_1) + l_2(w, y_2), \tag{16}$$

$$H = h_1(w, y_2) + h_2(w, y_2), \tag{17}$$

and in the product markets:

$$y_1 = x_1(p_1,\, p_2,\, I), \tag{18}$$

$$y_2 = x_2(p_1,\, p_2,\, I). \tag{19}$$

Equations (16)-(19) coupled with the firms' optimal price choice (12) and the definition of the marginal cost (15), yield a solution for all the endogenous variables in the economy, namely, prices $\{w, p_1, p_2\}$, quantities $\{y_1, y_2, l_1, l_2, h_1, h_2\}$, and income $\{I\}$. Moreover, one can show that this equilibrium is unique (see lemma A.2 in the appendix A.2).

Focusing on the high-skill wage premium, w, the next proposition characterizes its equilibrium interactions with variations in the SBTC and the relative supply of high-skill labor.

Proposition 3.1. *Given firm technologies $z_1 > z_2$ and $\sigma \geq \zeta$, the equilibrium high-skill wage premium w is increasing in z_i and decreasing in the relative supply of high-skill labor H/L.*

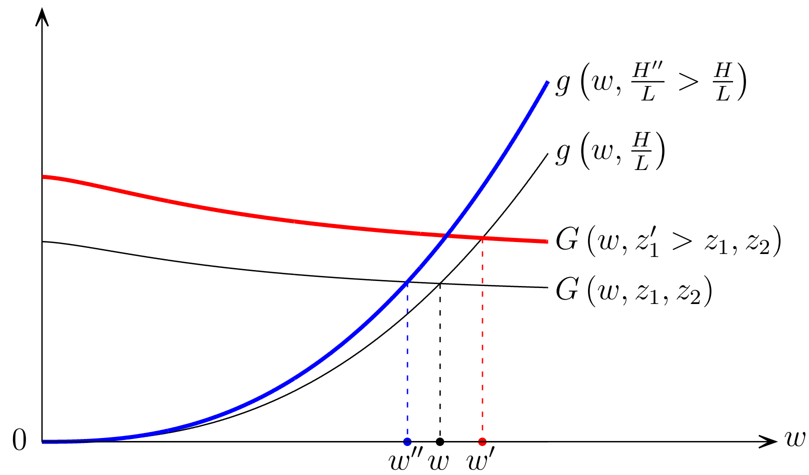
Proof. In the appendix A.2. □

This result, depicted graphically in figure 3, generalizes the Tinbergen (1974) model for an environment with two goods and monopolistic competition. In fact, when the consumer does not value good 2, the equilibrium condition becomes

$$\left(\frac{1-\alpha}{\alpha}w\right)^{\zeta}\frac{H}{L} = z_1^{\zeta-1}.$$

The intuition for how w changes in equilibrium is the same in both environments. An improvement in the efficiency of the high-skill labor through an increase in z_i, implies excess demand in the market that is resolved with a higher price of that factor. A similar argument can be made for an increase in H/L that generates negative excess demand and therefore a lower w.

Figure 3: Comparative statics of the equilibrium high-skill premium as a function of $(z_1,\, z_2, H/L)$.



Notes: This figure plots the functions $g(w, H/L)$ and $G(w, z_1, z_2)$ characterized in lemma A.2 for a parameterization where $z_1 > z_2$ and $\sigma > \zeta$. H/L refers to the relative supply of high-skill labor, while the z_i's refer to the SBTC of firms $i = 1, 2$. The lemma shows that the unique solution in w is given by $g(w, H/L) = G(w, z_1, z_2)$.

3.4 The equilibrium bidding for managers

In stage 1, firms bid for managers by posting managerial wages $(r_1,\, r_2)$. Let m_i denote the managerial employment in firm $i = 1,\, 2$. Further, let

$$\pi_i(m_1,\, m_2) \equiv \tilde{\pi}_i(z(m_1),\, z(m_2))$$

be the profit of firm i when firms 1 and 2 employ m_1 and m_2 managers, respectively.[10] Importantly, the Hicks-neutral technological change, A has no impact in the determination of the model equilibrium that is characterized in the following proposition.

Proposition 3.2. *For every equilibrium allocation of managers across the firms,* $(m_1^*,\, m_2^*)$*, the unique wage for managers is given by*

$$r_1 = r_2 = \frac{\pi_1(M,\, 0) - \pi_1(0,\, M)}{M} = \frac{\pi_2(0,\, M) - \pi_2(M,\, 0)}{M} \equiv r(M). \tag{20}$$

Two equilibrium allocations are $(m_1^*,\, m_2^*) = (M,\, 0)$ *and* $(m_1^*,\, m_2^*) = (0,\, M)$.

[10] Analytical expressions for the profit functions $\pi_i(m_1,\, m_2)$ are derived in lemma A.3, appendix A.2.

Proof. In the appendix A.2. □

Given that managers enhance the efficiency of high-skill labor in the production function, the corresponding profit also changes. In particular, holding all else constant, a firm's profit increases with the number of managers employed. Firms bid for managers by offering transfer that equals the profit opportunity cost of not securing managers. Because one manager not employed in one firm is one manager employed in the other, an equilibrium allocation ensures that all managers ultimately find employment in a leading firm. The total payment for managers, $r(M)\cdot M$, is thus equal to the difference in profits, $\pi_i(M,\, 0)-\pi_i(0,\, M)$ when firm i lures all managers. This indicates that a firm's profit, net of the transfer to the managers, remains the same whether the firm employs their services or not because, in equilibrium, $\pi_i(M,\, 0) - r(M)\cdot M = \pi_i(0,\, M)$. It also means that the competition forces present in this environment make managers being residual claimants of the excess profit generated by their ability to increase the efficiency of high-skill workers.

It is worth noting that the equilibrium managerial wage, $r(M)$ is unique, and is independent of the equilibrium managerial allocations across firms. However, the equilibrium allocation of managers across firms is not unique as we establish in Proposition 3.2 that there are two equilibria wherein all managers work for one of the two firms.

Having established that the managerial wage is unique, we analyze its behavior with respect to managerial supply when $\sigma = \zeta$.[11]

Proposition 3.3. *Let $\sigma = \zeta$. There is a unique $\bar{\gamma}(\sigma) \equiv \frac{\sigma}{\sigma-1} > 1$ such that*

(a) *If $0 < \gamma \leq 1$, $r(M)$ is decreasing in M;*

(b) *If $1 < \gamma \leq \bar{\gamma}(\sigma)$, $r(M)$ is hump-shaped, i.e., there is a unique $M^* > 0$ such that $r(M)$ is increasing (decreasing) in M according as $M < (>)\, M^*$;*

(c) *If $\gamma > \bar{\gamma}(\sigma)$, for low values of γ, $r(M)$ is non-monotonic in that there are two values of M, M^* and M^{**} with $0 < M^* < M^{**}$ such that $r(M)$ is increasing for $0 \leq M \leq M^*$ and $M \geq M^{**}$, and is decreasing for $M^* < M < M^{**}$. On the other hand, for high values of γ, $r(M)$ is increasing in M.*

[11] The analysis when $\sigma \neq \zeta$ is cumbersome. See the quantitative exercise in Section 4 for the analysis of this case.

Proof. In the appendix A.2. □

To understand the intuition behind the above proposition, Assume without loss of generality that $(m_1^*,\, m_2^*) = (M,\, 0)$.[12] At this equilibrium allocation of managers, we thus have $z_1 = z(M) \equiv z_0 + M^\gamma$ and $z_2 = z(0) \equiv z_0$. For $\sigma = \zeta$, from (28), it follows that the equilibrium skill-premium is given by

$$w = \frac{\alpha}{1-\alpha}\left(\frac{L}{2H}\right)^{\frac{1}{\sigma}}\left(z(M)^{\sigma-1} + z_0^{\sigma-1}\right)^{\frac{1}{\sigma}} \equiv w(M).$$

Substituting the last expression into (42), the expression of the equilibrium managerial wage, we obtain that

$$r(M) \equiv B(L,\, H)\cdot\frac{1}{M}\cdot\underbrace{\left(z(M)^{\sigma-1} - z_0^{\sigma-1}\right)}_{\phi_Z(M)}\cdot\underbrace{\left(z(M)^{\sigma-1} + z_0^{\sigma-1}\right)^{-\frac{\sigma-1}{\sigma}}}_{\phi_W(M)}, \tag{21}$$

where

$$B(L,\, H) \equiv \frac{\alpha}{1-\alpha}\cdot\frac{1}{2^{1/\sigma}(\sigma-1)}\cdot L^{\frac{1}{\sigma}}H^{1-\frac{1}{\sigma}} > 0,$$

which is independent of M.

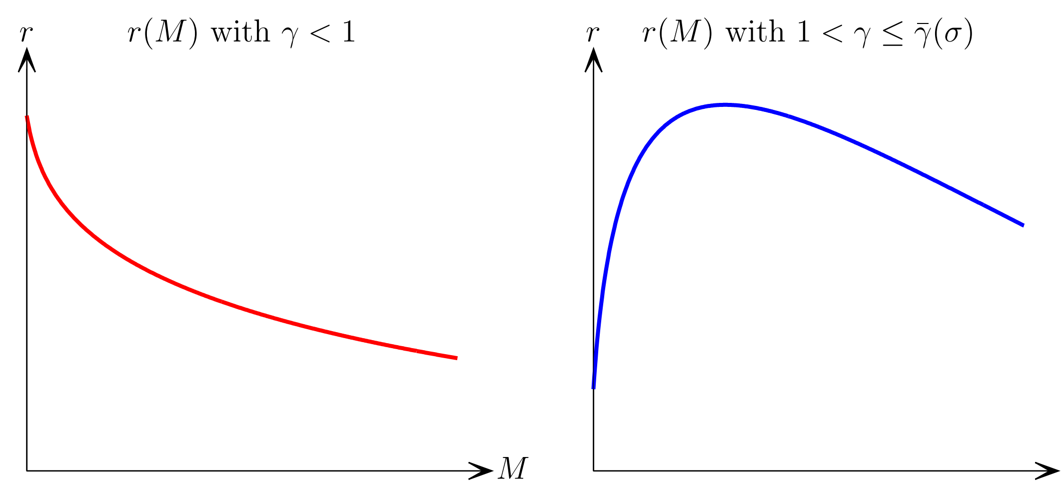
We decompose the equilibrium managerial wage, $r(M)$ into three terms. The first term, $1/M$ represents the direct negative effect (i.e., $r(M)$ decreases with M) of an increase in the aggregate managerial stock. The second term, $\phi_Z(M)$ is the indirect positive effect (because $\phi_Z'(M) > 0$) of an increase in the managerial supply. This term reflects the enhanced productivity of the firm that outbids the other in hiring all managers. Finally, the third term, $\phi_W(M)$ is the indirect negative effect (because $\phi_W'(M) < 0$) of an increase in managerial supply, which works through the change skill premium. Note, from Lemma A.3, that an increase in skill premium, w decreases a firm's profit because w enters the expression of profits as $w^{-(\sigma-1)}$ (at $\zeta = \sigma$). Higher managerial supply, on the other hand, increases the relative demand for high-skill workers in firm 1 (the one that gets all managers), and hence, a higher skill premium which lowers firm profits. Because all the three effects do not point in the same direction, the effect of increased managerial supply on $r(M)$ is in general indeterminate. Whether $r(M)$ increases or decreases with managerial supply depends on the relative strength of the countervailing effects.

[12]The proof is the same for $(m_1^*,\, m_2^*) = (0,\, M)$ because of symmetry.

Recall, from (4), that the firm-specific SBTC depends on γ through $z(m_i) = z_0 + m_i^{\gamma}$. Proposition 3.3 demonstrates that when $\gamma \leq 1$, that is, the SBTC has decreasing returns to managerial supply, the direct and indirect negative effects together dominate the indirect positive effect of an increase in managerial stock. As a result, managerial wage, $r(M)$ decreases with their supply in the general equilibrium (left panel of Figure 4). This result aligns with the implications of Proposition 3.1, which establishes a relationship between the supply of high-skill labor H and its compensatory wage w. Both results encapsulate the general intuition that prices reflect relative scarcity.

However, the above intuition disrupts when $\gamma > 1$, that is, the SBTC exhibits increasing returns to managerial supply. For intermediate values of γ, that is, $1 < \gamma \leq \bar{\gamma}(\sigma)$, for low levels of M (i.e., $M < M^*$), the indirect positive effect of an increase in managerial supply dominates the negative effects. For high levels of M, on the other hand, the negative effects together dominate the positive effect. Consequently, the equilibrium relationship between the managerial wage and their supply exhibits a hump-shape, as depicted in the right panel of Figure 4.

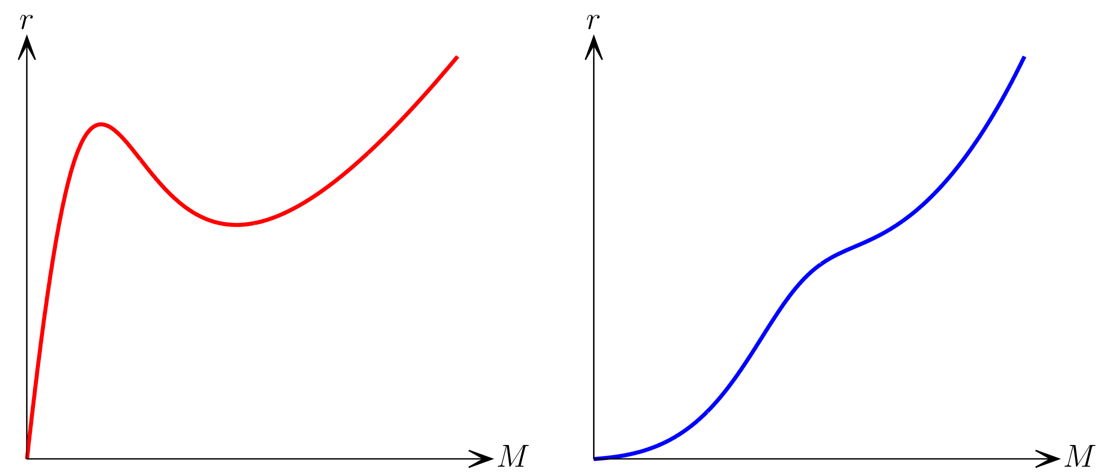
Figure 4: Equilibrium managerial wage with respect to supply



Notes: This figure plots the equilibrium managerial wage from Proposition 3.2(a)-(b) for different values of the managerial supply in the economy. The aggregate supply of managers in the economy is denoted by M, and r refers to their equilibrium wage. Both lines are generated with a parameterization with $\sigma = \zeta = 2$, and hence, $\bar{\gamma}(\sigma) = 2$. The left panel uses a $\gamma \leq 1$ while the right panel uses a $1 < \gamma \leq 2$. Different parameterizations with $\sigma > \zeta$ generate similar patterns.

When the increasing returns of the SBTC with respect to managerial supply is sufficiently strong, that is, $\gamma > \bar{\gamma}(\sigma)$, $r(M)$ can be non-monotonic or even increasing for all levels of M. For $\bar{\gamma}(\sigma) < \gamma \leq \gamma^*$, $r(M)$ is non-monotonic in that for low and high values of M, $r(M)$ is increasing as the indirect positive effect is stronger than the negative effects together. In contrast, for intermediate values of M (i.e., for $M^* < M < M^{**}$), the negative effects dominate, and hence $r(M)$ decreases with the managerial supply. This non-monotonicity is depicted in the left panel of Figure 5. However, for $\gamma > \gamma^*$, the positive effect dominates throughout, and hence, $r(M)$ is increasing for all M, which is depicted in the right panel of Figure 5.

Figure 5: Equilibrium managerial wage with respect to supply



Notes: This figure plots the equilibrium managerial wage from Proposition 3.2(c) for different values of the managerial supply in the economy. The aggregate supply of managers in the economy is denoted by M, and r refers to their equilibrium wage. Both lines are generated with a parameterization with $\sigma = \zeta =?$, and hence, $\bar{\gamma}(\sigma) =?$. The left panel uses a $\bar{\gamma}(\sigma) < \gamma \leq \gamma^*$ while the right panel uses a $\gamma^* < \gamma$. Different parameterizations with $\sigma > \zeta$ generate similar patterns.

We conclude this section by emphasizing that the different behavior of the manager's wage with its supply, as depicted in figure 4, offers relevant insights for parameterizing γ in order to align this model with real-world observations. Indeed, using the data outlined in section 2, we can project the managers wages against the supply of managers and the square of this variable to examine the evidence of a hump-shaped relationship. Table 3 presents the results of these regression. Both specifications (I) and (II) yield a negative and significant negative coefficient on the square of the managers' labor supply. This outcome provides suggestive evidence supporting the validity of the presented model when parameterized with $\gamma > 1$.

Table 3: Regression models for the managerial wage

Dependent variable: log relative wage manager to high school	(I)	(II)
Regressors:		
relative employment of managers	2.50***	2.10***
	(0.35)	(0.55)
square of the relative employment of managers	-2.26***	-1.69*
	(0.77)	(0.97)
relative employment of college workers		0.023
		(0.026)
sectoral fixed effects	Yes	Yes
Observations	96	96
R-squared	0.68	0.69

Source: US Census samples and the American Community Survey.
Notes: Section 2.2 explains the methodology used to extract average wages and hours supply of different groups following the methodology used in Acemoglu and Autor (2011). The wage premium relative to high school workers is defined as the ratio of the average wage of a particular group relative to the average wage of high-school workers. The employment relative to the total is the ratio of the total hours worked in a group relative to total hours worked in the economy. This figure uses a baseline definition of managers that includes the occupations listed in table 6 of the appendix.

4 Quantitative exercise

The preceding section unveiled a model underscoring the general equilibrium implications between the supply of managers and the wages of both managers and non-managers in the economy. We showed that an environment with a relative increase in the number of managers is consistent with a growing premium of both managers and high-skill workers. In this section, we present a simple extension of that model to allow for the quantification of the main innovation put forth in this paper. Specifically, we aim to address the question: how has the change in the supply of managers from 1950 to 2019 contributed to the evolution of the skill premium in the US economy?

From proposition 3.2, we know that an equilibrium is characterized by a corner solution where a single firm, which we can call the leading firm, hires all managers of the economy. We assume this is firm 1. The level of skill-biased technological change (SBTC), induced by managers in this leading firm, takes the form imposed by equation (4). Specifically, at time t, we have

$$z_{1t} = z_0 + b \cdot m_t^{\gamma}, \tag{22}$$

where m_t represents the number of managers hired, and b is a time-invariant parameter that scales the effect of the manager in the production function but has otherwise no baring in the equilibrium definition used in proposition 3.2. Substituting equation (22) into the production function results in:

$$y_{1t} = A\left(\alpha S_t\left[\left(z_0 + b \cdot m_t^{\gamma}\right)h_{1t}\right]^{\frac{\zeta-1}{\zeta}} + (1-\alpha)\, l_{1t}^{\frac{\zeta-1}{\zeta}}\right)^{\frac{\zeta}{\zeta-1}}, \tag{23}$$

where h_{1t} and l_{1t} represent the amount of high-skill and low-skill workers hired, and S_t is an additional exogenous variable. We use S_t, common to both firms, as an additional free variable that allows the model to absorb all the variation in the equilibrium wages of the high-skill workers not explicitly accounted for in the current environment. That is, given a sequence of labor supplies $\{L_t, H_t, M_t\}$, one can always select a S_t that matches a particular value of w_t in the equilibrium. In this sense, S_t can be interpreted as a residual SBTC or a model wedge, as originally implemented in Tinbergen (1974) and in subsequent studies (Katz and Murphy, 1992; Card and DiNardo, 2002; or Acemoglu and Autor, 2011). In our quantification, we interpret S_t as encompassing all other forces that may affect the high-skill premium but are not included in the current model. These might relate to, but are not exclusively limited to capital-skill complementarities (Krusell, Ohanian, Ríos-Rull, and Violante, 2000), quality-adjusted high-skill labor supply (Carneiro and Lee, 2011), changes in the quality of goods consumed (Jaimovich, Rebelo, Wong, and Zhang, 2020), or structural changes of the economy (Buera, Kaboski, Rogerson, and Vizcaino, 2021).

One additional useful transformation of the production function implies re-writing equation (23) in terms of relative factor input variables. Let the total labor force of the economy at time t be given by the auxiliary variable $F_t \equiv L_t + H_t + M_t$. Then the production function becomes

$$y_{1t} = AF_t\left(\alpha S_t\left[\left(z_0 + b \cdot F_t^{\gamma}\tilde{m}_t^{\gamma}\right)\tilde{h}_{1t}\right]^{\frac{\zeta-1}{\zeta}} + (1-\alpha)\, \tilde{l}_{1t}^{\frac{\zeta-1}{\zeta}}\right)^{\frac{\zeta}{\zeta-1}}, \tag{24}$$

where $\tilde{m}_t$, $\tilde{h}_{1t}$, and $\tilde{l}_{1t}$ are the input factors normalized to the total labor force in the economy. Provided with particular calibration of parameters $\{\sigma, \zeta, \alpha, A, z_0, b, \gamma\}$, the equilibrium definition described in section 3.1 allows for a structure where we can determine endogenously the premium of the high-skill workers w_t and the premium of the managers r_t using a set of relative aggregate input factors $\{\tilde{L}_t, \tilde{H}_t, \tilde{M}_t\}$, the size of the labor force F_t, and S_t as the residual SBTC. We now proceed to describe the calibration strategy of the model.

4.1 Calibration strategy

The parameters of the model are calibrated based on the aggregate data observations presented in figure 6 from section 2. Specifically, we utilize the observed sequence of labor shares in the US economy, $\{\tilde{L}_t, \tilde{H}_t, \tilde{M}_t\}$ for $t = 1950, 1960, ..., 2019$, along with an index capturing the size of the labor force, $\{F_t\}$, as model inputs. Given these inputs and a chosen set of parameters, the skill-biased technological change (SBTC) variable, S_t, is then adjusted to provide a model solution that generates an exact match with the sequence of data observations for the high-skill premium $\{w_t\}$. At this stage, we are left with an untargeted model generated sequence of managers' premia $\{r_t^{model}\}$ which can be compared with the counterpart data-observed sequence $\{r_t^{data}\}$. The calibration is simply the outcome of choosing parameters that minimizes the mean-square distance between the model and data observations:

$$\Psi = (1/T) \sum_{t=1950}^{2019} \left[\log r_t^{model} - \log r_t^{data}\right]^2 \tag{25}$$

We incorporate additional discipline into the model by externally calibrating certain parameters. The elasticity of substitution between goods in the household utility function, as defined in equation (2), is fixed at $\sigma = 2.5$. This enables our model, which does not explicitly incorporate capital, to yield a labor share of 60%. For the elasticity of substitution between high-skill and low-skill labor in the production function (equation 24), we choose $\zeta = 2$. This is an intermediate value within the estimated range of 1.6 to 2.9 from Acemoglu and Autor (2011).[13] It is important to note that the parameter α is unidentified alongside S_t, since the latter is used to rationalize the observed sequence of w_t. For this reason, we use a simple normalization, setting $\alpha = 0.5$. Similarly, the common Hicks-neutral productivity does not influence the equilibrium outcome of either premium, so it is also normalized to $A = 1$.

One final externally set parameter is the constant z_0 from the manager-induced SBTC equation (22). Following Gabaix and Landier (2008) and Bao, De Loecker, and Eeckhout (2022), we calibrate this parameter to reflect the share of manager wages in firms' total sales. Because we do not use data on firm total sales, we convert it info a share to wages with respect

[13] Our choice is also close to the estimated elasticity of 1.8 in Ohanian, Orak, and Shen (2021), that revisits the estimation from Krusell, Ohanian, Ríos-Rull, and Violante (2000) using 20 additional years of data. Different from our model, the production function in these papers allows for additional substitution between low-skill labor and capital.

to value added. Another key difference is that while these studies focus on compensation of very top-level managers such as CEOs, our notion of managers is broader and includes mid-level management positions. Consequently, we target the aggregate managers' wage share to gross output at the beginning of our sample, which gives us a z_0 value of 2.4%.

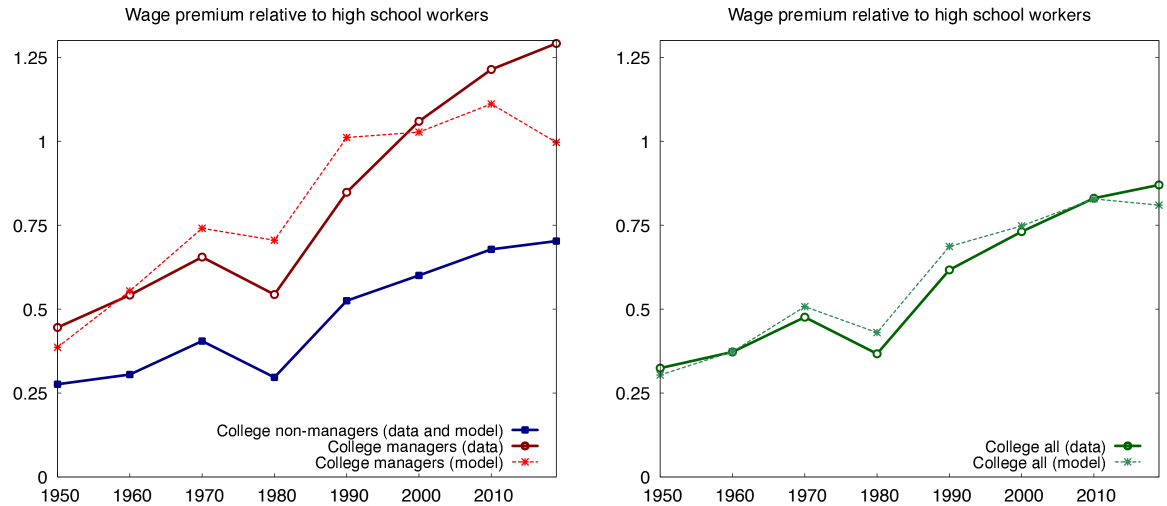
The remaining parameters b, γ are selected to minimize the distance criterion introduced in equation (25). Specifically, they are chosen to minimize the discrepancy between the manager's pay generated by the model and the corresponding data. Table 4 provides a summary of the calibration.

Table 4: Model calibration

Parameter		Value	Target
Elast. of sub. between goods	σ	2.5	Economy-wide labor share of 60%
Elast. of sub. between high and low-skill labor	ζ	2.0	Acemoglu and Autor (2011)
Hicks-neutral productivity	A	1.0	Normalization
Production intensity of high-skill labor	α	0.5	Normalization
Size effect in SBTC induced by managers	z_0	0.024	Share of managers wages on sales
Importance of additional managers in SBTC	b	0.88	Minimum distance Ψ in (25)
Convexity of additional managers in SBTC	γ	1.04	Minimum distance Ψ in (25)

Figure 6 illustrates how well the model fits the data. It's important to note that the blue line in the left panel of the figure, by design, represents both the data and the model evolution of the wage premium for high-skill labor, owing to the inclusion of the free SBTC variables S_t. However, the same cannot be said for the fit of the managers' premium, represented by the red line, where the fit is based on parameters that are held constant for the entire period. Despite this, the model successfully captures the general trend and level of managers' pay, generating a 39% premium in 1950 that peaks at 111% in 2010 and falls back to 100% in 2019.

Figure 6: Model fit of managers premium: data vs. model



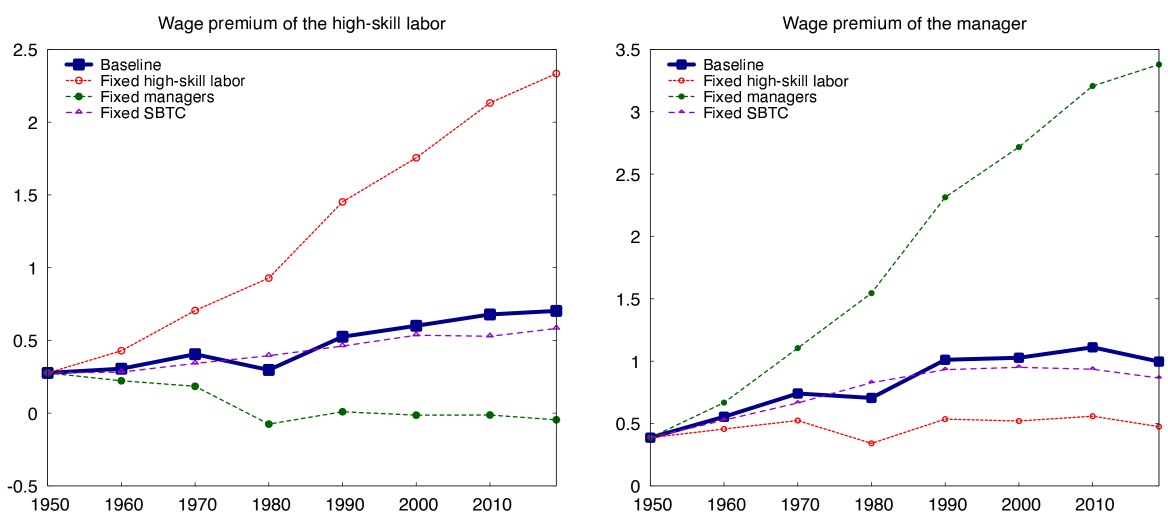
Overall, the calibration strategy results in a mean square error across all data points of 7.3% (calculated as the square root of equation 25), corresponding to an average distance between the model predictions and data observations of 6.1% for the analyzed period. Despite providing a good fit, the manager's premium predicted in the model diverges from the data in the last observation of the time window. The calibration results and the internal mechanism of the model explained in proposition 3.3 explain this outcome. Specifically, our parameterization of $\gamma = 1.04$ is consistent with convexity in the managers induced SBTC equation (22). Furthermore, the data show an uninterrupted increase in the relative supply of managers in the labor force. Combined, these two factors generate a model environment in which the premium for managers declines due to general equilibrium effects when their supply in the economy becomes sufficiently large. Our calibration suggests that tipping point occurred at around 2010.

To complement the results, the right panel of figure 6 also plots the model prediction for the combined high-skill and manager premium, which corresponds to the overall college premium in the data. This combined premium is generated by computing the weighted average of the model predicted wage premium of managers with college education and non-managers with college education. Against the data counterpart of the overall college premium, the model delivers predictions that average a deviation of just 2.1%. The proximity of the predicted college worker premium with the data counterpart, shows that the model can capture some of the general trends related with wage inequality in the US economy.

4.2 Decomposing the expansion in the high-skill labor and managers' premium

Drawing on the results generated by the calibration delineated in table 4, we can employ the model to dissect the sources of wage inequality. This is accomplished by running counterfactual scenarios where certain variables are kept at a constant level throughout the analysis period. Specifically, we investigate how the wage premium would alter if either the relative supply of managers had remained constant at 1950s levels, or if the relative supply of high-skill labor had remained static. These counterfactuals are conducted while keeping the same calibration and sequence of exogenous SBTC, $\{S_t\}$, as implied in figure 10. For further comparability, we also run a counterfactual where we let the relative supply of managers and high-skill workers to evolve as observed in the data, but the exogenous SBTC is kept at 1950s levels. Figure 7 shows the implied wage premiums of these counterfactuals and table 5 summarizes the results.

Figure 7: Model counterfactuals under fixed high-skill labor, fixed managers, and fixed SBTC



The figure underscores the role that the shifting composition of the labor force plays in wage premiums. Fixing the share of high-skill labor at its 1950s levels, while maintaining the influence of the sequence of exogenous SBTC, changes the balance of supply and demand forces towards the later, a result that is common in the skill-premium literature. The increase in efficiency induced by the SBTC generates higher demand for high-skill labor that is resolved in the market with higher wages (red line in the left panel of the figure). Interestingly, the increase of the managers premium (depicted in the red line at the right panel of the figure) remains comparatively subdued, with an uptick of only 8 percentage points. This outcome stems from the fact that, in the model, most of the benefit of hiring managers are realized through a more intense use of high-skill labor. However, the increase in a firm's profit from using additional high-skill labor is relatively small due to the high wages of high-skill workers.

Table 5: Change in wage premium between 1950-2019 in the model counterfactuals

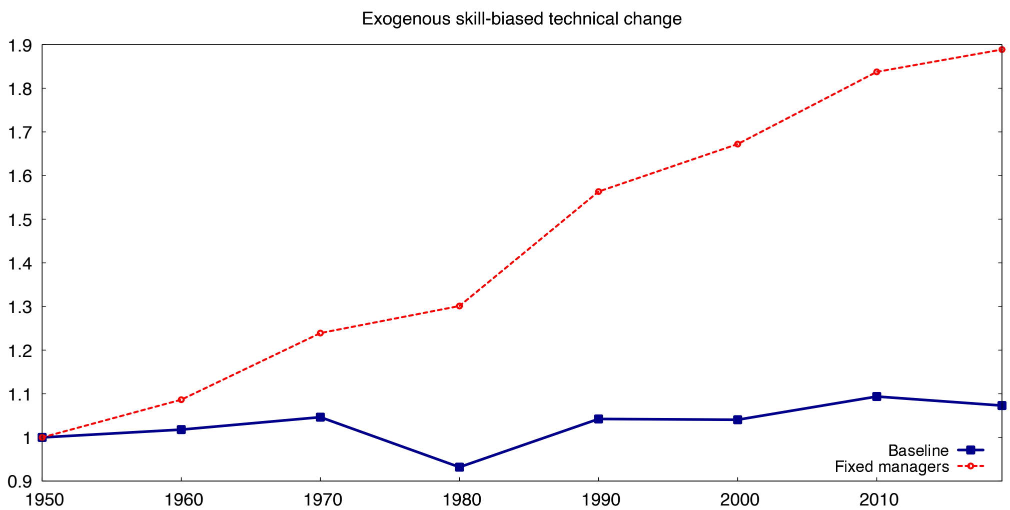
	change of the premium 1950-2019	
	for high-skill workers	for managers
Scenario:		
Baseline	42pp	61pp
Under fixed high-skill labor	205pp	8pp
Under fixed managerial labor	-32pp	299pp
Under fixed SBTC	30pp	47pp

Keeping the share of managers constant in the model (the green lines in the figure) provides a mirror image of this result. We recall that an increase in the share of managers in the economy improves the production efficiency of high-skill workers through equation (22). Therefore, with a constant share of managers in the economy, demand for high-skill labor due to higher efficiency does not increase as much as it would otherwise. However, the marginal impact of an additional manager in a firm's profit increase due to suppression of high-skill wages. This translates into a larger premium of managers in the economy.

Additionally, a comparison between the blue and purple lines in the figure shows that fixing the exogenous SBTC to its 1950s levels does not substantially alter the dynamics of the high-skill or managerial premium over the period. In particular, the increase in the high-skill premium decreases from 42 to 30 percentage points, while the increase in the managers' premium decreases from 61 to 47 percentage points. This suggests that the inclusion of managers in our model provides a mechanism that largely mitigates the reliance of an exogenous model wedge, or in our case, the SBTC, to match the evolution of the skill-premium of high-skill workers.

An alternative way of uncovering this result involves in recalculating the required SBTC wedge that rationalizes the observed path in the high-skill wages when the share of managers in the economy is held constant. Figure 8 contrasts the evolution of the SBTC under the baseline calibration with this counterfactual scenario. Notably, under the baseline scenario where the share of managers varies in line with the data, the necessary increase in the SBTC that matches the evolution of the high-skill premium is a mere 7%. Instead, when the increase in the managers share is shutdown, the required change in the SBTC escalates to 88% over the period. This substantial difference underscores that accounting for the relationship between production, availability of managers, and firm competition for their services can provide an important channel that explains the skill-premium puzzle evident in the data.

Figure 8: Skill-biased technical change required to match the high-skill premium in the data



5 Conclusion

This paper explores how the availability and competition for managerial services contributes to the rise of the skill premium in the US economy. An environment where firms hold market power and managers enhance production efficiency leads to competition for the services of managers where their compensation is associated with the profit opportunity cost of not hiring managers. When the effect of managers on firms' production is sufficiently convex, this leads to a compensation of managers that is hump-shaped, increasing when the supply of managers is scarce and decreasing when it becomes abundant. At the same time, an increase of the share of managers in the labor force generates an increase in demand for the high-skill labor if the managerial services provided increase disproportionally the relative efficiency of high-skill labor instead of low-skill labor. This effect contributes further to an increase in the high-skill premium.

A full characterization of a model that highlights how these relationships can emerge as economic equilibrium outcomes is presented. The model extends the canonical Tinbergen (1974) environment by explicitly incorporating firm competition under market power. By enabling managers to influence the production efficiency as in Gabaix and Landier (2008), we show that a Nash equilibrium exists where firms use profits to bid for managerial services. Furthermore, the characterization of the equilibrium reveals that, under some parameterizations, the model generates outcomes that are consistent with observations on the high-skill and managerial premium for the US economy in the past 70 years. Using census ACS data, we document a concurrent increase in both the relative supply of managers (from 4% to 20%) and high-skill workers (from 12% to 33%). During the same period, the high-skill wage premium surged by 42 percentage points, while the managers premium rose 61 percentage points.

Rather than categorizing these data patterns as puzzling, a reasonable calibration of the model can account for these observed dynamics. In our framework, the increase in the relative supply of managers can almost entirely explain the high-skill premium observed in the data, thereby eliminating the need to rely on exogenous model wedges such as skill-biased technical change.

Another important conclusion of this paper centers on the evolving pattern of managers' compensation over time. In contrast to theories that attribute an increase in managerial pay to frictions related to the misalignment of managers and shareholder interests or other forms of misbehavior, our model uniquely relies on competitive markets and firm competition to yield comparable outcomes. Furthermore, the results from our calibration exercise indicate that, as the relative supply of managers increases in the economy, the rate of their pay increase is anticipated to slow down and potentially invert. Importantly, these trends emerge without necessitating additional changes in public policies related to taxation or firm governance. This insight underscores some self-regulating dynamics inherent in our proposed model.

It is important to note that the results were derived using a stylized model that was left purposely simple to highlight the mechanisms at play. Notably, we have disregarded other forces that may also contribute to the increase in the high-skill premium, such as, capital-skill complementarities, quality-adjusted high-skill labor supply, changes in the quality of goods consumed, or structural changes of the economy.[14] Nevertheless, one advantage of using a simple model lies its capacity to incorporate additional features. Model extensions along these lines and deeper data analyses are left for future research.

[14] These forces have been studied in Krusell, Ohanian, Ríos-Rull, and Violante (2000); Carneiro and Lee (2011); Jaimovich, Rebelo, Wong, and Zhang (2020); Buera, Kaboski, Rogerson, and Vizcaino (2021).

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A Appendix

A.1 Additional details of the empirical section

In this section, we include additional details about the empirical section 2. These include data sources, worker category definitions, sample selection, methodology to compute wage premia and hours, and alternative regression results.

A.1.1 Data sources

All statistics presented in this paper use publicly available datasets. The information extracted from these sources is used to compute variables related with wage income and hours of work across across different groups of the US population, as well as across-sector productivity dynamics. The following paragraphs provide information about the data sources.

US Census Data for individual wages and hours or work between 1950 and 1990 use sample information of the US census provided by the IPUMS.[15] In some instances, survey variables are recoded to insure uniformity across years (see IPUMS website for details). Our analysis includes the following waves of the survey:

- 1950 1% SAMPLE,
- 1960 1% SAMPLE,
- 1970 1% FORM 1 METRO SAMPLE (this is not a sample only of metro areas, but rather a 1-in-100 national random sample of the population)
- 1980 1% METRO SAMPLE (this is not a sample only of metro areas, but rather a 1-in-100 national random sample of the population)
- 1990 1% METRO SAMPLE (this is not a sample only of metro areas, but rather a 1-in-100 national random sample of the population)

American Community Survey (ACS) Past 1999 we use the ACS survey which is conducted by the US Census Bureau and was designed to replace the Census long form. The dataset is also available at the IPUMS website.[15] Most variable definition are kept

[15] The dataset can be accessed in the address: https://usa.ipums.org/usa/.

unchanged between the US Census and the ACS survey, with occasional recoding to insure uniformity (see IPUMS website for details). To maintain a similar frequency as in the data available in the US Census, we use the years of 2000, 2010, and 2019 of the ACS survey. We use the 2019 instead of the 2020 ACS survey since, due to the effects of the COVID-19 pandemic on the 2020 ACS data collection, some experimental weights on sample variables were introduced that may affect their comparison with other years.

Current Population Survey (CPS) We complement the analysis that uses the US Census and ACS data using the alternative dataset CPS.[16] This consists in a yearly survey of U.S. households, available since 1962. Many of the questions of the CPS survey overlap with the ones of the ACS/US Census. However, the sample size of the CPS (around 65 thousand) is substantially lower than the present in the ACS/US Census (around 3 million). For that reason, we just use the CPS data to complement and validate some of the aggregate statistics generated using the ACS/US Census.

A.1.2 Educational attainment and occupation definitions

The empirical section 2 defines individuals in the sample between high-skill and low-skill workers. The category of high-skill workers includes individuals that report having completed college or more education, while low-skill workers include everyone else.

As for the occupation category, we focus mostly on managerial related occupations. The distinction between managerial and non-managerial occupations uses the harmonized coding scheme provided by IPUMS based on the Census Bureau's 2010 ACS. Throughout the paper, we use a relatively broad definition of managerial occupation but, for robustness, we also choose more narrower categorizations of managers. The particular mapping used can be found in table 6.

The broad definition of managers captures the main idea of the paper as these occupations that are not directly related with production, but instead with the organization of the firm. These include not just standard occupations related with management such as administrative services (code 100), but also adjacent occupations such as human resources (code 620) or accountants and auditors (code 800).

[16]The CPS is publicly available in the IPUMS website: https://cps.ipums.org/cps/.

Table 6: Definitions of managerial occupations using the IPUMS code scheme

	IPUMS code (2010)	Managers definition: Broad	Narrow	Narrower
MANAGEMENT, BUSINESS, SCIENCE, AND ARTS				
Chief executives and legislators/public administration	10	x	x	x
General and Operations Managers	20	x	x	x
Managers in Marketing, Advertising, and Public Relations	30	x	x	x
Administrative Services Managers	100	x	x	
Computer and Information Systems Managers	110	x	x	
Financial Managers	120	x	x	
Human Resources Managers	130	x	x	
Industrial Production Managers	140	x	x	
Purchasing Managers	150	x	x	
Transportation, Storage, and Distribution Managers	160	x	x	
Farmers, Ranchers, and Other Agricultural Managers	205	x	x	
Constructions Managers	220	x	x	
Education Administrators	230	x	x	
Architectural and Engineering Managers	300	x	x	
Food Service and Lodging Managers	310	x	x	
Funeral Directors	320	x	x	
Gaming Managers	330	x	x	
Medical and Health Services Managers	350	x	x	
Natural Science Managers	360	x	x	
Property, Real Estate, and Community Association Managers	410	x	x	
Social and Community Service Managers	420	x	x	
Managers, nec (including Postmasters)	430	x	x	
BUSINESS OPERATIONS SPECIALISTS				
Agents and Business Managers of Artists, Performers, and Athletes	500	x		
Buyers and Purchasing Agents, Farm Products	510	x		
Wholesale and Retail Buyers, Except Farm Products	520	x		
Purchasing Agents, Except Wholesale, Retail, and Farm Products	530	x		
Claims Adjusters, Appraisers, Examiners, and Investigators	540	x		
Compliance Officers, Except Agriculture	560	x		
Cost Estimators	600	x		
Human Resources, Training, and Labor Relations Specialists	620	x		
Logisticians	700	x		
Management Analysts	710	x		
Meeting and Convention Planners	720	x		
Other Business Operations and Management Specialists	730	x		
FINANCIAL SPECIALISTS				
Accountants and Auditors	800	x		
Appraisers and Assessors of Real Estate	810	x		
Budget Analysts	820	x		
Credit Analysts	830	x		
Financial Analysts	840	x		
Personal Financial Advisors	850	x		
Insurance Underwriters	860	x		
Financial Examiners	900	x		
Credit Counselors and Loan Officers	910	x		
Tax Examiners and Collectors, and Revenue Agents	930	x		
Tax Preparers	940	x		
Financial Specialists, nec	950	x		

For the more narrower definitions, we choose either all occupations related with management, business, science, and arts (IPUMS codes 10-430), or just Chief executives and legislators/public administration (IPUMS codes 10-30). We highlight that some statistics and regression results become severely underpowered when using narrower definitions of managers.

A.1.3 Sample selection and methodology to compute wage premia and hours supplied

Following the literature on the skill premium (Katz and Murphy, 1992; Autor, Katz, and Kearney, 2008; or Carneiro and Lee, 2011), we restrict our sample to include only individuals with ages between 25 and 60 years old. Additionally, we exclude individuals that are unemployed, not in the labor force, or that report working less than 40 weeks per year. Observations without education, or income information are dropped. Also excluded are observations in which an individual reports a weekly wage income that is below \$50 (at 1982 prices), or that is 50% below the federal minimum wage.

After imposing these data restrictions, a measure of individual yearly supply of hours is computed as the product of reported total weeks of work in a year and the average week length in hours. For those census years where there's no continuous variable for the usual work week length or number of weeks per year, we use instead the corresponding discrete variables (with intervals) to generate an estimate based on averaging the continuous variable for the same interval in the years of the survey for which both discrete and continuous are simultaneously available. With this variable, we compute each individual hourly wage as the ratio of total yearly wage income to total hours worked within a year.

To generate average wages and the supply of hours across the broadly defined groups of interest in our paper (low-skill, high-skill, managers), we follow the same methodology as used in Acemoglu and Autor (2011). In particular, we compute composition-adjusted wage premia that hold constant the relative employment shares of different demographic characteristics such as gender, education (narrowly defined), potential experience, and occupation, across all years of the sample. In our analysis we use 2 groups for gender (male and female), 4 groups for potential experience (9 or less years, 10 to 19 years, 20 to 29 years, and more than 29 years), 5 groups for education (less than high school, high school, some college, college, and more than college), and 2 groups for occupation (manager, and non-manager accordingly to table 6).

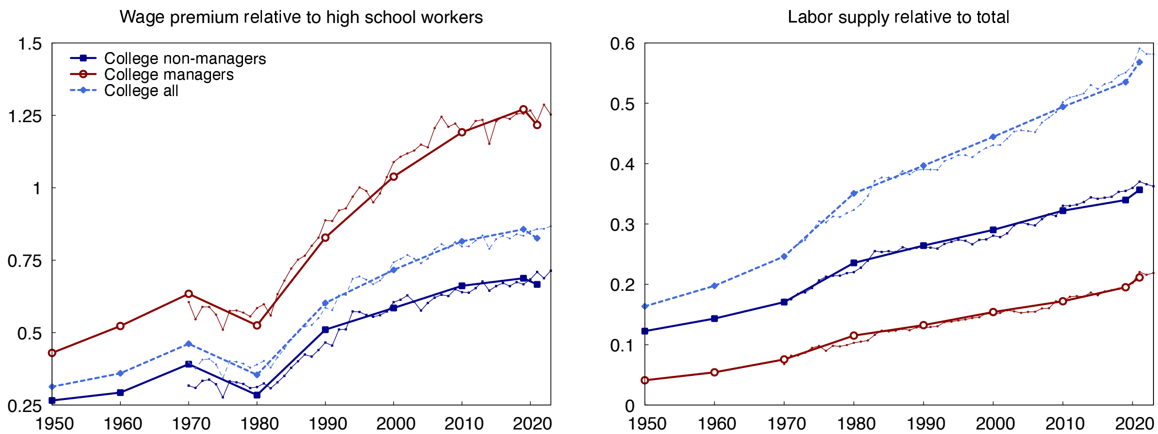
The adjustment computes first the mean wages as predicted component of a regression of the log hourly wages against dummies for each group in gender, education, potential experience, and experience with additional interactions for each group dummy and using the individual weights provided by the census survey. Then, we derive mean wages for broader groups as the fixed-weighted predicted average for each broad group studied: 'low-skill workers' as all the individuals with less than completed college, 'high-skill workers' as all the individuals with completed college or more education that are not managers, and 'manager' as all the individuals with completed college or more education that are managers. The fixed-weights used to generate these means are computed as the average share of total hours worked for each group over the entire time-window of the sample.

Our measure of hours supplied for each broadly defined group are just the sum of yearly hours worked using the individual weights provided by the census survey. Both our measures of hours supply and average wages are used to generate figure 6 in the main text of the paper and related statistics. Moreover, to analyze trends of hours supply and wage premia across sectors, we repeat this methodology by using only sample selections associated with each sector of activity. The resulting measures are used to generate the results presented in figure 2, as well as in tables 1 and 2 of the paper.

Additionally, we also compare our measures of the wage premia and labor supply with similar ones computed using the CPS. This allow us to ascertain the reliability of the estimates presented throughout the paper. One advantage of the CPS over the Census/ACS datasets relies on the availability of yearly data since 1963. However, the sample size of the CPS is also much smaller, thus imposing limitations on how precise one can measure certain statistics. For this exercise, we use either the CPS or the Census/ACS dataset to compute average wages and hours supplied for two broad groups: low-skill workers (individuals with less than completed college), and high-skill workers (individuals with with completed college or more education). In both cases, we follow the exact same methodology as described in the preceding paragraphs.

Figure 9 shows how the measures in the two datasets compare with each other. Although, at the aggregate, both datasets display very similar trends, we opted to ACS data to perform our analysis as this dataset due to the fact that the larger sample size of the ACS allows to include additional sectoral controls.

Figure 9: Composition-adjusted wages and relative labor supply across education and occupations: Census/ACS vs. CPS



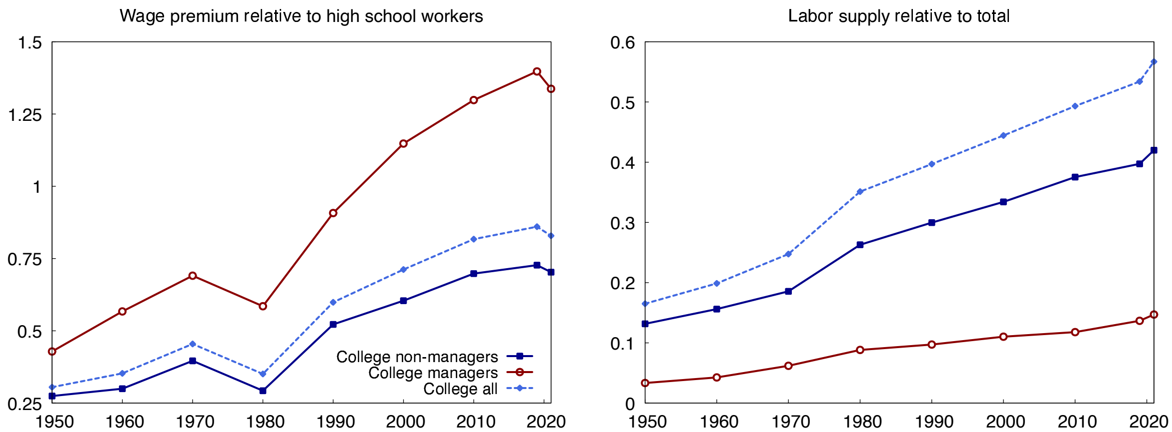
Source: US Census samples, the American Community Survey, and Current Population Survey.
Notes: The two panels replicates figure 6 in the main text (that use the Census/ACS dataset) in which counterpart measures using the CPS dataset are also plotted. The time-window for the CPS series is from 1970 to 2023 and the time window for the Census/ACS is from 1950 to 2019 to which 2021 is also added (the last ACS available to date).

A.1.4 Robustness of empirical results to different managers categorization

In this section we redo figure 6 and the regressions presented in table 2 to evaluate how robust are the results when changing the definition of occupations that fall into managers accordingly to the described in table 6. To this end we apply the same methodology to extract relative wages and hours of work as described in this appendix.

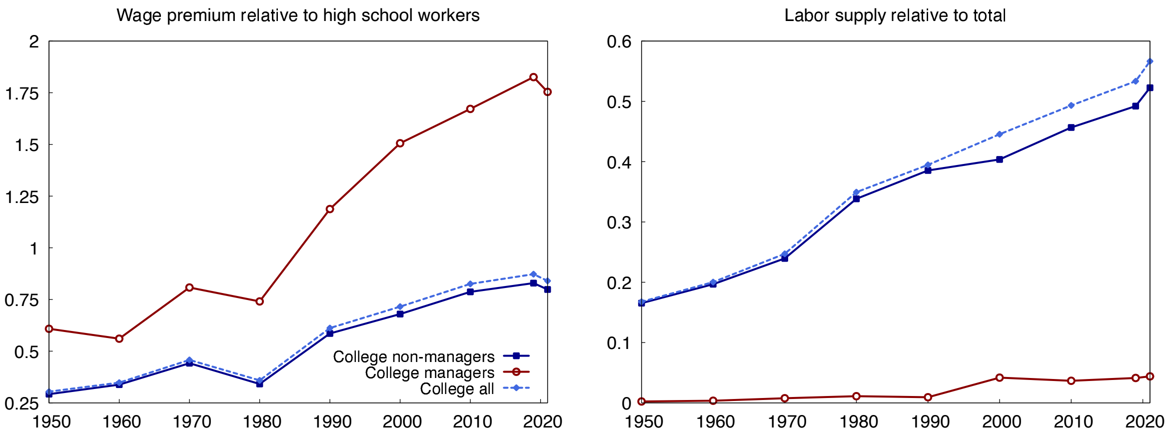
Figures 10 and 11 describe the composition-adjusted observed wages and relative labor supply across education and occupations applying the 'narrow' and the 'narrower' definition of a manger, respectively. One can see that the patterns documented are qualitatively equivalent. The main differences are mostly in quantitative magnitudes. Not surprisingly, narrowing the definition of a manager decreases the relative supply of this occupation. Nevertheless, whether we use the 'narrow' or the 'narrower' definition of the manager, we observe a relative growth in this occupation, an important feature of our theory that resists a different categorization. Similarly, we observe an increase in the wage premium of managers that is stronger that the observed for the overall college workers, irrespectively of the definition used for managers occupations. In particular, as the definition becomes narrower, the managers' premium increases in level. Nevertheless, over the span of the period 1950 to 2020 we still have a considerable increase in this premium despite the increase in the relative supply of workers operating in these occupations. To summarize, while we observe an increase in the relative labor supply of managers from 4% to 19% between 1950 and 2019 in our baseline, in observe an increase from 3% to 15% and from 0.2% to 4% when using the 'narrow' and the 'narrower' definitions of managers, respectively. As for the managers wage premium, we observe an increase from 45% to 129% in the same period for the baseline, while an increase from 43% to 138% and 61% to 175% in the two definitions used.

Figure 10: Composition-adjusted wages and relative labor supply across education and occupations: using the 'narrow' definition of managers



Source: US Census samples and the American Community Survey.
Notes: The average wages and hours supply of different groups follows the methodology used in Acemoglu and Autor (2011) as explained in section 2.2. The wage premium relative to high school workers is defined as the ratio of the average wage of a particular group relative to the average wage of high-school workers normalized to zero (when $w_t^i/w_t^{\text{high}school} = 1$, the wage premium is 0). The labor supply relative to total just gives the ratio of the total hours worked in a group relative to total hours worked in the economy. This figure uses the 'narrow' definition of managers that includes the occupations listed in table 6.

Figure 11: Composition-adjusted wages and relative labor supply across education and occupations: using the ‘narrower’ definition of managers



Source: US Census samples and the American Community Survey.
Notes: The average wages and hours supply of different groups follows the methodology used in Acemoglu and Autor (2011) as explained in section 2.2. The wage premium relative to high school workers is defined as the ratio of the average wage of a particular group relative to the average wage of high-school workers normalized to zero (when $w_t^i/w_t^{\mathrm{high}school} = 1$, the wage premium is 0). The labor supply relative to total just gives the ratio of the total hours worked in a group relative to total hours worked in the economy. This figure uses the ‘narrower’ definition of managers that includes the occupations listed in table 6.

Also using the two alternative definition of managers summarized in table 6, we reestimate the statistical model from equation (1) that augments the Tinbergen (1974) regressions to include the relative supply of managers. The results are presented in table 7.

Column (I) presents the same results from table 6 while columns (II) and (III) present the new results. Reassuringly, the main conclusions for the regression presented in the main text hold against using the different definitions of managers. In particular, the regressions associated with the ‘narrow’ and ‘narrower’ managers have positive and significant coefficients for the relative supply of managers on the relative wage of college educated workers. At the same time, both regressions present negative and significant coefficients on the relative employment of college workers (capturing a supply effect) and significantly positive coefficients on the time trend (capturing a demand effect that is skill biased). Furthermore, and similar to the interpretation made with the original version of these regressions explained in section 2.2, omitting the manager relative employment regressor generates statistically lager effects on the time trend for all the models (I)-(III). A potential interpretation that is consistent to our model is that part of the increase in demand for college educated workers that drives the increase in the college premium is explained by the intensity of managers utilization in each sector of the economy.

Table 7: Regression models for the college non-manager wage premium between 1950 and 2019 with alternative definition of managers: baseline, 'narrow', 'narrower'

Dependent variable: log relative wage college to high school	baseline (I)	'narrow' (II)	'narrower' (III)
Regressors:			
relative employment of college workers	-.075***	-.080***	-.096**
	(0.022)	(.024)	(.022)
relative employment of managers	.541**	.734**	2.07**
	(.228)	(.356)	(.475)
time trend	.0040***	.0045***	.0047***
	(.0008)	(.0008)	(.0006)
sectoral fixed effects	Yes	Yes	Yes
Observations	96	96	96
R-squared	0.80	0.79	0.85

Source: US Census samples and the American Community Survey.
Notes: The average wages and hours supply of different groups follows the methodology used in Acemoglu and Autor (2011) as explained in section 2.2. The wage premium relative to high school workers is defined as the ratio of the average wage of a particular group relative to the average wage of high-school workers normalized to zero (when $w_t^i/w_t^{\text{high}school} = 1$, the wage premium is 0). The labor supply relative to total just gives the ratio of the total hours worked in a group relative to total hours worked in the economy. All the occupations that are considered as managers can be found in the appendix A.1.2.

A.2 Proofs of lemmas and propositions

The claims made here are all associated with the equations and definitions of section 3. We start with a simple lemma (A.1) that just re-writes product demands and profits as functions of marginal costs and total income.

Lemma A.1. *Given the constant marginal costs of the two firms, (c_1, c_2), the optimal consumption of each good, and the profit of each firm are given by*

$$x_1(c_1, c_2) = \frac{(\sigma-1)I}{\sigma c_1^{\sigma}(c_1^{1-\sigma}+c_2^{1-\sigma})}, \quad x_2(c_1, c_2) = \frac{(\sigma-1)I}{\sigma c_2^{\sigma}(c_1^{1-\sigma}+c_2^{1-\sigma})}, \tag{26}$$

$$\tilde{\pi}_1(c_1, c_2) = \frac{I}{\sigma}\cdot\frac{c_1^{1-\sigma}}{c_1^{1-\sigma}+c_2^{1-\sigma}}, \quad \tilde{\pi}_2(c_1, c_2) = \frac{I}{\sigma}\cdot\frac{c_2^{1-\sigma}}{c_1^{1-\sigma}+c_2^{1-\sigma}}. \tag{27}$$

Proof. Using (11), the composite is given by

$$P(c_1, c_2) = \left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\left(c_1^{1-\sigma}+c_2^{1-\sigma}\right).$$

Substituting the above and (11) into (8) and (9), we the expressions in (26). Firm i's profit is given by

$$\tilde{\pi}_i(c_1,\, c_2) = (p_i - c_i)x_i(c_1,\, c_2) = \left(\frac{\sigma c_i}{\sigma - 1} - c_i\right) x_i(c_1,\, c_2).$$

Substituting $x_i(c_1,\, c_2)$ for $i = 1,\, 2$, we obtain the expressions in (27). □

The next lemma (3.1) uses the firms' production functions to determine factor demand functions and marginal costs.

Lemma 3.1. *The (conditional) factor demand and cost functions of firm $i = 1,\, 2$ are given by*

$$h_i(w,\, y_i) = \frac{1}{A}\left(\frac{\alpha}{w}\right)^{\zeta} z_i^{\zeta-1} c_i^{\zeta} y_i, \tag{13}$$

$$l_i(w,\, y_i) = \frac{1}{A}\,(1-\alpha)^{\zeta}\, c_i^{\zeta} y_i. \tag{14}$$

Firm i's cost function is linear in y_i, and the associated constant marginal cost is given by

$$c_i \equiv c(w,\, z_i) = \frac{1}{A}\left(\alpha^{\zeta} z_i^{\zeta-1} w^{1-\zeta} + (1-\alpha)^{\zeta}\right)^{\frac{1}{1-\zeta}} \tag{15}$$

with c_i increasing in w and decreasing in z_i.

Proof. The first-order conditions with respect to h_i and l_i are respectively given by:

$$A\alpha z_i^{\frac{\zeta-1}{\zeta}} h_i^{-\frac{1}{\zeta}} \left(\alpha(z_i h_i)^{\frac{\zeta-1}{\zeta}} + (1-\alpha) l_i^{\frac{\zeta-1}{\zeta}}\right)^{\frac{1}{\zeta-1}} = w,$$

$$A(1-\alpha) l_i^{-\frac{1}{\zeta}} \left(\alpha(z_i h_i)^{\frac{\zeta-1}{\zeta}} + (1-\alpha) l_i^{\frac{\zeta-1}{\zeta}}\right)^{\frac{1}{\zeta-1}} = 1.$$

The above conditions imply

$$l_i = \left(\frac{(1-\alpha)w}{\alpha}\right)^{\zeta} z^{1-\zeta} h_i$$

Substituting the above into the production function of firm i, we obtain (13). The steps to obtain (14) are similar. The cost function of firm i is given by

$$C_i(w,\, z_i,\, y_i) = [w h_i(w,\, y_i) + l_i(w,\, y_i)] y_i = \underbrace{\frac{1}{A}\left(\alpha^{\zeta} z_i^{\zeta-1} w^{1-\zeta} + (1-\alpha)^{\zeta}\right)^{\frac{1}{1-\zeta}}}_{c(w,\, z_i)} \cdot y_i.$$

This completes the proof of the lemma. $\square$

The unique determination of the second stage equilibrium, taking z_1 and z_2 as given, is characterized in lemma A.2 by solving the model for the relative price of the high-skill labor w.

Lemma A.2. *Given firm technologies, (z_1, z_2) and $\sigma \geq \zeta$, the equilibrium high-skill premium, denoted by $w(z_1, z_2)$, is uniquely determined by*

$$G(w, z_1, z_2) = g(w), \tag{28}$$

where

$$G(w, z_1, z_2) \equiv \frac{z_1^{\zeta-1}\left(\alpha^\zeta z_1^{\zeta-1} w^{1-\zeta} + (1-\alpha)^\zeta\right)^{\frac{\zeta-\sigma}{1-\zeta}} + z_2^{\zeta-1}\left(\alpha^\zeta z_2^{\zeta-1} w^{1-\zeta} + (1-\alpha)^\zeta\right)^{\frac{\zeta-\sigma}{1-\zeta}}}{\left(\alpha^\zeta z_1^{\zeta-1} w^{1-\zeta} + (1-\alpha)^\zeta\right)^{\frac{\zeta-\sigma}{1-\zeta}} + \left(\alpha^\zeta z_2^{\zeta-1} w^{1-\zeta} + (1-\alpha)^\zeta\right)^{\frac{\zeta-\sigma}{1-\zeta}}},$$

$$g(w) \equiv \frac{H}{L}\left(\frac{(1-\alpha)w}{\alpha}\right)^\zeta.$$

If $z_i > z_j$ for $i, j = 1, 2$ and $i \neq j$, the equilibrium high-skill premium, $w(z_i, z_j)$ is a increasing in z_i. The aggregate income, $I(z_1, z_2)$ is determined by

$$I = \frac{\sigma L}{(\sigma-1)(1-\alpha)^\zeta} \cdot \frac{\left(\alpha^\zeta z_1^{\zeta-1} w^{1-\zeta} + (1-\alpha)^\zeta\right)^{\frac{1-\sigma}{1-\zeta}} + \left(\alpha^\zeta z_2^{\zeta-1} w^{1-\zeta} + (1-\alpha)^\zeta\right)^{\frac{1-\sigma}{1-\zeta}}}{\left(\alpha^\zeta z_1^{\zeta-1} w^{1-\zeta} + (1-\alpha)^\zeta\right)^{\frac{\zeta-\sigma}{1-\zeta}} + \left(\alpha^\zeta z_2^{\zeta-1} w^{1-\zeta} + (1-\alpha)^\zeta\right)^{\frac{\zeta-\sigma}{1-\zeta}}}.$$

Proof. Using $y_i = x_i(c_1, c_2)$, it follow from (13) and (14) that

$$L = l_1(w, y_1) + l_2(w, y_2) = A^{\zeta-1}(1-\alpha)^\zeta \left(c_1^\zeta x_1(c_1, c_2) + c_2^\zeta x_2(c_1, c_2)\right), \tag{29}$$

$$H = h_1(w, y_1) + h_2(w, y_2) = A^{\zeta-1}(\alpha/w)^\zeta \left(z_1^{\zeta-1} c_1^\zeta x_1(c_1, c_2) + z_2^{\zeta-1} c_2^\zeta x_2(c_1, c_2)\right). \tag{30}$$

Substitute $c_i = c(w, z_i)$ as in (15) into the expressions of $x_i(c_1, c_2)$ for $i = 1, 2$ in order to express the consumption of the two goods as functions of (w, z_1, z_2). Then, divide (29) by

(30) to get the equilibrium skill-premium equation in (28). Note that

$$\frac{\partial G}{\partial w}=-\frac{(\sigma-\zeta)(\alpha(1-\alpha))^{\zeta}(\sigma-1)\left[\left(\alpha^{\zeta}(\frac{w}{z_1})^{1-\zeta}+(1-\alpha)^{\zeta}\right)\left(\alpha^{\zeta}(\frac{w}{z_1})^{1-\zeta}+(1-\alpha)^{\zeta}\right)\right]^{\frac{\sigma-\zeta}{\zeta-1}-1}}{w^{\zeta}\left(z_1^{\zeta-1}-z_2^{\zeta-1}\right)^{-2}\left[\left(\alpha^{\zeta}(\frac{w}{z_1})^{1-\zeta}+(1-\alpha)^{\zeta}\right)^{\frac{\sigma-\zeta}{\zeta-1}}+\left(\alpha^{\zeta}(\frac{w}{z_2})^{1-\zeta}+(1-\alpha)^{\zeta}\right)^{\frac{\sigma-\zeta}{\zeta-1}}\right]^{2}}.$$

Thus, $G(w,\, z_1,\, z_2)$ is decreasing in w because, by assumption, $\sigma \geq \zeta$.[17] Next, notice that $\lim_{w\to\infty} G(w,\, z_1,\, z_2) = \frac{1}{2}\left(z_1^{\zeta-1}+z_2^{\zeta-1}\right) \in (0,\, \infty)$, and because G decreases with w, we have $G(0,\, z_1,\, z_2) > 0$. On the other hand, $g(w)$ is a strictly increasing and convex function with $g(0) = 0$ and $\lim_{w\to\infty} g(w) \to \infty$. Thus, by the Intermediate value theorem, the intersection between $G(w,\, z_1,\, z_2)$ and $g(w)$ is unique, which defines a unique $w(z_1,\, z_2)$.

Let $a(w,\, z_i) \equiv \left(\alpha^{\zeta} z_i^{\zeta-1} w^{1-\zeta} + (1-\alpha)^{\zeta}\right)^{\frac{\sigma-\zeta}{\zeta-1}}$. It is immediate to show that $\partial a(w,\, z_i)/\partial z_i > 0$. Note that we can write

$$G(w,\, z_i,\, z_j) \equiv z_j^{\zeta-1} + \frac{a(w,\, z_i)}{a(w,\, z_i)+a(w,\, z_j)}\left(z_i^{\zeta-1}-z_j^{\zeta-1}\right).$$

From the above it follows that $G(w,\, z_i,\, z_j)$ is increasing in z_i if $z_i > z_j$. Because $g(w)$ is an increasing function of w, which does not depend on z_1 and z_2, $w(z_i,\, z_j)$ is increasing in z_i. The first part of the Lemma is described in Figure 3.

Finally, note that

$$I = p_1 x_1(c_1,\, c_2) + p_2 x_2(c_1,\, c_2) = \frac{\sigma}{\sigma-1}\{c_1 x_1(c_1,\, c_2) + c_2 x_2(c_1,\, c_2)\}$$

which implies the expression of aggregate household income in the lemma. □

With the results from lemma A.2 one can further characterize some comparative statics implications on the high-skill wage premium w. This is shown in proposition 3.1.

[17]When $\sigma < \zeta$, $G(w,\, z_1,\, z_2)$ is increasing in w. Because

$$\lim_{w\to 0} G(w,\, z_1,\, z_2) = \frac{z_1^{\sigma-1}+z_2^{\sigma-1}}{z_1^{\sigma-\zeta}+z_2^{\sigma-\zeta}} > 0$$

for any $(\sigma,\, \zeta)$ and $\lim_{w\to\infty} G(w,\, z_1,\, z_2) < \infty$, the equilibrium $w(z_1,\, z_2)$ exists. However, the unicity cannot be guaranteed. If there are multiple equilibrium premia, choose the highest one, and our results in Lemma A.2 hold.

Proposition 3.1. *Given firm technologies $z_1 > z_2$ and $\sigma \geq \zeta$, the equilibrium high-skill wage premium w is increasing in z_i and decreasing in the relative supply of high-skill labor H/L.*

Proof. From lemma A.2, we have that the equilibrium w is the unique solution of:

$$G(w,\, z_1,\, z_2) = g(w, H/L). \tag{31}$$

The lemma also shows that $\partial G/\partial w < 0$, $\partial g/\partial w > 0$, $\partial g/\partial (H/L) > 0$, and $\partial G/\partial z_1 > 0$. Then, applying the implicit function theorem gives:

$$\begin{aligned} \frac{dw}{d\left(H/L\right)} &= \frac{\partial g/\partial\left(H/L\right)}{\partial G/\partial w - \partial g/\partial w} < 0, \\ \frac{dw}{dz_1} &= -\frac{\partial G/\partial z_1}{\partial G/\partial w - \partial g/\partial w} > 0. \end{aligned}$$

□

Finally, in the following lemma we describe the profits of the firms in the equilibrium of the second stage.

Lemma A.3. *Given firm technologies, $(z_1,\, z_2)$, the equilibrium profit of firm $i = 1,\, 2$ is*

$$\tilde{\pi}_i(z_1,\, z_2) = \frac{L(1-\alpha)^{-\zeta}}{\sigma - 1} \cdot \frac{\left(\alpha^\zeta z_i^{\zeta-1} w^{1-\zeta} + (1-\alpha)^\zeta\right)^{\frac{1-\sigma}{1-\zeta}}}{\left(\alpha^\zeta z_1^{\zeta-1} w^{1-\zeta} + (1-\alpha)^\zeta\right)^{\frac{\zeta-\sigma}{1-\zeta}} + \left(\alpha^\zeta z_2^{\zeta-1} w^{1-\zeta} + (1-\alpha)^\zeta\right)^{\frac{\zeta-\sigma}{1-\zeta}}}.$$

Proof. Immediately follows from Lemmas A.1 and A.2. □

With the results on the stage 2 of the equilibrium, that is, conditional on z_1 and z_2, we move now to the equilibrium allocation of managers across firms. The next proposition determines the payment for managers in the economy and an allocation between firm 1 and 2 resulting from the bidding process defined in section 3.

Proposition 3.2. *For every equilibrium allocation of managers across the firms, $(m_1^*,\, m_2^*)$, the unique wage for managers is given by*

$$r_1 = r_2 = \frac{\pi_1(M,\, 0) - \pi_1(0,\, M)}{M} = \frac{\pi_2(0,\, M) - \pi_2(M,\, 0)}{M} \equiv r(M). \tag{20}$$

Two equilibrium allocations are $(m_1^*, m_2^*) = (M, 0)$ *and* $(m_1^*, m_2^*) = (0, M)$.

Proof. Take any equilibrium allocation of managers in firms 1 and 2, (m_1^*, m_2^*), and let M be the aggregate supply of managers. Let $m_1^* \equiv m^*$ and $m_2^* \equiv M - m^*$. Further, let r_i be the wage for each manager offered by firm $i = 1, 2$. In a Nash equilibrium we have

$$\pi_1(m^*, M - m^*) - r_1 \cdot m^* \geq \pi_1(0, M), \tag{32}$$

$$\pi_1(m^*, M - m^*) - r_1 \cdot m^* \geq \pi_1(M, 0) - r_1 \cdot M. \tag{33}$$

The first inequality asserts that firm 1 has higher net profit by employing $m^* \in [0, M]$ managers rather than employing no managers. The second inequality, on the other hand, asserts that employing all M managers is less profitable for firms 1 than hiring $m^* \in [0, M]$ managers. Similarly, for firm 2 we have

$$\pi_2(m^*, M - m^*) - r_2 \cdot (M - m^*) \geq \pi_2(M, 0), \tag{34}$$

$$\pi_2(m^*, M - m^*) - r_2 \cdot (M - m^*) \geq \pi_2(0, M) - r_2 \cdot M. \tag{35}$$

We first show that, in equilibrium, $r_1 = r_2 = r^*$. Suppose on the contrary that $r_1 < r^*$. In this case, all the M managers go to firm 2, and hence, firm 1's profit is given by $\pi_1(0, M)$. From (32) it follows that $\pi_1(0, M) \leq \pi_1(m^*, M - m^*) - r^* \cdot m^*$. Thus, $r_1 < r^* = r_2$ is not a profitable deviation. Next, consider a deviation $r_1 > r^*$. In this case, all managers go to firm 1, and its profit becomes $\pi_1(M, 0) - r_1 \cdot M$. Note that $\pi_1(M, 0) - r_1 \cdot M \leq \pi_1(m^*, M - m^*) - r_1 \cdot m^* < \pi_1(m^*, M - m^*) - r^* \cdot m^*$. The first inequality follows from (33), and $r_1 > r^*$ implies the second inequality. Therefore, $r_1 > r^* = r_2$ is not a profitable deviation for firm 1. Similar argument goes for firm 2. Therefore, in equilibrium, we have $r_1 = r_2 = r^*$.

Using $r_1 = r^*$, (33) and (32) are written as

$$\pi_1(m^*, M - m^*) - r^* \cdot m^* \geq \pi_1(0, M), \tag{36}$$

$$\pi_1(m^*, M - m^*) - r^* \cdot m^* \geq \pi_1(M, 0) - r^* \cdot M. \tag{37}$$

Next, from Lemma A.3 we have that $\pi_1(m_1^*, m_2^*) + \pi_2(m_1^*, m_2^*) = L(1 - \alpha)^{-\zeta}/(\sigma - 1)$.

Therefore, (34) and (35) can respectively written as

$$r^* \cdot (M - m) \leq \pi_1(M,\, 0) - \pi_1(m,\, M - m), \tag{38}$$

$$r^* \cdot m \geq \pi_1(m,\, M - m) - \pi_1(0,\, M). \tag{39}$$

Therefore, from (36) and (39) it follows that

$$r^* \cdot m^* = \pi_1(m^*,\, M - m^*) - \pi_1(0,\, M). \tag{40}$$

On the other hand, (37) and (38) together imply that

$$r^* \cdot (M - m^*) = \pi_1(M,\, 0) - \pi_1(m^*,\, M - m^*). \tag{41}$$

Adding (40) and (41), we obtain

$$r^* = \frac{\pi_1(M,\, 0) - \pi_1(0,\, M)}{M} = \frac{\pi_2(0,\, M) - \pi_2(M,\, 0)}{M} \equiv r(M). \tag{42}$$

The second equality in (42) follows from the fact that $\pi_1(M,\, 0) + \pi_2(M,\, 0) = \pi_2(0,\, M) + \pi_2(0,\, M) = L(1-\alpha)^{-\zeta}/(\sigma - 1)$.

To show that $(m_1^*,\, m_2^*) = (M,\, 0)$ and $(m_1^*,\, m_2^*) = (0,\, M)$ are two equilibrium allocations, note that both $m^* = M$ and $m^* = 0$ solve (40) and (41), the two equilibrium conditions at the bidding stage. □

Proposition 3.3. *Let $\sigma = \zeta$. There is a unique $\bar{\gamma}(\sigma) \equiv \frac{\sigma}{\sigma-1} > 1$ such that*

(a) *If $0 < \gamma \leq 1$, $r(M)$ is decreasing in M;*

(b) *If $1 < \gamma \leq \bar{\gamma}(\sigma)$, $r(M)$ is hump-shaped, i.e., there is a unique $M^* > 0$ such that $r(M)$ is increasing (decreasing) in M according as $M < (>)\, M^*$;*

(c) *If $\gamma > \bar{\gamma}(\sigma)$, for low values of γ, $r(M)$ is non-monotonic in that there are two values of M, M^* and M^{**} with $0 < M^* < M^{**}$ such that $r(M)$ is increasing for $0 \leq M \leq M^*$ and $M \geq M^{**}$, and is decreasing for $M^* < M < M^{**}$. On the other hand, for high values of γ, $r(M)$ is increasing in M.*

Proof. In order to analyze the behavior of $r(M)$ we shall use the inverse function $M =$

$(z_1 - z_0)^{\frac{1}{\gamma}}$, which makes the proof simpler. Then, the expression in (21) becomes

$$\tilde{r}(z_1) = B(L,\, H) \cdot \underbrace{(z_1 - z_0)^{-\frac{1}{\gamma}}}_{\phi_M(z_1)=\frac{1}{M}} \cdot \underbrace{\left(z_1^{\sigma-1} + z_0^{\sigma-1}\right)^{-\frac{\sigma-1}{\sigma}}}_{\phi_W(z_1)} \cdot \underbrace{\left(z_1^{\sigma-1} - z_0^{\sigma-1}\right)}_{\phi_Z(z_1)} \tag{21'}$$

Clearly, $\text{sign}[r'(M)] = \text{sign}[\tilde{r}'(z_1)]$ as z_1 is strictly increasing in M. Let $x \equiv \frac{z_0}{z_1} \in (0,\, 1]$. Note that $x \to 0$ as $z_1 \to \infty$ (or equivalently, $M \to \infty$), and $x = 1$ when $z_1 = z_0$ (or equivalently, $M = 0$). Differentiating $\tilde{r}(z_1)$, we obtain

$$\begin{aligned}
\tilde{r}'(z_1) &= \phi'_M(z_1)\phi_W(z_1)\phi_Z(z_1) + \phi'_W(z_1)\phi_M(z_1)\phi_Z(z_1) + \phi'_Z(z_1)\phi_M(z_1)\phi_W(z_1) \\
&= \underbrace{\frac{B(L,\, H)}{\gamma\sigma} \cdot (z_1 - z_0)^{-\frac{\gamma+1}{\gamma}} \left(z_1^{\sigma-1} + z_0^{\sigma-1}\right)^{-\frac{2\sigma-1}{\sigma}} z_1^{2(\sigma-1)}}_{>0} \cdot H(x,\, \gamma,\, \sigma),
\end{aligned}$$

where $H(x,\, \gamma,\, \sigma) \equiv \gamma(\sigma - 1)(1 - x)\left(1 + (2\sigma - 1)x^{\sigma-1}\right) - \sigma\left(1 - x^{2(\sigma-1)}\right)$. Therefore,

$$\text{sign}[r'(M)] = \text{sign}[\tilde{r}'(z_1)] = \text{sign}[H(x,\, \gamma,\, \sigma)].$$

Note the following properties of $H(x,\, \gamma,\, \sigma)$:

P1. $\lim_{x\to 0} H(x,\, \gamma,\, \sigma) = \gamma(\sigma - 1) - \sigma < (>)\, 0$ according as $\gamma < (>)\, \bar{\gamma}(\sigma) \equiv \frac{\sigma}{\sigma-1}$; $H(x,\, \gamma,\, \sigma)$ is continuous at $x = 1$ with $H(1,\, \gamma,\, \sigma) = 0$.

P2. $H_x(x,\, \gamma,\, \sigma) = (\sigma - 1)\left(2\sigma x^{2\sigma-3} - \gamma\sigma(2\sigma - 1)x^{\sigma-1} + \gamma(\sigma - 1)(2\sigma - 1)x^{\sigma-2} - \gamma\right)$ with

$$\lim_{x\to 0} H_x(x,\, \gamma,\, \sigma) = \begin{cases} \infty & \text{if } \gamma > 0 \text{ and } 1 < \sigma < 2, \\ 2\gamma & \text{if } \gamma > 0 \text{ and } \sigma = 2, \\ -\gamma(\sigma - 1) & \text{if } \gamma > 0 \text{ and } \sigma > 2, \end{cases}$$

and $H_x(1,\, \gamma,\, \sigma) = 2\sigma(\sigma - 1)(1 - \gamma) \geq (<)\, 0$ according as $\gamma \leq (>)\, 1$.

Let us write

$$M = (z_1 - z_0)^{\frac{1}{\gamma}} = \left(z_0\left(\frac{1}{x} - 1\right)\right)^{\frac{1}{\gamma}} \equiv M(x).$$

Clearly, $\lim_{x\to 0} M(x) \to \infty$, $M(1) = 0$ and $M'(x) < 0$. We first prove the following useful result.

Lemma A.4. *For any $x \in (0, 1]$ and $\gamma \geq 1$,*

(a) *if $1 < \sigma \leq 2$, $H(x, \gamma, \sigma)$ is strictly concave in x;*

(b) *If $\sigma > 2$, there is a unique $\tilde{x} \in (0, 1)$ such that $H(x, \gamma, \sigma)$ is strictly convex (concave) in x according as $x < (>)\tilde{x}$.*

Proof. Note that

$$H_{xx}(x, \gamma, \sigma) = (\sigma - 1)x^{\sigma-3} \underbrace{[2\sigma(2\sigma - 3)x^{\sigma-1} - \gamma\sigma(\sigma - 1)(2\sigma - 1)x - \gamma(\sigma - 1)(2\sigma - 1)(2 - \sigma)]}_{h(x, \gamma, \sigma)},$$

and hence, $\text{sign}[H_{xx}(x, \gamma, \sigma)] = \text{sign}[h(x, \gamma, \sigma)]$ because $x > 0$ and $\sigma > 1$.

First, consider $1 < \sigma \leq \frac{3}{2}$, that is $2\sigma - 3 \leq 0$. Then, $h(x, \gamma, \sigma) < 0$ for all $x \in (0, 1]$, and hence, $H_{xx}(x, \gamma, \sigma) < 0$. In other words, $H(x, \gamma, \sigma)$ is strictly concave on $(0, 1]$.

Next, consider $\frac{3}{2} < \sigma \leq 2$. Note that

$$\begin{aligned} h_x(x, \gamma, \sigma) &= \sigma(\sigma - 1)[2(2\sigma - 3)x^{\sigma-2} - \gamma(2\sigma - 1)], \\ h_{xx}(x, \gamma, \sigma) &= -2\sigma(\sigma - 1)(2\sigma - 3)(2 - \sigma)x^{\sigma-3} \leq 0. \end{aligned}$$

The last inequality holds because $\sigma \leq 2$. Because $h(x, \gamma, \sigma)$ is concave in x, it is maximized at $x = \bar{x}$ which is given by

$$h_x(\bar{x}, \gamma, \sigma) = 0 \quad \Longleftrightarrow \quad \bar{x}^{\sigma-2} = \frac{\gamma(2\sigma - 1)}{2(2\sigma - 3)} \quad \Longleftrightarrow \quad \bar{x} = \left(\frac{2(2\sigma - 3)}{\gamma(2\sigma - 1)}\right)^{\frac{1}{2-\sigma}}. \tag{43}$$

Note that $3/2 < \sigma \leq 2$ implies that $\bar{x} \in (0,\, 1)$.[18] Now,

$$\begin{aligned}
h(\bar{x},\,\gamma,\,\sigma) &= 2\sigma(2\sigma-3)\bar{x}^{\sigma-1} - \gamma\sigma(\sigma-1)(2\sigma-1)\bar{x} - \gamma(\sigma-1)(2\sigma-1)(2-\sigma)\\
&= \sigma\bar{x}\{2(2\sigma-3)\bar{x}^{\sigma-2} - \gamma(\sigma-1)(2\sigma-1)\} - \gamma(\sigma-1)(2\sigma-1)(2-\sigma)\\
&= \sigma\bar{x}\left\{2(2\sigma-3)\cdot\frac{\gamma(2\sigma-1)}{2(2\sigma-3)} - \gamma(\sigma-1)(2\sigma-1)\right\} - \gamma(\sigma-1)(2\sigma-1)(2-\sigma)\\
&= \sigma\bar{x}\left\{\gamma(2\sigma-1) - \gamma(\sigma-1)(2\sigma-1)\right\} - \gamma(\sigma-1)(2\sigma-1)(2-\sigma)\\
&= \gamma\sigma(2\sigma-1)(2-\sigma)\bar{x} - \gamma(\sigma-1)(2\sigma-1)(2-\sigma)\\
&= \gamma(2\sigma-1)(2-\sigma)\{\sigma\bar{x} - (\sigma-1)\}\\
&= \gamma(2\sigma-1)(2-\sigma)\left\{\sigma\left(\frac{2(2\sigma-3)}{\gamma(2\sigma-1)}\right)^{\frac{1}{2-\sigma}} - (\sigma-1)\right\}\\
&\leq \gamma(2\sigma-1)(2-\sigma)\underbrace{\left\{\sigma\left(\frac{2(2\sigma-3)}{\gamma(2\sigma-1)}\right)^{2} - (\sigma-1)\right\}}_{\tilde{h}(\sigma)}.
\end{aligned}$$

The last inequality holds because $\bar{x}^{2-\sigma} = \frac{2(2\sigma-3)}{\gamma(2\sigma-1)} < 1$, and $3/2 < \sigma$ implies that $\frac{1}{2-\sigma} > 2$. So, $\text{sign}[h_x(\bar{x},\,\gamma,\,\sigma)] = \text{sign}[\tilde{h}(\sigma)]$ because $\gamma(2\sigma-1)(2-\sigma) > 0$. Note that $\tilde{h}(\sigma)$ is a strictly convex function as $\tilde{h}''(\sigma) = 192/\gamma^2(2\sigma-1)^4 > 0$, which is maximized at $\sigma = 2$. It is easy to see that $\tilde{h}(2) = \frac{8}{9\gamma^2} - 1$ which is strictly negative because $\gamma \geq 1$, and hence, $\tilde{h}(\sigma) < 0$ for all $\sigma \in (3/2,\, 2]$. Therefore, $h(x,\,\gamma,\,\sigma) \leq h(\bar{x},\,\gamma,\,\sigma) < 0$ which imply that $H_{xx}(x,\,\gamma,\,\sigma) < 0$, i.e., $H(x,\,\gamma,\,\sigma)$ is strictly concave in x.

Finally, consider the case when $\sigma > 2$. Note that $\lim_{x\to 0} h(x,\,\gamma,\,\sigma) = \gamma(\sigma-1)(2\sigma-3)(\sigma-2) > 0$ and $h(1,\,1,\,\sigma) = -2(\gamma+\sigma(2\sigma-3)(\gamma-1)) < 0$, and hence, by the Intermediate Value Theorem, there is $\tilde{x} \in (0,\, 1)$ such that $h(\tilde{x},\,\gamma,\,\sigma) = 0$. Moreover, $h(x,\,\gamma,\,\sigma)$ is strictly convex in x for $\sigma > 2$, and hence, $\tilde{x}$ is unique. Thus, $h(x,\,\gamma,\,\sigma) > (<)\, 0$ according as $x < (>)\, \tilde{x}$, and hence, $H(x,\,\gamma,\,\sigma)$ is strictly convex (concave) in x according as $x < (>)\, \tilde{x}$. $\square$

Now we proceed to prove Proposition 3.3. To prove part (a), note that $H(x,\,\gamma,\,\sigma)$ is strictly increasing in γ, and hence, in order to show $H(x,\,\gamma,\,\sigma) \leq 0$ for all $\gamma \leq 1$, it suffices to prove that $H(x,\,1,\,\sigma) \leq 0$. First, consider $1 < \sigma \leq 2$. Because, by Lemma A.4, $H(x,\,1,\,\sigma)$ is strictly concave, $H_x(x,\,1,\,\sigma)$ is strictly decreasing in x, and thus it reaches a minimum at $x = 1$. Therefore, $H_x(1,\,1,\,\sigma) = 0$ (cf. property P2) implies that $H(x,\,1,\,\sigma)$ is increasing

[18] The fact that $\bar{x} < 1$ is equivalent to $\sigma < 5/2$, which is implied by $\sigma \leq 2$.

on $(0,\, 1]$, which achieves a maximum at $x = 1$. Thus, $H(x,\, 1,\, \sigma) \le 0 = H(1,\, 1,\, \sigma)$. This case is depicted in the left panel of Figure 12. Next, consider $\sigma > 2$. Because, by Lemma A.4, $H(x,\, 1,\, \sigma)$ is strictly convex (concave) for $x < (>)\tilde{x}$, $H_x(x,\, 1,\, \sigma)$ is strictly increasing (decreasing) in x for $x < (>)\tilde{x}$. Given that $\lim_{x\to 0} H_x(x,\, 1,\, \sigma) < 0$ and $H_x(1,\, 1,\, \sigma) = 0$, there is a unique $\hat{x} \in (0,\, \tilde{x})$ such that $H_x(x,\, 1,\, \sigma) < (>)\, 0$, i.e., $H(x,\, 1,\, \sigma)$ is decreasing (increasing) for $x < (>)\, \hat{x}$. Because $\lim_{x\to 0} H(x,\, 1,\, \sigma) < 0$, $H(x,\, 1,\, \sigma)$ must be negative on $(0,\, \hat{x}]$. On the other hand, $H(x,\, 1,\, \sigma)$ being increasing on $[\hat{x},\, 1]$ implies that it reaches the maximum at $x = 1$. Therefore, $H(1,\, 1,\, \sigma) = 0$ implies that $H(x,\, 1,\, \sigma) \le 0$ for all $x \in (0,\, 1]$. This case is depicted in the right panel of Figure 12. Having proven that $H(x,\, 1,\, \sigma) \le 0$ for all $x \in (0,\, 1]$, we conclude that $r(M)$ is decreasing in M for $0 < \gamma \le 1$ and $\sigma > 1$ because $\mathrm{sign}[r'(M)] = \mathrm{sign}[H(x,\, \gamma,\, \sigma))]$.

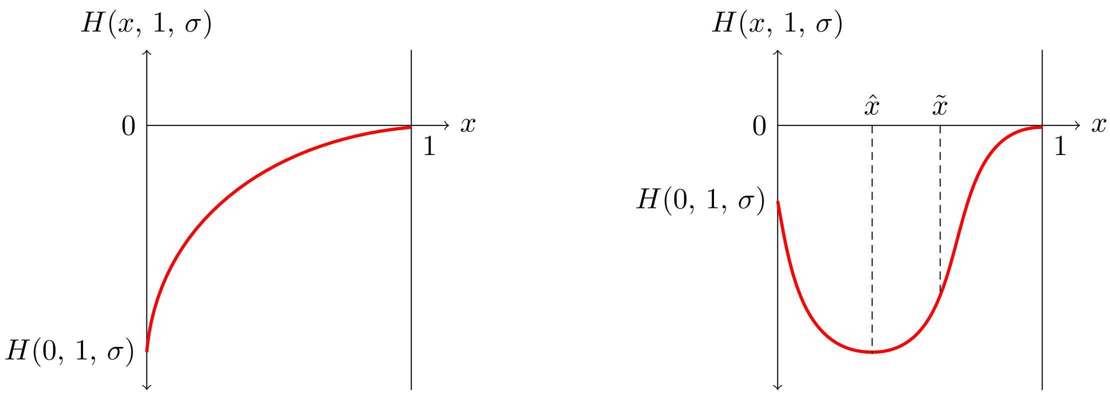


Figure 12: The left panel depicts $H(x,\, 1,\, \sigma)$ for $\sigma \in (1,\, 2]$, while the right panel depicts $H(x,\, 1,\, \sigma)$ for $\sigma > 2$ when $0 < \gamma \le 1$.

To prove part (b), first consider $1 < \sigma \le 2$. The strict concavity of $H(x,\, \gamma,\, \sigma)$ in x implies that $H_x(x,\, \gamma,\, \sigma)$ is strictly decreasing on $(0,\, 1]$. Because $\lim_{x\to 0} H_x(x,\, \gamma,\, \sigma) > 0$ and $H_x(1,\, \gamma,\, \sigma) < 0$ (cf. property P2), by the Intermediate Value Theorem, there is a unique $\hat{x} \in (0,\, 1)$ such that $H_x(x,\, \gamma,\, \sigma) > (<)\, 0$ or $H(x,\, \gamma,\, \sigma)$ strictly increasing (decreasing) in x according as $x < (>)\, \hat{x}$. Note that $H(\hat{x},\, \gamma,\, \sigma) > 0 = H(1,\, \gamma,\, \sigma)$ because $H(x,\, \gamma,\, \sigma)$ is strictly decreasing on $[\hat{x},\, 1]$. Because $\lim_{x\to 0} H(x,\, \gamma,\, \sigma) < 0$, $H(\hat{x},\, \gamma,\, \sigma) > 0$ and $H(x,\, \gamma,\, \sigma)$ is strictly increasing on $(0,\, \hat{x}]$, by the Intermediate Value Theorem, there is a unique $x^* \in (0,\, \hat{x})$ such that $H(x^*,\, \gamma,\, \sigma) = 0$. On the other hand, $H(x,\, \gamma,\, \sigma) \ge 0 = H(1,\, \gamma,\, \sigma)$ on $[\hat{x},\, 1]$ because $H(x,\, \gamma,\, \sigma)$ is strictly decreasing in x. Therefore, $H(x,\, \gamma,\, \sigma) < (\ge)\, 0$ according as $x < (>)\, x^*$ for $1 < \sigma \le 2$. This case is depicted in the left panel of Figure 13. Next, consider $\sigma > 2$. From Lemma A.4, it follows that $H_x(x,\, \gamma,\, \sigma)$ is strictly increasing (decreasing) in x according as $x < (>)\, \tilde{x}$. Note that $H_x(x,\, \gamma,\, \sigma)$ is maximized at $x = \tilde{x}$ with $H_x(\tilde{x},\, \gamma,\, \sigma) > 0$.

Suppose not, i.e., let $H_x(\tilde{x},\, \gamma,\, \sigma) \leq 0$. Then, it must be the case that $H_x(x,\, \gamma,\, \sigma) \leq 0$ for all $x \in (0,\, 1]$, i.e., $H(x,\, \gamma,\, \sigma)$ is non-increasing in x for all x which contradicts the fact that $\lim_{x\to 0} H(x,\, \gamma,\, \sigma) < 0 = H(1,\, \gamma,\, \sigma)$ [cf. property P2]. The above together with the facts that $\lim_{x\to 0} H_x(x,\, \gamma,\, \sigma) < 0$ and $H_x(1,\, \gamma,\, \sigma) < 0$ imply that $H_x(x,\, \gamma,\, \sigma)$ intersects the horizontal axis at exactly two points, $\hat{x}_1$ and $\hat{x}_2$ with $0 < \hat{x}_1 < \tilde{x} < \hat{x}_2 < 1$ such that $H(x,\, \gamma,\, \sigma)$ is decreasing on $(0,\, \hat{x}_1]$, increasing on $(\hat{x}_1,\, \hat{x}_2)$ and is again decreasing on $[\hat{x}_2,\, 1]$. It follows that $H(\hat{x}_1,\, \gamma,\, \sigma) < \lim_{x\to 0} H(x,\, \gamma,\, \sigma) < 0$ and $H(\hat{x}_2,\, \gamma,\, \sigma) > 0 = H(1,\, \gamma,\, \sigma)$. Therefore, by the Intermediate Value Theorem, there is a unique $x^* \in (\hat{x}_1,\, \hat{x}_2)$ such that $H(x,\, \gamma,\, \sigma) < (>)\, 0$ according as $x < (>)\, x^*$ for all $\gamma \in (1,\, \bar{\gamma}(\sigma)]$ and $\sigma > 2$. Moreover, $H(x,\, \gamma,\, \sigma)$ is strictly increasing on $(\hat{x}_1,\, \hat{x}_2)$ which implies that x^* is unique. This case is depicted in the right panel of Figure 13. To complete the proof of part (b), write $M^* = M(x^*)$. Because $M'(x) < 0$, the result follows from the fact that $H(x,\, \gamma,\, \sigma) > 0$, i.e., $r'(M) > 0$ if and only if $x < x^*$ (equivalently, $M > M^*$).

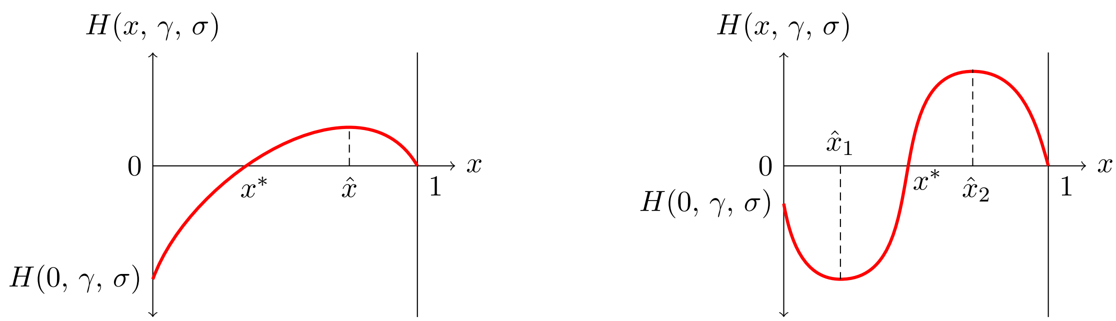


Figure 13: The left panel depicts $H(x,\, 1,\, \sigma)$ for $\sigma \in (1,\, 2]$. The right panel depicts $H(x,\, \gamma,\, \sigma)$ for $\sigma > 2$.

Finally, to prove part (c), first consider $1 < \sigma \leq 2$. Because $H(x,\, \gamma,\, \sigma)$ is strictly concave, the function is minimized at $x = 1$ as $\lim_{x\to 0} H(x,\, \gamma,\, \sigma) > 0 = H(1,\, \gamma,\, \sigma)$. Therefore, $H(x,\, \gamma,\, \sigma) \geq 0$ on $(x,\, 1]$. This case is depicted in the left panel of Figure 14. Next, consider $\sigma > 2$. This case is similar to part (b) for $\sigma > 2$ in that $H(x,\, \gamma,\, \sigma)$ is decreasing on $(0,\, \hat{x}_1]$, increasing on $(\hat{x}_1,\, \hat{x}_2)$ and is again decreasing on $[\hat{x}_2,\, 1]$. However, for $\gamma > \bar{\gamma}(\sigma)$ we have $\lim_{x\to 0} H(x,\, \gamma,\, \sigma) > 0$. Hence, there are two possibilities: (i) $H(x,\, \gamma,\, \sigma)$ intersects the horizontal axis exactly twice at x^* and x^{**} with $0 < x^* < x^{**} < 1$ as shown by the Blue curve in the right panel of Figure 14, and (ii) $H(x,\, \gamma,\, \sigma)$ is positive for all $x \in (0,\, 1)$ as depicted by the Red curve in the right panel of Figure 14. We shall show that the former situation emerges for low values of γ while the latter, for high its values. Note that $\lim_{x\to 0} H(x,\, \bar{\gamma}(\sigma),\, \sigma) = 0$, and $H(x,\, \bar{\gamma}(\sigma),\, \sigma) < 0$ for all $x \in (0,\, x^*)$. Because for any $\gamma > \bar{\gamma}(\sigma)$, $\lim_{x\to 0} H(x,\, \gamma,\, \sigma) > 0$, $\lim_{x\to 0} H_x(x,\, \gamma,\, \sigma) < 0$ and $H_x(1,\, \gamma,\, \sigma) < 0$, $H(x,\, \gamma,\, \sigma)$ must intersect the horizontal axis exactly twice at x^* and x^{**} for γ close to $\bar{\gamma}(\sigma)$ ($H(x,\, \gamma,\, \sigma)$ is linear, and hence, continuous in γ). On the other hand, $\lim_{\gamma \to \infty} H(x,\, \gamma,\, \sigma) \to \infty$, and hence, we can choose $\gamma_{max} < \infty$ such that $H(x,\, \gamma_{\max},\, \sigma) > 0$. Then, by the Intermediate Value Theorem, there is a $\gamma^* > \bar{\gamma}(\sigma)$ such that $H(x,\, \gamma,\, \sigma) > 0$ if and only if $\gamma > \gamma^*$. Moreover, γ^* is unique because $H(x,\, \gamma,\, \sigma)$ is linearly increasing in γ. To complete the proof of part (c), define $M^* = M(x^{**})$ and $M^{**} = M(x^*)$ so that $0 < M^* < M^{**} < \infty$. The proof follows as $M'(x) < 0$.

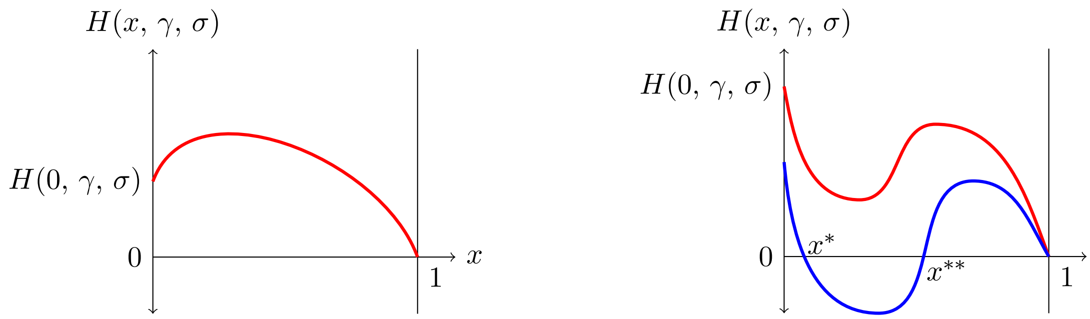


Figure 14: The left panel depicts $H(x,\, 1,\, \sigma)$ for $\sigma \in (1,\, 2]$. The right panel depicts $H(x,\, 1,\, \sigma)$ for $\sigma > 2$. The Blue curve corresponds to $\gamma \leq \gamma^*$, while the Red curve corresponds to $\gamma > \gamma^*$.

This completes the proof of the proposition. □

A.3 Managers and the Hicks-neutral technological change

In this section, we present an alternative model of production technology with Hicks-neutral technological change (HNTC) that is a function of managerial employment in each firm. In particular, let the production function of firm $i = 1,\, 2$ is given by:[19]

$$y_i = z_i \left(\alpha h_i^{\frac{\sigma-1}{\sigma}} + (1-\alpha) l_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}},$$

where the HNTC is given by

$$z_i = z_0 + m_i^{\gamma}, \quad \gamma > 0.$$

The managerial employment in firm i is given by m_i. We shall show that our results in Propositions 3.2 and 3.3 are robust to this modification. We shall omit the detailed calculations as they are similar to the ones in Section 3. It is well-established that, under the HNTC, change in z_i or equivalently, in m_i does not alter the relative demand of high-skill

[19]For simplicity, we use the same substitution parameter, σ for both the production and utility functions.

workers, and hence, the skill premium. In particular, unlike Lemma A.2, skill premium depends only on the relative supply of high-skill workers, H/L, and is independent of the managerial supply, M, which is given by

$$w = \frac{\alpha}{1-\alpha} \cdot \left(\frac{L}{H}\right)^{\frac{1}{\sigma}}. \tag{44}$$

Proposition 3.2 continues to hold even under the HNTC. However, the expression of the equilibrium managerial wage, $r(M)$ is now different as the production function is different under the HNTC, which is given by

$$r(M) \equiv \tilde{r}(z_1) = B(L,\, H) \cdot (z_1 - z_0)^{-\frac{1}{\gamma}} \cdot \frac{z_1^{\sigma-1} - z_0^{\sigma-1}}{z_1^{\sigma-1} + z_0^{\sigma-1}}, \tag{45}$$

where

$$B(L,\, H) \equiv \frac{L^{\frac{1}{\sigma}}\left(\alpha H^{\frac{\sigma-1}{\sigma}} + (1-\alpha)L^{\frac{\sigma-1}{\sigma}}\right)}{(\sigma-1)(1-\alpha)}.$$

The following proposition analyzes the effect of an increase in M on $r(M)$.

Proposition A.1. *Let $r(M)$ be the unique equilibrium managerial wage.*

(a) *If $0 < \gamma \leq 1$, $r(M)$ is decreasing in M;*

(b) *If $\gamma > 1$, $r(M)$ is hump-shaped, i.e., there is a unique $M^* > 0$ such that $r(M)$ is increasing (decreasing) in M according as $M < (>)\, M^*$.*

Proof. The proof is very similar to that of Proposition 3.3, and hence, we present the sketch of the proof. Let $x \equiv \frac{z_0}{z_1}$. Differentiating $\tilde{r}(z_1)$ with respect to z_1, we obtain

$$\tilde{r}'(z_1) = \underbrace{\frac{B(L,\, H) z_1^{\sigma+1} z_0^{\sigma+1} x^{1-\sigma}}{(z_1 - z_0)^{-\frac{1+\gamma}{\gamma}} (z_1 z_0^{\sigma} + z_0 z_1^{\sigma})^2}}_{>0} \cdot \underbrace{\left[2\gamma(\sigma-1)(1-x)x^{\sigma-1} - \left(1 - x^{2(\sigma-1)}\right)\right]}_{H(x,\gamma,\sigma)},$$

and hence, $\text{sign}[\tilde{r}'(z_1)] = \text{sign}[H(x,\, \gamma,\, \sigma)]$. Note that $\lim_{x\to 0} H(x,\, \gamma,\, \sigma) = -1$ and $H(1,\, \gamma,\, \sigma) = 0$. So, unlike Proposition 3.3, it is never the case that $H(x,\, \gamma,\, \sigma) > 0$ for all $x \in (0,\, 1]$. Moreover,

$$\lim_{x\to 0} H_x(x,\, \gamma,\, \sigma) = \begin{cases} \infty & \text{for } 1 < \sigma \leq 2, \\ 0 & \text{for } \sigma > 2, \end{cases}$$

and $H_x(1, \gamma, \sigma) = 2(\sigma - 1)(1 - \gamma) > (<)\, 0$ according as $\gamma < (>)\, 1$. The rest of the proof is similar to that of Proposition 3.3(a)-(b). Following similar steps as in the proof of Lemma A.4, it is easy to show that (i) $H(x, \gamma, \sigma)$ is strictly concave in x for $1 < \sigma \leq 2$, and (ii) there is a unique $\tilde{x} \in (0, 1)$ such that $H(x, \gamma, \sigma)$ is strictly convex (concave) according as $x < (>) \tilde{x}$. Then, we can show that $H(x, \gamma, \sigma) \leq 0$ for $\gamma \leq 1$, and $H(x, \gamma, \sigma) < (>)\, 0$ according as $x < (>)\, x^*$ for $x^* \in (0, 1)$ is a unique threshold of x. Thus, Proposition A.1 follows. □

A.4 Additional results of the quantitative exercise

This section evaluates the robustness of the quantitative section 4 by applying the calibration strategy to a sequence of the labor supply using a different definition for the occupations that fall into managers. The main difference is that the time series that is imputed into the model changes: $\{\tilde{M}_t\}$ for $t = 1950, 1960, ..., 2019$. The definition that we use in this exercise uses all occupations in the 'narrow' column from table 6 instead of the ones from the column 'broad'. Applying this change in the evolution of labor supply of managers implies a different calibration for some of the parameters shown in table 4. In particular, because of this more narrow definition of managers (in the 1950s the share of managers in the labor force changes from 4% to 3%), the managers' wages with respect to gross output of firms decreases at the beginning of our sample thus yielding a calibration for z_0 of 0.016 (in the baseline calibration we found $z_0 = 2.4$). The remaining two internally calibrated parameters are selected to minimize the distance criterium described in equation (25). This yields a $b = 0.58$ and $\gamma = 1.02$ (in the baseline $b = 0.88$ and $\gamma = 1.04$). All the remaining parameters remain the same as in the baseline ($\sigma = 2.5$, $\zeta = 2.0$, $A = 1$, $\alpha = 0.5$). Figure 15 summarizes the results of this exercise by showing the how the model fits the untargeted sequence of the wage premium for the managers.

We note that the main conclusions taken in the paper hold when changing the definition used for managers occupations. The model seems to provide a good fit of the observed data with a larger deviation occurring only at the end of the period (the average distance between the model predictions and data observations yields 4.6%). Moreover, finding a calibration with $\gamma > 1$ stresses the importance of the model characterization highlighted in section 3.4.

Figure 15: Robustness of the model fit of managers premium: data vs. model