Incentives and Income Distribution in Tenancy Relationships

by

Kaniṣka Dam*

Received July 14, 2011; in revised form February 21, 2015;
accepted February 21, 2015

I analyze a problem of partnership formation between heterogeneous principals and agents, where each partnership is subject to a moral-hazard problem. In a Walrasian equilibrium of the economy, wealthier agents work in more productive lands following a positively assortative matching pattern, since higher wealth has greater marginal effect in high-productivity lands. Consequently, the matches that consist of less wealthy agents are not able to implement the efficient effort and contracts. Agents' share of the match output is in general nonmonotone with respect to initial wealth. If wealth is more unequally distributed than land quality, then the equilibrium share (of the agents) is a monotonically increasing function of wealth. (JEL: O12, D31)

1 Introduction

While a plethora of writings on the theory of sharecropping have stressed the role of the agent's wealth endowment in determining his output share in a tenancy relationship, the roles of land quality and outside option have been paid little attention. It has been argued that share tenancy emerges as an incentive device when the agent's liability is limited by his initial wealth (Shetty, 1988; Laffont and Matoussi, 1995; Ray and Singh, 2001), and wealth has a positive effect on the agent's output share because higher wealth implies the possibility of greater rent extraction by the principal without weakening incentives.[1] Rao (1971) and Braido (2008), among few

* Centro de Investigación y Docencia Económicas, Mexico City. I owe thanks to an anonymous referee for insightful comments.

[1] Basu (1992), Sengupta (1997), and Ghatak and Pandey (2000) have been important contributions to the literature that argue that share tenancy emerges due to limited liability even if the agent may have zero wealth. There is also a large literature that claims that sharecropping emerges because of pure risk-sharing motives (e.g., Stiglitz, 1974; Newbery, 1977), or as an incentive device to mitigate moral hazard (e.g., Eswaran and Kotwal, 1985).

Journal of Institutional and Theoretical Economics – ISSN 0932-4569
DOI: 10.1628/093245615X14274466953446 –

others, have emphasized the role of land quality in share contracts.[2] On the other hand, the role of the agent's outside option in determining his share is also important. Banerjee, Gertler, and Ghatak (2002) argue that higher outside options following the introduction of *Operation Barga*, the land reform act of 1978 in the Indian state of West Bengal, had significant favorable effects on the output share and productivity of the sharecroppers. In the present paper I propose a unified framework that analyzes the joint effects of land and wealth heterogeneities on the tenancy contracts through endogenous outside option.

Most of the theoretical works on share tenancy employ variants of the partial-equilibrium agency model (e.g., Grossman and Hart, 1983) where a principal (landlord) of given characteristics leases her land to an agent (tenant) of given characteristics, and offers a tenancy contract that consists of a fixed rent component and a given share of output. The optimal contract determines the incentive structure of the final payoff to the agent. In such models, the level of income of the agent is determined entirely by his exogenously given outside option. Endogenous determination of the agent's outside option thus calls for a general-equilibrium framework. The present paper starts with this motivation. It extends the *differential rent* model of Sattinger (1979) to a situation where agents are assigned to principals, and each principal–agent relationship is subject to limited liability and moral hazard in effort choice. In particular, I consider a model where principals are heterogeneous with respect to the quality or productivity of the lands they own, and agents differ in wealth endowment. In an equilibrium, each principal of a given type chooses an agent optimally by maximizing her expected profits, taking the expected wages as given. A Walrasian equilibrium implies that each agent must receive an expected wage equal to his outside option, defined as the maximum of the payoffs that could be obtained by switching to alternative partnerships. As wealthier agents and more productive principals have absolute advantages in any partnership, equilibrium wage and profit are increasing respectively in wealth and land quality. Given the assumptions of the model, it turns out that agents with higher initial wealth have greater marginal effect (comparative advantage) in more productive lands. In any principal–agent relationship, the optimal contract determines how much surplus is to be transferred from the principal to the agent, and it is the case that transferring surplus to wealthier agents is more costly for any principal. Since higher wealth has comparative advantage in more productive lands, their owners have higher marginal willingness to pay for the wealthier agents. Therefore, the optimality of matching implies that wealthier agents work in high-productivity lands following a positively assortative matching (PAM) pattern.

A PAM between the principals and agents emerges as a result of the complementarity between the principal types and agent types because "better" agents have comparative advantage in forming partnerships with "better" principals. This has been the central premise of the assignment models when utility is fully transferable

[2] Rao (1971), using farm management data from India, shows that the quality of land has explained 90% of variations in contracts offered to sharecroppers.

(e.g., Sattinger, 1979). When utility is not fully transferable, Legros and Newman (2007) argue that such *type–type complementarity* may not be enough to generate a PAM; one must also consider the *type–payoff complementarity*, i.e., it is less costly at the margin for the better principals to transfer utility to better agents.[3] Legros and Newman (2007) propose a general complementarity condition, called the *generalized increasing differences*, under which a PAM is an optimal matching. I assume a slightly stronger condition than the generalized difference, and show that PAM along with a "local" first-order condition is a necessary as well as a sufficient condition for an optimal matching.

The equilibrium relationship between output share and initial wealth (of the agents) determines the equilibrium share function. The equilibrium output share of an agent depends on his initial wealth, on the productivity of the land he cultivates through the equilibrium matching function, and on his outside option via the equilibrium wage function. A principal can extract more surplus from a wealthy agent in the form of fixed rent without distorting the incentives much, and hence higher wealth implies greater output share. On more productive lands, on the other hand, less incentive is required in order to induce a given effort level, and hence lower shares are associated with high-quality lands. Finally, a higher outside option, which implies greater incentives to shirk, implies a higher output share. Because of these countervailing effects, the output shares of the agents are in general nonmonotone with respect to initial wealth. It is shown that when greater relative market power lies on the agent side of the market, the positive effects dampen the negative effect of land quality on the output shares of the agents, and the equilibrium share is thus an increasing function of wealth.

Similarly to the assignment model of Sattinger (1979), the present work also implies that the final distribution of equilibrium wages is skewed to the right relative to the distribution of initial wealth. With heterogeneous lands, agents with greater wealth are assigned to more productive lands, which boosts their expected incomes above what they would be earning if lands were identical.

The present paper contributes to the recent literature on the problems of two-sided matching between principals and agents when partnerships are subject to incentive problems. It is worth noting that a common feature of the models of assignment between principals and agents is the endogenous determination of the reservation payoffs of individuals. In a model where the partnerships are not subject to the incentive problem, i.e., when utilities are fully transferable, endogenous reservation payoffs are not that relevant for the determination of the organizational forms, since a fixed transfer may achieve efficiency. One of the main focuses of the present paper

---

[3] By fully transferable utility I mean that the utility possibility frontier (UPF) for a given principal–agent pair is a straight line with slope -1, whereas the UPF is a downward-sloping concave function when utility is not fully transferable. Legros and Newman (2007) refer to the latter situation as "nontransferable utility," whereas in the current context nontransferability should be understood as a situation where utility cannot be transferred between two individuals at all, i.e., when there do not exist any side payments between them.

is to analyze the effect of market imperfections on the equilibrium organizational forms. In particular, I analyze how the equilibrium contracts between the principals and the agents are affected by the differences in initial wealth of the agents. To this end, the current model is related to the following earlier contributions to the literature. Legros and Newman (1996) analyze a model of firm formation under credit market imperfections and wealth heterogeneity. When borrowing is costly and wealth is unequally distributed among the members of a firm, the equilibrium organizations typically consist of heterogeneous agents. In the present model, differences in wealth are not associated with the differences in access to credit markets; rather they affect the incentive problems at each partnership through the differences in liability of the agents. In this respect, the current work can be seen as a generalization of that of Dam and Pérez-Castrillo (2006), who show, in an assignment model with one-sided wealth heterogeneity, that wealth differences typically imply more efficient organizations. The present paper considers two-sided heterogeneity, namely, in land quality and initial wealth, and show that constrained efficient organizational forms crucially depend on the extent of heterogeneity of each side of the market. Chakraborty and Citanna (2005) analyze a model of occupational choice under wealth differences among individuals, and show that less wealth-constrained individuals choose to take up projects in which incentive problems are more important due to endogenous sorting effects.

In the context of share tenancy, this paper is related to that of Ghatak and Karaivanov (2014), who analyze a model of partnership based on double-sided moral hazard, and show that partnerships may not emerge in equilibrium if the individuals differ in their degrees of comparative advantage in accomplishing specific tasks and the matching is endogenous. They also found conditions under which a matching is assortative. One major contribution of their work is that the principal–agent economy is characterized by less sharecropping under endogenous matching than is an economy where matching is random. As opposed to the present model, Ghatak and Karaivanov (2014) analyze an agency model under unlimited liability, and hence comparative advantage and not wealth is the main source of heterogeneity.

As regards other types of market imperfections, Ackerberg and Botticini (2002) make an important contribution to the literature by analyzing landlord–tenant contracts in renaissance Tuscany, and show that contracts are influenced in a significant way by the endogenous nature of matching between the landlords and tenants. While more risk-averse agents are expected to receive lower output shares, in the presence of endogenous matching they may end up receiving a higher share because they are assigned less risky tasks. Serfes (2005) formalizes the arguments of Ackerberg and Botticini (2002) and considers an assignment model that analyzes the trade-off between risk-sharing and incentives, and shows a nonmonotone relationship between risk and incentive. Kremer (1993) considers production of output by agents heterogeneous with respect to ability, and shows that if the production function exhibits complementarity in ability, the firms are formed by homogeneous agents. The present work, as opposed to Kremer (1993), assumes incentive problems in all partnerships, and shows that heterogeneity matters when incentive problems are severe. Finally, Alonso-Paulí and Pérez-Castrillo (2012) examine an environment in which managers may choose between incentive contracts and contracts that include a code of best practice. They show that firms with the best projects may hire less talented managers by offering code-of-best-practice contracts even if a positive sorting is optimal when standard incentive contracts are chosen. Alonso-Paulí and Pérez-Castrillo (2012) do not characterize the equilibrium contracts as in the present paper; rather they assert that if the code-of-best-practice contracts are adopted, then a negatively assortative matching is more efficient than a positive sorting.

2 A Model of Principal–Agent Assignment

2.1 Description

Consider an economy with a continuum $[0, 1]$ of heterogeneous risk-neutral principals and a continuum $[0, 1]$ of heterogeneous risk-neutral agents, both being endowed with Lebesgue measure 1. The positive real numbers $\lambda \in \Lambda \equiv [\lambda_{\min}, \lambda_{\max}]$ and $\omega \in \Omega \equiv [\omega_{\min}, \omega_{\max}]$ denote the *qualities* of the principals and the agents, respectively. Quality or type of an individual may be interpreted as productivity, efficiency, wealth, etc., which would influence final payoffs. For example, higher values of λ could imply more productive (better) principals. The distributions of qualities are exogenous to the model. Let $G(\lambda)$ be the cumulative distribution of λ, which gives the fraction of principals with qualities lower than λ, and $g(\lambda)$ be the corresponding density function. Similarly, let $F(\omega)$ be the cumulative distribution of ω with the corresponding density $f(\omega)$. I denote by $\xi \equiv (F, G)$ the principal–agent economy.

Principals and agents are assigned to each other to form partnerships or matches. Each individual in a given match has to take a set of actions, which are inputs to the final match output. Some of these actions, such as effort or investment decision, may not be publicly verifiable, which induces incentive problems in each partnership. This section extends the *differential rents* model of Sattinger (1979) to situations where matches are subject to incentive problems.[4] Individuals of identical quality will be perfect substitutes, and hence only qualities and not the names matter. The principal–agent assignment can be described by a one-to-one mapping $l : \Omega \longrightarrow \Lambda$ or its inverse $w \equiv l^{-1}$. Therefore, if $\lambda = l(\omega)$, or equivalently $\omega = w(\lambda)$, then (λ, ω) denotes a typical match or partnership.

2.2 Equilibrium Assignment when Utilities Are Not Fully Transferable

Consider an arbitrary match (λ, ω) where the type ω agent receives an expected wage $u(\omega)$. For the purpose of this section, the utility possibility frontier $\phi(\lambda, \omega, u(\omega))$ for the match (λ, ω) will be taken as the primitive. How this is determined will be discussed at the end of this section. The function $\phi(\lambda, \omega, u(\omega))$ denotes the maximum payoffs the principal obtains, given $u(\omega)$. Further, let $\Phi(\omega, \lambda, v(\lambda))$ denote the quasi-

[4] Edmans, Gabaix, and Landier (2009) and Dam (2014) also build on the assignment model presented in Sattinger (1979) to incorporate incentive problems.

inverse of ϕ, which is the maximum payoff to a type ω agent when the type λ principal receives $v(\lambda)$. Then by definition,

$$u(\omega) = \Phi(\omega, \lambda, \phi(\lambda, \omega, u(\omega)))\,.$$

I assume that $\phi_1, \phi_2 > 0$ and $\phi_3 < 0$. The last assumption implies that the frontier is strictly decreasing, and hence the inverse function $\Phi(\omega, \lambda, \cdot)$ exists. I also assume that utilities are not fully transferable, i.e., the utility possibility frontier is nonlinear. Under transferable utilities, the function $\phi(\lambda, \omega, u(\omega))$ is additively separable of the form $\phi(\lambda, \omega, u(\omega)) = S(\lambda, \omega) - u(\omega)$, where $S(\lambda, \omega)$ is the aggregate match surplus, which is given for the match and does not depend on the transfer.

I first describe how the matches are optimally formed in such a principal–agent economy. An allocation of this economy is an assignment rule l, together with the vectors v and u of payoffs such that $v(\lambda) \in$ v for each type λ principal, and $u(\omega) \in$ u for each type ω agent. An allocation (l, v, u) is a Walrasian equilibrium allocation if the following properties are satisfied: (a) Given $u(\omega)$ for each type ω agent, we have

$$w(\lambda) = \arg\max_{\omega}\{\phi(\lambda, \omega, u(\omega))\}, \qquad (1)$$

$$v(\lambda) = \max_{\omega}\{\phi(\lambda, \omega, u(\omega))\}$$

for each type λ principal. (b) If $[\lambda_1, \lambda_2] = l([\omega_1, \omega_2])$ for any subintervals $[\omega_1, \omega_2]$ of Ω and $[\lambda_1, \lambda_2]$ of Λ, then it must be the case that $G(\lambda_2) - G(\lambda_1) = F(\omega_2) - F(\omega_1)$.

The first condition implies that each type λ principal chooses her partner's type optimally. The second is a measure consistency requirement, which says that if an interval of agent types $[\omega_1, \omega_2]$ is mapped by the rule l into an interval of principal types $[\lambda_1, \lambda_2]$, then these two sets cannot have different measures. This implies the market clearing for each type.

Now suppose that the equilibrium assignment is given by $\lambda = l(\omega)$. The next step is to determine the sign of $l'(\omega)$. The following definition introduces the notion of assortative or monotone matching.

DEFINITION 1 *If $l'(\omega) \geq (\leq)\ 0$, then the assignment is said to be positively (negatively) assortative.*

Let me first describe how an equilibrium allocation is determined in the market. First, each type λ principal solves the maximization problem (1) in order to choose a partner type taking the wage $u(\omega)$ as given. This determines the optimal assignment rule $l(\omega)$. Next, from the market-clearing condition for each type of agent, the wage level $u(\omega)$ is determined. Finally, once the optimal matching and wages are known, the profit $v(\lambda)$ of each type λ principal is determined from $v(\lambda) = \phi(\lambda, l^{-1}(\lambda), u(l^{-1}(\lambda)))$. The following proposition provides the necessary and sufficient conditions for the optimality of $l(\omega)$ or $l^{-1}(\lambda)$ that solves the maximization problem (1).

PROPOSITION 1 *Suppose that $\phi_1, \phi_2 > 0$, $\phi_3 < 0$, and $\phi_2\phi_{31} - \phi_3\phi_{21} \geq 0$. The assignment rule $\lambda = l(\omega)$ or $\omega = l^{-1}(\lambda)$ is optimal, i.e., solves the maximization prob-*

lem (1)*, if and only if the following two conditions are satisfied: (a) For each* $\omega \in \Omega$*,*

$$u'(\omega) = -\frac{\phi_2(\lambda, \omega, u(\omega))}{\phi_3(\lambda, \omega, u(\omega))} \quad \textit{for } \lambda = l(\omega)\,; \tag{FOC}$$

(b) the assignment is positively assortative, i.e., $l'(\omega) \geq 0$.

The proofs of the above and all subsequent results are relegated to the appendix, section A.2. Notice that $\phi_2\phi_{31} - \phi_3\phi_{21} \geq 0$ is a *single-crossing* property of the utility possibility frontier ϕ with respect to (λ, ω). This says that the marginal gain of a type λ principal, ϕ_1, is increasing in ω, and better principals have higher marginal valuation for better agents. As a consequence, the payoff of every principal is maximized by assigning higher ω to higher λ. It follows from the envelope theorem that

$$v'(\lambda) = \phi_1(\lambda, \omega, u(\omega)) \quad \text{for } \lambda = l(\omega)\,. \tag{E}$$

Therefore, in every match, an agent's wage and a principal's profit are paid according to their contributions to the match surplus.

Although the utility possibility (Pareto) frontier has been taken as a primitive for our analysis, it actually depends on the preferences of the principals and agents, technology, etc., and hence is an endogenous object. Formally,

$$\phi(\lambda, \omega, u(\omega)) = \max_{x \in X(\lambda,\omega)} V(x, \lambda, \omega),$$

$$\text{subject to} \quad U(x, \lambda, \omega) = u(\omega)\,,$$

where V and U represent the preferences of the principal and agent, respectively, over allocations x and types λ and ω, and $X(\lambda, \omega)$ is the set of feasible allocations. The frontier described in Proposition 1 is derived from the above maximization problem when the constraint is binding, and hence is an endogenous object. Therefore, conditions must be imposed on the model fundamentals such as V, U, X, etc., which would imply the desired properties of the Pareto frontier under which the above proposition holds.

At this juncture let me remark on the two assumptions for Proposition 1, namely the single-crossing property and the strictly downward sloping utility possibility frontier. The assumptions on the model fundamentals that imply sufficient conditions for $\phi_2\phi_{31} - \phi_3\phi_{21} \geq 0$ are often cumbersome, and hence are difficult to check.[5] One may write down a specific model of a principal–agent relationship, solve for the optimal contract, and derive the Pareto frontier to see whether it is downward sloping and satisfies the single crossing condition so that Proposition 1 applies to this particular context.[6] Often, this route is easier. The next section makes an attempt in this regard, where a particular type of agency problem between the principals (landlords) and agents (tenants) is considered. It is shown that the characteristics of the model fundamentals imply the assumptions of Proposition 1, and hence the equilibrium assignment pattern is derived as a consequence of the above proposition.

[5] See the appendix, section A.1, for details.

[6] The conditions on the utility possibility frontier under which the above proposition holds will be verified in section 3.

Another important observation one must make is that differential equation (FOC) holds only if the frontier is strictly downward sloping, i.e., $\phi_3 < 0$. In many principal–agent relationships the frontier is not entirely downward sloping.[7] First, when the agency relationships are subject to limited liability, the agent may require an *efficiency wage*, a wage over and above his reservation wage, and hence part of the frontier may be flat, i.e., $\phi_3 = 0$. This situation will be analyzed in section 3. Another situation where one may find that the Pareto frontier is not entirely downward sloping is repeated principal–agent relationships where contracts may be incomplete. Fudenberg, Holmstrom, and Milgrom (1990) show that a downward-sloping utility possibility frontier that is invariant over time guarantees that the long-term contracts in a repeated principal–agent model are renegotiation-proof. However, as shown by Quadrini (2004), one may have the frontier upward sloping, and yet the equilibrium contracts are renegotiation-proof as long as the utility allocation does not belong to the upward-sloping part.[8] In such models the principal often fails to commit to some ex post efficient contracts, and hence if the continuation Pareto frontier is upward sloping over some range, it makes renegotiation viable.

In the next section I analyze principal–agent contracts subject to limited liability, and hence the utility possibility frontier of a given principal–agent pair may be flat for some parameter values, since the agent may earn an efficiency wage. I will further argue that a flat Pareto frontier will not emerge in a Walrasian equilibrium, since it is purely a partial-equilibrium phenomenon.

3 Application to Tenancy Contracts under Limited Liability

3.1 Description

This subsection considers optimal share-tenancy contracts between risk-neutral principals (landlords) and risk-neutral agents (tenants) in an attempt to identify a situation to which the results of Proposition 1 apply. Principals own a plot of land apiece, which can be leased out to a sharecropper, the agent. Agents are heterogeneous with respect to their initial wealth $\omega \in \Omega \subset \mathbb{R}_{++}$, which is distributed according to the cumulative distribution function $F(\omega)$. Here initial wealth represents the type or quality of an agent. On the other hand, $\lambda \in \Lambda \subset \mathbb{R}_{++}$ represents the productivity of lands, whose distribution function is $G(\lambda)$. I assume that $f(\omega) > 0$ for all ω and $g(\lambda) > 0$ for all λ. There are no alternative markets for land and labor services. Hence, an unused plot of land generates zero profit to its owner, and an unemployed agent consumes his wealth endowment. A land with quality λ produces a stochastic output, which is given by

$$\tilde{y} = \begin{cases} y_S & \text{with probability } \pi(\lambda, e), \\ y_F & \text{with probability } 1 - \pi(\lambda, e) , \end{cases}$$

[7] I thank an anonymous referee for pointing this out.

[8] See also Phelan and Townsend (1991) and Zhao (2006) for further analyses of repeated agency models.

where $e \in [0, e_{\max}]$ is the nonverifiable effort exerted by an agent. Subscripts S and F stand for "success" and "failure," respectively. I assume, without loss of generality, that $y_S = 1$ and $y_F = 0$. I make the following assumptions on the probability-of-success function $\pi(\lambda, e)$:

(1) $\pi(0, e) = \pi(\lambda, 0) = 0$ for all (λ, e);
(2) $\pi_\lambda(\lambda, e) > 0$, $\pi_e(\lambda, e) > 0$, $\pi_{ee}(\lambda, e) \leq 0$, and $\pi_{eee}(\lambda, e) = 0$ for all (λ, e);
(3) $\pi(\lambda, e)$ is log-supermodular in (λ, e), i.e., $\pi\pi_{e\lambda} - \pi_e\pi_\lambda \geq 0$.

Log-supermodularity of $\pi(\lambda, e)$ implies that land productivity and effort are complementary in determining the probability of success, and such complementarity is strong enough. This property is equivalent to the fact that the likelihood ratio $\pi_e(\lambda, e)/\pi(\lambda, e)$ is nondecreasing in λ. Notice that log-supermodularity implies supermodularity, i.e., $\pi_{e\lambda}(\lambda, e) > 0$. The agent incurs the total cost of effort, which is given by the function $\psi(e)$ with $\psi(0) = 0$ and with $\psi'(e) > 0$, $\psi''(e) > 0$, and $\psi'''(e) = 0$ for all $e \in [0, e_{\max}]$. The choice of effort by the agent is not publicly verifiable, and hence there is a potential moral-hazard problem in each partnership (λ, ω).[9,10]

3.2 Optimal Contract for an Arbitrary Partnership

I first analyze the optimal tenancy contract for an arbitrary match (λ, ω). A contract is a vector $c(\lambda, \omega) = (\alpha(\lambda, \omega), R(\lambda, \omega))$ where α is the agent's share of the match output $\tilde{y}$, and R is the fixed rental payment made to the principal. I restrict attention to the class of contracts for which $\alpha \in [0, 1]$. If $\alpha = 1$ and $R > 0$, then the contract is a *pure rent contract*. A contract with $\alpha < 1$, on the other hand, is referred to as a *share contract*. Define by

$$S(\lambda, e) := \pi(\lambda, e)y_S + [1 - \pi(\lambda, e)]y_F - \psi(e) = \pi(\lambda, e) - \psi(e)$$

the aggregate expected surplus of a match. Given the assumptions on $\pi(\lambda, e)$ and $\psi(e)$, the above surplus function is strictly concave in e for each λ with $S(\lambda, 0) = 0$. Therefore, for each λ there exists a unique level of effort $e^*(\lambda) > 0$ that maximizes $S(\lambda, e)$. I assume an interior maximum $e^*(\lambda)$ in the interval $(0, e_{\max})$. This is the first-best effort level, which is defined by

$$S_e(\lambda, e^*(\lambda)) = \pi_e(\lambda, e^*(\lambda)) - \psi'(e^*(\lambda)) = 0\,.$$

At the first-best effort level, the agent's marginal contribution to the gain in production is equated with his marginal cost of effort.

When the agent's effort choice is not contractible, any principal–agent relationship (λ, ω) is subject to the following *incentive compatibility* constraint (IC):

$$e = \arg\max_{e'}\{\alpha\pi(\lambda, e') - R - \psi(e')\}\,.$$

[9] Notice that log-supermodularity of $\pi(\lambda, e)$ is not a necessary condition for the results to hold.
[10] All the subsequent results hold under $\pi_{eee} \leq 0$ and $\psi'''(e) \geq 0$.

Given that $\pi(\lambda, e)$ is concave and $\psi(e)$ is strictly convex in e, there is a unique interior solution to the above maximization problem, and the incentive compatibility constraint can be replaced by the following first-order condition (IC1):

$$\alpha\pi_e(\lambda, e) = \psi'(e)\,.$$

Notice that the above equation implicitly defines e as a function of α. Call it $e(\alpha)$. It is easy to show that $e'(\alpha) > 0$, i.e., the agent's effort is increasing in his output share. Therefore, the share α of the agent measures the power of incentives in a given landlord–tenant match. Within each principal–agent relationship (λ, ω), the principal therefore solves the following maximization problem:

$$\text{(M)}\qquad \phi(\lambda, \omega, \bar{u}(\omega)) = \max_{\{\alpha(\lambda,\omega), R(\lambda,\omega), e(\lambda,\omega)\}} \{[1-\alpha(\lambda,\omega)]\pi(\lambda, e(\lambda,\omega)) + R(\lambda,\omega)\}$$

$$\text{(IR)}\qquad \text{subject to}\quad \alpha(\lambda,\omega)\pi(\lambda, e(\lambda,\omega)) - R(\lambda,\omega) - \psi(e(\lambda,\omega)) \geq \bar{u}(\omega)\,,$$

$$\text{(IC1)}\qquad \alpha(\lambda,\omega)\pi_e(\lambda, e(\lambda,\omega)) = \psi'(e(\lambda,\omega))\,,$$

$$\text{(LL)}\qquad R(\lambda,\omega) \leq \omega\,,$$

$$\text{(F)}\qquad 0 \leq \alpha(\lambda,\omega) \leq 1\,.$$

Call the above maximization problem $\mathcal{M}$. The first constraint is the *individual rationality* constraint (IR) of the agent. The agent must receive in expectation no less than his outside option $\bar{u}(\omega)$. I assume that $\bar{u}'(\omega) \geq 0$.[11] The third constraint is the *limited liability* constraint (LL), which guarantees nonnegative final income to the agent even when the match output is low. The last constraint is the *feasibility* constraint (F), which implies that both the principal and the agent receive a nonnegative share of the final output. Let $\alpha^0(\lambda)$ and $\hat{\alpha}(\lambda, \omega, \bar{u}(\omega))$ solve the following two equations, respectively:

$$[1-\alpha^0(\lambda)]\pi_e(\lambda, e(\alpha^0(\lambda)))e'(\alpha^0(\lambda)) = \pi(\lambda, e(\alpha^0(\lambda)))\,,$$

$$\hat{\alpha}(\lambda,\omega,\bar{u}(\omega))\pi(\lambda, e(\hat{\alpha}(\lambda,\omega,\bar{u}(\omega)))) - \psi(e(\hat{\alpha}(\lambda,\omega,\bar{u}(\omega)))) = \omega + \bar{u}(\omega)\,.$$

The following lemma characterizes the optimal effort and contract associated with an arbitrary partnership (λ, ω).

LEMMA 1 *Consider an arbitrary match (λ, ω). There exist positive real numbers $A^0(\lambda)$, $A^*(\lambda)$, and $\bar{A}(\lambda)$ with $A^0(\lambda) < A^*(\lambda) < \bar{A}(\lambda)$ such that the optimal share, which solves the maximization problem $\mathcal{M}$, is given by*

$$\alpha(\lambda,\omega) = \begin{cases} \alpha^0(\lambda) & \text{if } \omega < A^0(\lambda), \\ \hat{\alpha}(\lambda,\omega,\bar{u}(\omega)) & \text{if } A^0(\lambda) \leq \omega \leq A^*(\lambda), \\ 1 & \text{if } A^*(\lambda) < \omega \leq \bar{A}(\lambda)\,. \end{cases}$$

The optimal rental payment is given by

$$R(\lambda,\omega) = \min\{\omega, \pi(\lambda, e^*(\lambda)) - \psi(e^*(\lambda)) - \bar{u}(\omega)\}\,.$$

[11] The agent may be assumed to have a backyard technology from which he earns $\bar{u}(\omega)$ using the resources ω, which is nondecreasing in ω.

And the optimal effort of the agent is given by

$$\alpha(\lambda, \omega)\pi_e(\lambda, e(\alpha(\lambda, \omega))) = \psi'(e(\alpha(\lambda, \omega)))\,.$$

When the wealth level of the agent is very low, and so is his outside option $\bar{u}(\omega)$, the agent requires to be paid an expected wage over and above this level in order to be induced to exert the lowest incentive-compatible effort level $e(\alpha^0(\lambda))$. Therefore, his individual rationality constraint does not bind. In other words, the agent earns *efficiency wages*. For the intermediate levels of wealth and outside option, both the limited-liability and the individual-rationality constraint bind. Since the incentive problem is still costly for the principal in view of the moral-hazard problem, the optimal effort is less than the first-best level, and the optimal share is less than 1. Finally, when the wealth level of the agent is very high, the limited-liability constraint does not bind, and the optimal effort is at its first-best level. The corresponding share is equal to 1, i.e., it is optimal for the principal to sell the firm to the agent in exchange for a fixed price $R(\lambda, \omega) = \pi(\lambda, e^*(\lambda)) - \psi(e^*(\lambda)) - \bar{u}(\omega)$.

Example 1. Assume that $\pi(\lambda, e) = \lambda e$ and $\psi(e) = e^2/2$. Note that the probability-of-success function is not log-supermodular, but is supermodular. In this case, the optimal share is given by

$$\alpha(\lambda, \omega) = \begin{cases} \frac{1}{2} & \text{if } \omega + \bar{u}(\omega) < \frac{\lambda^2}{8}, \\ \frac{1}{\lambda}\sqrt{2[\omega + \bar{u}(\omega)]} & \text{if } \frac{\lambda^2}{8} \le \omega + \bar{u}(\omega) \le \frac{\lambda^2}{2}, \\ 1 & \text{if } \frac{\lambda^2}{2} < \omega + \bar{u}(\omega) \le \frac{\lambda^2}{2} + \omega\,. \end{cases}$$

The optimal rental payment is given by

$$R(\lambda, \omega) = \min\left\{\omega, \frac{\lambda^2}{2} - \bar{u}(\omega)\right\}.$$

Finally, the optimal effort is given by

$$e(\lambda, \omega) = \begin{cases} \frac{\lambda}{2} & \text{if } \omega + \bar{u}(\omega) < \frac{\lambda^2}{8}, \\ \sqrt{2[\omega + \bar{u}(\omega)]} & \text{if } \frac{\lambda^2}{8} \le \omega + \bar{u}(\omega) \le \frac{\lambda^2}{2}, \\ \lambda & \text{if } \frac{\lambda^2}{2} < \omega + \bar{u}(\omega) \le \frac{\lambda^2}{2} + \omega\,. \end{cases}$$

The limits on ω as in Lemma 1 are given by $A^0(\lambda) = \tilde{u}^{-1}(\lambda^2/8)$, $A^*(\lambda) = \tilde{u}^{-1}(\lambda^2/2)$, and $\bar{A}(\lambda) = \bar{u}^{-1}(\lambda^2/2)$, where $\tilde{u}(\omega) := \omega + \bar{u}(\omega)$.

The following lemma analyzes the comparative-statics results of the optimal effort and incentives with respect to λ and ω.

LEMMA 2 *Let $e(\lambda, \omega)$ and $\alpha(\lambda, \omega)$ be the optimal effort and output share described in Lemma 1. Then:*

(a) The optimal effort and share are increasing in ω, but nonmonotone in λ. In particular, there is a unique positive real number $B^0(\omega)$ such that the optimal effort and share are nonincreasing (nondecreasing) for $\lambda < (\ge)\ B^0(\omega)$.

(b) The optimal rental payment is nondecreasing in λ, but nonmonotone in ω. In particular, there is a unique positive real number $A^*(\lambda)$ such that the optimal rent is nondecreasing (nonincreasing) for $\omega < (\geq)\ A^*(\lambda)$.

Since $\pi_{e\lambda}(\lambda, e) > 0$ [implied by log-supermodularity of $\pi(\lambda, e)$], an increase in land productivity has favorable impact on the marginal effect of an increase in the agent's effort. As a consequence, the optimal incentives become less high-powered, since it is not necessary to offer stronger incentives to the agent to undertake higher effort. Since the share is used in order to compensate the agent for the increased marginal cost of effort, the resulting optimal effort is also lower following an increase in λ. However, when the agent's individual-rationality constraint does not bind at the second-best contract, the incentives may become more high-powered following an increase in λ, since it is necessary to offer stronger incentives to the agent as the incentive problem is very severe. When the agent's initial wealth is higher, his outside option is higher too, and hence he has greater bargaining power. Therefore, it is necessary for the principal to give up a greater share of output to the agent. Consequently, the agent exerts higher effort. Finally, when the incentive problem is important (i.e., the limited-liability constraint is binding), the optimal rental payments equal the agent's wealth, and hence higher wealth implies higher rent. When the first-best contracts are implemented under nonbinding limited liability, the firm is sold to the agent. The selling price R will be lower as the outside option of the type ω agent increases, for the principal now has to give up higher rent to the agent. Following an increase in the land productivity, the optimal rent increases, since the principal's role in surplus creation becomes more important.[12]

Finally, the following lemma states some important properties of the utility possibility frontier of a given match (λ, ω).

LEMMA 3 *Under the optimal contract $c(\lambda, \omega) = (\alpha(\lambda, \omega), R(\lambda, \omega))$ and effort $e(\lambda, \omega)$ associated with an arbitrary match (λ, ω), the Pareto frontier is given by*

$$\phi(\lambda, \omega, u(\omega))$$
$$= \begin{cases} [1-\alpha^0(\lambda)]\pi(\lambda, e(\alpha^0(\lambda))) + \omega & \text{if } \omega < A^0(\lambda), \\ [1-\hat{\alpha}(\lambda, \omega, \bar{u}(\omega))]\pi(\lambda, e(\hat{\alpha}(\lambda, \omega, \bar{u}(\omega)))) + \omega & \text{if } A^0(\lambda) \leq \omega \leq A^*(\omega), \\ \pi(\lambda, e^*(\lambda)) - \psi(e^*(\lambda)) - \bar{u}(\omega) & \text{if } A^*(\lambda) < \omega \leq \bar{A}(\lambda)\,. \end{cases}$$

[12] An alternative contracting scenario would be to consider $c = (x(1), x(0))$, where $x(\tilde{y})$ is the wage of the agent at state $\tilde{y} \in \{0, 1\}$, instead of (α, R). In this case the incentive component would be $x(1) - x(0)$, which can be interpreted as a bonus for better performance, instead of α. The limited-liability constraint would modify to $x(0) \geq -\omega$, which means that the monetary punishment that can be inflicted on the agent is bounded above by his initial wealth. All the subsequent results of the current paper would be the same with this alternative specification. This equivalence between the nonlinear and linear contracts is typical for a two-point distribution of output. See Laffont and Matoussi (1995) and Ray and Singh (2001) for the optimality of linear contracts under limited liability.

Also, $\phi_1 = [1 - \nu\alpha(\lambda, \omega)]\pi_\lambda > 0$*,* $\phi_2 = \nu \in [0, 1]$*, and* $\phi_3 = \nu - 1 \in [-1, 0)$*, where* ν *is the Lagrange multiplier associated with the limited-liability constraint in the maximization problem* $\mathcal{M}$*. Moreover,* $\phi_{21} \geq 0$ *and* $\phi_{31} \geq 0$*.*

The utility possibility frontier is nondecreasing in land productivity and wealth of the agent, since a higher value of each of them increases the expected match surplus. Notice that for values of the parameters (λ, ω) such that $\omega \geq A^0(\lambda)$, the individual-rationality constraint of the type ω agent binds, and hence $\phi_3 < 0$, i.e., the frontier is strictly downward sloping. Notice also that $\phi_{21} \geq 0$ and $\phi_{31} \geq 0$ together imply that the frontier satisfies the single-crossing property. Thus, the equilibrium matching pattern can be derived from Proposition 1.

Example 1 (continued). The Pareto frontier is given by

$$\phi(\lambda, \omega, \bar{u}(\omega)) = \begin{cases} \frac{\lambda^2}{4} + \omega & \text{if } \omega + \bar{u}(\omega) < \frac{\lambda^2}{8}, \\ \lambda\sqrt{2[\omega + \bar{u}(\omega)]} - \omega - 2\bar{u}(\omega) & \text{if } \frac{\lambda^2}{8} \leq \omega + \bar{u}(\omega) \leq \frac{\lambda^2}{2}, \\ \frac{\lambda^2}{2} - \bar{u}(\omega) & \text{if } \frac{\lambda^2}{2} < \omega + \bar{u}(\omega) \leq \frac{\lambda^2}{2} + \omega\,. \end{cases}$$

Notice that for $\omega + \bar{u}(\omega) < \lambda^2/8$ we have $\phi_1 = \lambda/2 > 0$, $\phi_2 = 1$, and $\phi_3 = 0$. For $\lambda^2/8 \leq \omega + \bar{u}(\omega) \leq \lambda^2/2$ we have $\phi_1 = [2(\omega + \bar{u}(\omega))]^{1/2} > 0$, $\phi_2 = \lambda[2(\omega + \bar{u}(\omega))]^{-1/2} - 1 > 0$, and $\phi_3 = \lambda[2(\omega + \bar{u}(\omega))]^{-1/2} - 2 < 0$. Finally, for $\omega + \bar{u}(\omega) > \lambda^2/2$ we have $\phi_1 = \lambda$, $\phi_2 = 0$, and $\phi_3 = -1$. Therefore, for $\omega + \bar{u}(\omega) \geq \lambda^2/8$, the single-crossing property of the Pareto frontier holds.

Notice that when $\omega < A^0(\lambda)$ the individual rationality constraint does not bind, i.e., the agent earns the efficiency wage. In this case the maximum expected payoff of the principal does not depend on the agent's outside option, i.e., $\phi_3 = 0$. In other words, the Pareto frontier is flat for $\omega < A^0(\lambda)$. This clearly violates the first assumptions of Proposition 1.

3.3 Wages, Profits, and Assignment in a Walrasian Allocation

The Walrasian equilibrium allocation of this landlord–tenant economy is determined as described in section 2.2. While solving the maximization problem (1), a principal must guarantee the agent his outside option. The outside option $\bar{u}(\omega)$ of a type ω agent who is matched with a type λ principal is the maximum payoff he can obtain by switching to other matches, i.e.,

$$\bar{u}(\omega) = \max_{\lambda' \in \Lambda} \Phi(\omega, \lambda', v(\lambda'))\,.$$

In a Walrasian equilibrium, the wage or income of a type ω agent is equal to his outside option, i.e., $u(\omega) = \bar{u}(\omega)$ for each $\omega \in \Omega$. I first argue that in every match (λ, ω) the individual rationality constraint of the agent must bind in a Walrasian equilibrium. Suppose an agent of type ω is offered $u(\omega)$ in an equilibrium allocation. Since there is a continuum of types, one can find an identical principal who would also offer $u(\omega)$ to the same agent, and hence $u(\omega)$ actually becomes the agent's outside option. Thus, any wage offer strictly above an agent's outside option cannot be a Walrasian wage.[13]

Notice also that in an equilibrium allocation, any given principal–agent pair (λ, ω) will choose a contract that generates a utility allocation that lies in the associated Pareto frontier. Thus, the equilibrium contract for such a match will be the one described in the previous subsection.

PROPOSITION 2 *The equilibrium profit $v(\lambda)$ is an increasing function of land quality, and the equilibrium wage $u(\omega)$ is a nondecreasing function of the initial wealth.*

At the end of this subsection I will determine which partnerships in a Walrasian equilibrium will implement the first-best contracts, and which are the matches that will sign the second-best contracts. For the time being, suppose that some principal–agent partnerships in equilibrium implement the first-best contracts, and the remaining ones implement the second-best contracts. When the first-best contracts are implemented, the Pareto frontier is independent of ω, as the limited liability constraints do not bind, i.e., the initial wealth of each agent does not influence the contract offered to him. As a consequence, one has $\phi_2 = 0$. On the other hand, the Pareto frontier is linear in $u(\omega)$ with slope ϕ_3 equal to -1 due to the absence of incentive problems. Therefore, from condition (FOC) one has $u'(\omega) = 0$. When the second-best contracts are implemented, it is the case that $\phi_2 > 0$ and $\phi_3 < 0$, and hence $u'(\omega) > 0$. However, under both the first- and second-best contracts, $\phi_1 > 0$, and hence $v'(\lambda) > 0$. Irrespective of whether the first-best or the second-best contracts are implemented, owners of more productive lands and wealthier agents have absolute advantages in producing surplus in any match. Therefore, high-quality principals obtain greater equilibrium profits, and wealthier agents consume higher wages in equilibrium.

In order to determine the equilibrium assignment, Proposition 1 implies that it is sufficient to check the signs of ϕ_{21} and ϕ_{31} in the present context. Clearly, $\phi_{21} = \phi_{31} = 0$ if the first-best contracts are implemented, and under the second-best, on the other hand, one has $\phi_{21} = \phi_{31} > 0$. Therefore the equilibrium matching pattern follows from Proposition 1, and is described in the following proposition.

PROPOSITION 3 *Let $\lambda = l(\omega)$ be an equilibrium assignment.*

(a) If the first-best efforts and contracts are implemented, then any matching pattern is consistent with an equilibrium allocation;

(b) if the second-best efforts and contracts are implemented, then the equilibrium assignment is positively assortative, i.e., wealthier agents cultivate more productive lands.

Notice that if the first-best contracts are implemented, then $\phi_{21} + \phi_{31}u'(\omega) = 0$. Since $u'(\omega) = 0$ and ϕ is independent of ω, any matching pattern would entail

[13] For an agent, a slack individual rationality constraint is a partial-equilibrium phenomenon where only a single principal–agent pair is considered, which cannot occur in a principal–agent market. A Walrasian allocation is often characterized by a *no-surplus condition*, i.e., $u(\omega) = \bar{u}(\omega)$ for each ω (see Ostroy, 1984).

the same expected profit to a given type λ principal in equilibrium, and therefore any matching pattern would be consistent with an equilibrium allocation. When the limited-liability constraints bind for some matches, the equilibrium contracts depend on the wealth endowment. As ϕ_{21} and ϕ_{31} are both strictly positive, higher wealth has greater impact on the match surplus when combined with more productive land. In other words, wealthier agents have comparative advantages in matches consisting of more productive principals, and hence the unique equilibrium matching is positively assortative.

A positively assortative matching is a consequence of complementarity between the principals' and agents' qualities. Complementarity implies that high-quality agents have comparative advantages over the low-quality ones in matches involving high-quality principals. This sort of comparative advantage determines that better agents must be assigned better principals. Notice that $\phi_{21} > 0$ and $\phi_{31} > 0$ are sufficient conditions for a positive sorting, since under these conditions the single-crossing property is satisfied, as $u'(\omega) > 0$. When utilities are not fully transferable, the complementarity has two aspects. First, $\phi_{21} > 0$ implies that a high-quality principal and a high-quality agent together produce higher aggregate surplus. This is the usual *type–type* complementarity, which also determines positive sorting in the standard assignment models (e.g., Rosen, 1974; Sattinger, 1979). Second, $\phi_{31} > 0$ implies that it is (marginally) less costly for a high-type principal to transfer surplus to a high-type agent. This *type–payoff* complementarity reinforces the conditions under which an equilibrium induces a positively assortative matching. It may be the case that $\phi_{21} < 0$ and $\phi_{31} > 0$, yet the single-crossing condition holds. Then also the matching will be positively assortative, as the type–payoff complementarity outweighs the type–type substitutability.

It is not so difficult to show, in a second-best contract, that

$$\begin{aligned}&\operatorname{sign}[\phi_{21}] = \operatorname{sign}[\phi_{31}]\\ &= \operatorname{sign}\left[(1-\hat{\alpha})(\pi\pi_{e\lambda} - \pi_e\pi_\lambda) + \frac{\hat{\alpha}\pi_\lambda\{\psi''\pi + (1-\hat{\alpha})\pi_e^3 - \pi\pi_{ee}(2-\hat{\alpha}+(1-\hat{\alpha})\pi_e)\}}{\pi(\psi'' - \hat{\alpha}\pi_{ee})}\right].\end{aligned}$$

Given that $\pi_{ee} \leq 0$ and $\psi'' > 0$, the second term of the above expression is strictly positive. Since $\pi(\lambda, e)$ is log-supermodular, i.e., $\pi\pi_{e\lambda} - \pi_e\pi_\lambda \geq 0$, one has $\phi_{21} > 0$ and $\phi_{31} > 0$. Note that log-supermodularity of $\pi(\lambda, e)$ is not necessary for the single-crossing condition to hold. In Example 1, the probability-of-success function is not log-supermodular, yet the utility possibility frontier satisfies the single-crossing condition.

If in a Walrasian equilibrium a given land quality λ is assigned to a given wealth level ω, then a positively assortative matching implies

$$F(\omega) = G(\lambda) \quad \Longrightarrow \quad l(\omega) = G^{-1}(F(\omega))\,.$$

The function $G(\cdot)$ is invertible, since $g(\lambda) > 0$ for each λ. From the above, it also follows that

$$l'(\omega) = \frac{f(\omega)}{g(l(\omega))}\,.$$

The above equation describes how the slope of the matching function at a given wealth level ω changes with respect to the dispersion of land quality relative to that of wealth at this point. The slope is greater (less) than 1 if land productivity is more (less) disperse than wealth around ω, i.e., $f(\omega) > (<)\ g(l(\omega))$. For example, when λ and ω are uniformly distributed, one has $l'(\omega) = \Delta\lambda/\Delta\omega$, where $\Delta\lambda := \lambda_{\max} - \lambda_{\min}$ and $\Delta\omega := \omega_{\max} - \omega_{\min}$, and hence the matching function has slope greater (less) than 1 if the distribution of λ has higher (lower) variance than that of the distribution of ω.[14]

Example 1 (continued). Let λ and ω be both uniformly distributed on $[2.5, 5]$. Then the matching function is given by

$$l(\omega) = \omega\,.$$

Now consider the equilibrium wage function when the second-best contracts are implemented, which is given by the following differential equation:

$$u'(\omega) = \frac{1 - \hat{\alpha}(l(\omega), \omega, u(\omega))}{2\hat{\alpha}(l(\omega), \omega, u(\omega)) - 1} = \frac{\omega - \sqrt{2(\omega + u(\omega))}}{2\sqrt{2(\omega + u(\omega))} - \omega}\,.$$

The equilibrium wage function $u(\omega)$ given by the above differential equation is drawn in the following figure.[15]

Notice that for $\hat{\alpha}(l(\omega), \omega, u(\omega))$ to lie in $[0.5, 1]$, one must specify $\omega \in (2, 8)$. The initial value of the wage $u(2.5)$ must also be low enough in order that $u'(\omega) > 0$. For the figure below, $u(2.5)$ is taken to be equal to 0.1.

Having determined the optimal assignment rule $l(\omega)$, now it is easy to determine which partnerships will implement the first-best contracts. From Lemma 1 it follows that, for any arbitrary partnership (λ, ω), the contracts are second-best contracts if $\omega \le A^*(\lambda)$. It is easy to show that $dA^*(\lambda)/d\lambda > 0$, which together with $l'(\omega) \ge 0$ implies the following result.

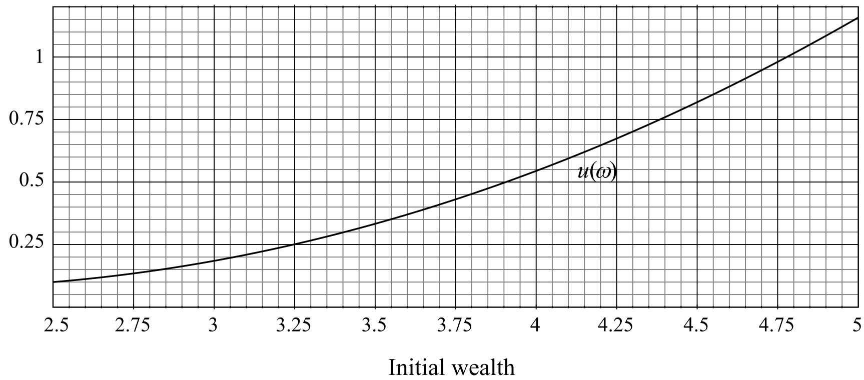
PROPOSITION 4 *There exists a unique ω^*, defined by $\omega^* = A^*(l(\omega^*))$, such that any equilibrium partnership involving an agent with wealth level $\omega \le (>)\ \omega^*$ implements the second-best (first-best) efforts and contracts.*

The agents with low initial wealth are matched with principals owning lands with low productivity. In such partnerships, the limited-liability constraints are more likely to bind, and hence such agents are more difficult to incentivize. Therefore, the partnerships involving agents with low initial wealth implement the second-best

[14] The probability density can be interpreted as a local measure of dispersion of a random variable. Suppose a random variable $\tilde{x}$ has a density function $h(x)$. Consider two distinct realizations x_1 and x_2 of the random variable such that it is more disperse around x_1 than around x_2. This implies that higher probability mass is concentrated in $[x_2 - \varepsilon, x_2 + \varepsilon]$ than in $[x_1 - \varepsilon, x_1 + \varepsilon]$ for $\varepsilon > 0$, i.e., $h(x_2) > h(x_1)$.

[15] The figure has been drawn by using the in-built *Grapher* application of Mac OSX, which uses the Runge–Kutta fourth-order method to draw graphs of solutions of an ordinary differential equation.

Figure 1
The Equilibrium Wage Function



Notes: The equilibrium income function defined by the differential equation (2). The initial distributions of wealth and land productivity are uniform over the support [2.5, 5]. The initial income is given by $u(\omega) = 0.1$.

contracts. High wealth levels, on the other hand, are assigned to lands with high productivity. As a consequence, in such matches incentive problems are not so costly, and the first-best contracts can be implemented. Since the first best implies a *franchise* contract, i.e., $\alpha = 1$ and $R \geq 0$, all the agents with initial wealth less than ω^* will become sharecroppers in equilibrium. This result conforms to the finding of Braido (2008) that lower-quality lands are typically leased to the sharecroppers.

4 *Testable Implications*

4.1 *Equilibrium Contracts*

The principal objective of this subsection is to analyze the behavior of the equilibrium share and effort with respect to initial wealth. In a partial-equilibrium setup where a principal–agent pair is treated in isolation, the contract for the pair in general depends on two parameters: the productivity of the land and the wealth of the agent. For instance, the optimal output share of the agent in an arbitrary pair (λ, ω) is given by $\alpha(\lambda, \omega)$. To analyze the behavior of α in this context with respect to ω, one must determine the sign of $\partial\alpha/\partial\omega$, which, following Lemma 2, is nonnegative (e.g., Ray and Singh, 2001, Proposition 3). I show that the monotonicity of the agents' output share with respect to initial wealth may not hold in a principal–agent market with two-sided heterogeneity, and look for sufficient conditions under which the equilibrium share is monotone in wealth.

I focus only on the equilibrium allocations that implement the second-best contracts. As I have shown earlier that the individual rationality of each agent binds in a Walrasian equilibrium, I only consider the second-best contracts when both the individual-rationality and limited-liability constraints bind. The initial wealth ω, apart from directly influencing the incentive-compatible contracts and effort, affects their optimal values in two indirect ways: through the matching and the wage functions. Thus in equilibrium, incentives and effort are solely functions of ω, which are given by

$$\alpha(\omega) = \hat{\alpha}(l(\omega), \omega, u(\omega)), \quad e(\omega) = \hat{e}(l(\omega), \omega, u(\omega)) . \tag{2}$$

Recall that $\hat{\alpha}$ and $\hat{e}$ are respectively the optimal share and effort of the type ω agent in a match (λ, ω) with $\lambda = l(\omega)$ when both the individual-rationality and limited-liability constraints of the agent bind. Consider the equilibrium share function $\alpha(\omega)$. Differentiation of (2) with respect to ω gives

$$\alpha'(\omega) = \underbrace{\frac{\partial\hat{\alpha}}{\partial\lambda} l'(\omega)}_{\text{matching effect}} + \underbrace{\frac{\partial\hat{\alpha}}{\partial\omega}[1 + u'(\omega)]}_{\text{wealth effect}} . \tag{3}$$

There are two channels through which ω affects the equilibrium share function. The first is a *wealth effect*, which is positive, since $\partial\hat{\alpha}/\partial\omega > 0$ and $u'(\omega) > 0$. The second is a *matching effect*, which works through the equilibrium matching function $\lambda = l(\omega)$. Agents with higher wealth are matched with principals owning lands with higher productivity. Since from the inequality (A10) in the appendix, section A.2, one has $\partial\hat{\alpha}/\partial\lambda < 0$, this matching effect dampens the positive impact of increased wealth on incentives. Therefore, the equilibrium share function is in general nonmonotone with respect to ω. These findings are summarized in the following proposition. A similar result holds for the equilibrium second-best effort, since

$$e'(\omega) = \frac{\partial\hat{e}}{\partial\lambda} l'(\omega) + \frac{\partial\hat{e}}{\partial\omega}[1 + u'(\omega)] .$$

Finally, notice that in the second-best contracts, the rental payment in each match depends only on the initial wealth of the tenant, and hence the equilibrium rent function is strictly increasing in wealth. Those findings are also summarized in the following proposition.

PROPOSITION 5 *Let $e(\omega)$, $\alpha(\omega)$, and $R(\omega)$ be the equilibrium effort, share, and rent functions, respectively. Then in a Walrasian equilibrium, (a) the equilibrium effort and share functions are in general nonmonotone in ω; (b) the rental payments are increasing in ω.*

As I have discussed earlier, nonmonotonicity of the equilibrium share with respect to the initial wealth of the agents is a consequence of the interaction of two countervailing forces. But most of the empirical studies, such as Laffont and Matoussi (1995) and Ackerberg and Botticini (2002), find evidence that tenants' output shares in general increase with their initial wealth. Therefore, it would be interesting to see under what conditions the equilibrium share function is monotone with respect to ω. In order to see that, consider the two limiting cases. First, suppose the lands are almost identical, i.e., $\lambda \to \hat{\lambda}$, but the agents are sufficiently heterogeneous. Then $l'(\omega) \to 0$ (almost flat matching function) and $u'(\omega) > 0$. Then from equation (3), clearly $\alpha'(\omega)$ remains strictly positive. Next consider the other limiting case, when the agents become almost homogeneous, i.e., $\omega \to \hat{\omega}$, but the lands remain sufficiently heterogeneous. In this case, $u'(\omega) \to 0$ but $l'(\omega) \to \infty$ (almost vertical matching function). Then from (3) it follows that $\alpha'(\omega) \to -\infty$, since $\partial\alpha/\partial\lambda < 0$. Therefore, by continuity of $\alpha'(\omega)$, one can conclude the following:

PROPOSITION 6 *In one limiting case, when the distribution of land quality becomes homogeneous (i.e., $\lambda \to \hat{\lambda}$ for some $\hat{\lambda} > 0$), the equilibrium share function $\alpha(\omega)$ is increasing in ω. In the other limiting case, when the distribution of initial wealth becomes homogeneous (i.e., $\omega \to \hat{\omega}$ for some $\hat{\omega} > 0$), the equilibrium share function $\alpha(\omega)$ is decreasing in ω.*

Standard contract theory would predict that the incentive should be more high-powered for high-wealth agents. Empirical evidence often suggests a positive relationship between incentive and wealth rather than the posited negative trade-off (see Ackerberg and Botticini, 2002). The standard explanation is that when contracts are negotiated in a principal–agent market rather than in an isolated principal–agent relationship, the relative market power of each side of the market influences the equilibrium contracts in an important way. This is because the agents may self-select themselves into different tasks, which explains the matching effect. Prasad and Salmon (2013) provide experimental evidence for favorable contract terms for the agents when they are on the short side of the market.

The sufficient condition in the above proposition goes along this line of argument. When the principals are almost homogeneous, the negative matching effect on incentives is dampened by the positive wealth effect. Therefore, higher output share for the agent is associated with higher initial wealth. In a principal–agent market with two-sided heterogeneity, the individuals on one side compete for the best individuals of the other side. When the agents are very heterogeneous relative to the principals, competition for the best agents is more fierce than that for the best principals. As a consequence, the wealthier agents end up receiving greater shares of the match output. On the other hand, if the distribution of wealth is too tight relative to the distribution of land quality, then a less wealthy agent receives a greater output share than the wealthier agents.

4.2 Wage Inequality

Now I analyze the shape of the wage function, and relate it to the nature of wage or income inequality in equilibrium. Notice that when the first-best contracts are implemented, $u'(\omega) = 0$ for all $\omega > \omega^*$, i.e., all the agents in partnerships that implement the first-best contracts receive the same expected wage. Therefore, there is no wage inequality among these agents. Let us focus on the matches that implement the second-best contracts. It is possible to show that $u''(\omega) \neq 0$ for $\omega \leq \omega^*$. Thus, one can conclude the following:

PROPOSITION 7 *There is no wage inequality for the agents with initial wealth $\omega > \omega^*$. For the partnerships consisting of agents with wealth $\omega \leq \omega^*$ the equilibrium wage function $u(\omega)$ exhibits wage inequality.*

The above proposition suggests that among the wealthiest of the tenants there is no wage inequality, since the partnerships involving these agents implement the first-best contracts. Since under the first-best contracts any matching may emerge in equilibrium, the wealthier individuals among the wealthiest agents are not able to exploit their comparative advantages in higher-quality land. On the other hand, since for low wealth level the partnerships implement the second-best contracts, the wealthier agents over this range of wealth distribution now obtain higher marginal returns because of comparative advantage, and hence, the equilibrium wage function exhibits wealth inequality.

4.3 Skewness of the Income Distribution

Proposition 7 relates the initial distributions of types to the changes in wage inequality, but does not provide any conclusions regarding the final distribution of wages. A well-known result in assignment models (e.g., Sattinger, 1979; Kremer, 1993; Teulings, 1995) is that the distribution of wages is positively skewed relative to the distribution of initial wealth. The same is true in the present context, as is implied by the single-crossing property of the Pareto frontier.[16]

First, notice that the distribution of wages would have the same shape as the wealth distribution if all agents had chosen to work in the lands with identical productivity. When the first-best contracts are implemented in equilibrium, the wage of any type ω agent does not vary with respect to λ, and hence the distribution of wages has the same shape as the distribution of initial wealth for wealth levels $\omega > \omega^*$. This result does not hold when the second-best contracts are implemented, i.e., $\omega \leq \omega^*$. With two-sided heterogeneity, in a second-best equilibrium, wealthier agents are assigned to lands with greater productivity. Therefore, the distribution of equilibrium wages will never have the same shape as the distribution of wealth. Differentiation of (FOC) with respect to λ gives

$$\frac{\partial u'(\omega)}{\partial \lambda} = \frac{1}{\phi_3^2}[\phi_2\phi_{31} - \phi_3\phi_{21}]\,. \tag{4}$$

The above derivative is positive, given the single-crossing condition. This implies that positive sorting enhances the wages of the agents above what they would have earned by working for the principals of the same quality. Therefore, the distribution of equilibrium wages will be positively skewed compared to the distribution of initial wealth.

[16] The proof of this assertion, which is easily adapted from Sattinger (1975) and Teulings (1995), is presented in the appendix, section A.3.

5 Discussion

One of the main objectives of an assignment model is to offer a unified framework that is able to simultaneously determine the (incentive) structure as well as the level of compensation of the agents. Consider first the partial-equilibrium framework described in section 3.2, where neither an agent nor a principal has the option to seek a partner other than the one he or she is assigned to. Due to such restrictions, the outside option of a given type ω agent, $\bar{u}(\omega)$, is exogenously given. Since the individual-rationality constraint of the agent binds at the optimum, the level of his compensation is completely determined by his exogenously given outside option. Therefore, the optimal contracting problem determines only the structure of incentives, i.e., the optimal rent and the output share of the agent. In an assignment model, on the other hand, since each individual is free to choose a partner, the outside option, and hence the level of compensation of each type ω agent, is endogenously determined in equilibrium. One of the main differences between a partial-equilibrium approach and an assignment model is reflected in Lemma 2 and Proposition 5. Lemma 2 analyzes the way in which the optimal incentive scheme responds to a change in ω, and shows that an agent's optimal output share is monotone with respect to initial wealth. Such monotonicity breaks down in an assignment model, due to the presence of a general-equilibrium effect. A change in ω not only has a positive direct effect, but also has an indirect dampening effect on α through the equilibrium principal–agent assignment. Due to the presence of these two countervailing effects, the equilibrium share function is in general nonmonotone with respect to ω, as is analyzed in Proposition 5.

6 Conclusion

Incentive contracts may be quite different in a market with many heterogeneous principals and agents from the contracts for an isolated principal–agent partnership. In the equilibrium, individual contracts are influenced by the two-sided heterogeneity via principal–agent matching. In this paper, I have developed a simple assignment model of incentive contracting between principals and agents. Agents who differ in their wealth endowment are assigned to lands differing in productivity. In a Walrasian equilibrium of the market, wealthier agents work on more productive lands, since they have comparative advantages in high-quality lands. Optimal tenancy contracts are share contracts when incentive problems are important in all matches. It is shown that when wealth is more heterogeneously distributed than land productivity, higher output shares for the agents are associated with higher initial wealths, although share is in general nonmonotone in wealth. It is also shown that when the second-best contracts are implemented, the equilibrium wage function exhibits wage inequality among the tenants. Moreover, because of positively assortative matching, the distribution of the equilibrium wage is positively skewed relative to the distribution of initial wealth.

In the present model the first-best contracts may not be implemented, due to informational asymmetries. In particular, the market failure stems from the fact that, in the presence of limited liability, less wealthy agents cannot be expected to exert high effort, as they cannot be forced to share losses with the principals in the event of failure. An important assumption in this paper is that the relationship between a principal and an agent lasts only for one period. Possibly, such a relationship usually involves dynamic considerations too, which in turn implies some degree of relaxation on the limited-liability constraint, and the conclusions of the current paper may alter. Ray (2005) considers a dynamic landlord–tenant relationship where the tenant has to make land-specific investments in order to maintain the land quality, and shows that share tenancy arises because of this sort of multiple tasks. Extension of the present model to a dynamic principal–agent market, which incorporates the above-mentioned features, would be an interesting research agenda for the future.

Appendix

A.1 Sufficient Conditions for the Single-Crossing Property

The utility possibility frontier $\phi(\lambda, \omega, u(\omega))$ in Proposition 1 is derived by solving the following maximization problem:

$$\phi(\lambda, \omega, u(\omega)) = \max_x V(x, \lambda, \omega)$$
$$\text{subject to} \quad U(x, \lambda, \omega) = u(\omega)\,,$$

where x is the consumption allocation of the agent.

ASSUMPTION A1 $V_x < 0$, $V_{xx} < 0$, $U_x > 0$, $U_{xx} < 0$.

ASSUMPTION A2 $V_\lambda > 0$, $V_\omega < 0$, $U_\lambda < 0$, $U_\omega > 0$.

ASSUMPTION A3 $V_{x\lambda}$, $V_{x\omega}$, $V_{\lambda\omega} \geq 0$ *and* $U_{x\lambda}$, $U_{x\omega}$, $U_{\lambda\omega} \geq 0$.

Assumption A2 implies that utilities are increasing in own types, but decreasing in the types of the others. Assumption A3 asserts that both V and U are supermodular. I also assume that the principal's utility is decreasing in the consumption allocation x, whereas the agent's utility is increasing. The Lagrange function is given by

$$\mathcal{L}(x, \nu; \lambda, \omega, u(\omega)) = V(x, \lambda, \omega) + \nu[U(x, \lambda, \omega) - u(\omega)]\,.$$

The first-order conditions are given by

$$V_x(x, \lambda, \omega) + \nu U_x(x, \lambda, \omega) = 0, \qquad \text{(A1)}$$
$$U(x, \lambda, \omega) - u(\omega) = 0, \qquad \text{(A2)}$$
$$\nu > 0\,.$$

The optimal consumption $x^* = x(\lambda, \omega, u(\omega))$ is determined from (A2), which is given by

$$U(x^*, \lambda, \omega) = u(\omega)\,.$$

Differentiating the above, one gets

$$x_1(\lambda, \omega, u(\omega)) = -\frac{U_\lambda}{U_x} > 0, \quad x_2(\lambda, \omega, u(\omega)) = -\frac{U_\omega}{U_x} < 0,$$
$$\text{and} \quad x_3(\lambda, \omega, u(\omega)) = \frac{1}{U_x} > 0\,.$$

Notice also that, from (A1), $\nu = -V_x/U_x > 0$, since $V_x < 0$. The value function is given by $\phi(\lambda, \omega, u(\omega))$. From the envelope theorem it follows that

$$\phi_1(\lambda, \omega, u(\omega)) = \mathcal{L}_\lambda = V_\lambda + \nu U_\lambda = V_\lambda - \frac{V_x}{U_x} U_\lambda,$$
$$\phi_2(\lambda, \omega, u(\omega)) = \mathcal{L}_\omega = V_\omega + \nu U_\omega = V_\omega - \frac{V_x}{U_x} U_\omega,$$
$$\phi_3(\lambda, \omega, u(\omega)) = \mathcal{L}_u = -\nu = \frac{V_x}{U_x} < 0\,.$$

The signs of ϕ_1 and ϕ_2 are ambiguous because V_τ and U_τ are of opposite signs for $\tau = \lambda, \omega$. So in order to have both $\phi_1 > 0$ and $\phi_2 > 0$, one requires to assume

(A3) $$U_x V_\lambda - V_x U_\lambda > 0 \quad \text{and} \quad U_x V_\omega - V_x U_\omega > 0\,.$$

Differentiating ϕ_1 with respect to ω and u, respectively, one gets

$$\phi_{21}(\lambda, \omega, u(\omega)) = (V_{\lambda\omega} + \nu U_{\lambda\omega}) + \frac{U_\lambda U_\omega (V_{xx} + \nu U_{xx})}{U_x^2}$$
$$+ \frac{1}{U_x}[-U_\omega(V_{x\lambda} + \nu U_{x\lambda}) - U_\lambda(V_{x\omega} + \nu U_{x\omega})],$$
$$\phi_{31}(\lambda, \omega, u(\omega)) = \frac{V_{x\lambda} + \nu U_{x\lambda}}{U_x} - \frac{U_\lambda(V_{xx} + \nu U_{xx})}{U_x^2}\,.$$

Given our assumptions, both of the above terms are ambiguous in sign. Therefore, for the single-crossing property to hold, i.e., $\phi_2\phi_{31} - \phi_3\phi_{21} \geq 0$, it will suffice to look for conditions on V and U such that both $\phi_{21} \geq 0$ and $\phi_{31} \geq 0$. First consider ϕ_{21}. Since V and U are both assumed to be supermodular, and $U_\lambda < 0$, $U_\omega > 0$, and $V_{xx}, U_{xx} < 0$, the first two terms in the expression for ϕ_{21} are positive. Therefore, a sufficient condition for $\phi_{21} \geq 0$ is given by

$$-U_\omega(V_{x\lambda} + \nu U_{x\lambda}) - U_\lambda(V_{x\omega} + \nu U_{x\omega}) \geq 0 \quad \Longleftrightarrow \quad \frac{U_x V_{x\omega} - V_x U_{x\omega}}{U_x V_{x\lambda} - V_x U_{x\lambda}} \geq -\frac{U_\omega}{U_\lambda}\,.$$

Next, consider ϕ_{31}. A sufficient condition for $\phi_{31} \geq 0$ is given by

(A4) $$\frac{V_{x\lambda} + \nu U_{x\lambda}}{U_x} - \frac{U_\lambda(V_{xx} + \nu U_{xx})}{U_x^2} \geq 0 \quad \Longleftrightarrow \quad \frac{U_x V_{x\lambda} - V_x U_{x\lambda}}{V_x U_{xx} - U_x V_{xx}} \geq -\frac{U_\lambda}{U_x}\,.$$

Therefore, the conditions (A3), (A4) guarantee the properties of the Pareto frontier that must be verified for Proposition 1 to hold. It is worth noting that the above analysis can easily be extended to where the allocation x is a vector instead of a real number.

A.2 Proofs

A.2.1 Proof of Proposition 1

I first prove the necessity of (a) and (b). Notice that if a type λ principal hires a type ω agent (i.e., $\lambda = l(\omega)$) in order to maximize her payoff, then the first-order condition of the maximization problem is given by

$$\text{(FOC1)} \qquad \Gamma(\omega) := \phi_2(\lambda, \omega, u(\omega)) + \phi_3(\lambda, \omega, u(\omega))u'(\omega) = 0\,,$$

which is equivalent to the condition (FOC). If ω is a maximum, then it must also satisfy the following second-order condition:

$$\begin{aligned} & [\phi_{22}(\lambda, \omega, u(\omega)) + \phi_{23}(\lambda, \omega, u(\omega))u'(\omega)] \\ & + [\phi_{23}(\lambda, \omega, u(\omega)) + \phi_{33}(\lambda, \omega, u(\omega))u'(\omega)]u'(\omega) \\ \text{(SOC)} \qquad & + \phi_3(\lambda, \omega, u(\omega))u''(\omega) \leq 0\,. \end{aligned}$$

For any given λ, (FOC1) holds with equality for some particular $\omega = w(\lambda)$, i.e., one has $\Gamma(w(\lambda)) = 0$ for all λ. Because this is true for all λ, one can differentiate the first-order condition (FOC1) with respect to λ, which yields

$$\begin{aligned} & \big\{[\phi_{22}+\phi_{23}u'(\omega)]+[\phi_{23}+\phi_{33}u'(\omega)]u'(\omega)+\phi_3 u''(\omega)\big\}w'(\lambda)+[\phi_{21}+\phi_{31}u'(\omega)] = 0 \\ \iff & [\phi_{22}+\phi_{23}u'(\omega)]+[\phi_{23}+\phi_{33}u'(\omega)]u'(\omega)+\phi_3 u''(\omega) = -[\phi_{21}+\phi_{31}u'(\omega)]l'(\omega)\,. \end{aligned}$$

Substituting the above in (SOC), the second-order condition reduces to

$$[\phi_{21} + \phi_{31}u'(\omega)]l'(\omega) = -\frac{1}{\phi_3}[\phi_2\phi_{31} - \phi_3\phi_{21}]l'(\omega) \geq 0\,.$$

Since $\phi_3 < 0$ and $\phi_2\phi_{31} - \phi_3\phi_{21} \geq 0$, the above inequality implies that $l'(\omega) \geq 0$. This proves the necessity of (b).

Next I prove that conditions (a) and (b) are sufficient for $\omega = l^{-1}(\lambda)$ to be maximal for each λ. Suppose that (a) and (b) hold for all ω, but there is a particular ω that does not solve the maximization problem (1). Then there exists $\omega' \neq \omega$ such that

$$\phi(l(\omega), \omega', u(\omega')) - \phi(l(\omega), \omega, u(\omega)) > 0\,.$$

Integrating the above, one gets

$$\text{(A5)} \qquad \int_{\omega}^{\omega'} [\phi_2(l(\omega), \hat{\omega}, u(\hat{\omega})) + \phi_3(l(\omega), \hat{\omega}, u(\hat{\omega}))u'(\hat{\omega})]d\hat{\omega} > 0\,.$$

Suppose without loss of generality that $\omega' > \omega$. Since $l'(\omega) \geq 0$, it follows that

$$l(\omega) \leq l(\hat{\omega}) \leq l(\omega') \quad \text{for all } \hat{\omega} \in [\omega, \omega']\,.$$

Notice that $\phi_2\phi_{31} - \phi_3\phi_{21} \geq 0$ is equivalent to $\phi_{21} + \phi_{31}u'(\omega) \geq 0$, since $\phi_2 + \phi_3 u'(\omega) = 0$ from the first-order condition. Since $\phi_{21} + \phi_{31}u'(\omega) \geq 0$, $\phi_2 + \phi_3 u'$ is nondecreasing in $l(\omega)$, which implies

$$\begin{aligned} & \phi_2(l(\omega), \hat{\omega}, u(\hat{\omega})) + \phi_3(l(\omega), \hat{\omega}, u(\hat{\omega}))u'(\hat{\omega}) \\ & \qquad \leq \phi_2(l(\hat{\omega}), \hat{\omega}, u(\hat{\omega})) + \phi_3(l(\hat{\omega}), \hat{\omega}, u(\hat{\omega}))u'(\hat{\omega}) = 0\,. \end{aligned}$$

Then integrating the above, one gets

(A6) $$\int_{\omega}^{\omega'} [\phi_2(l(\omega), \hat{\omega}, u(\hat{\omega})) + \phi_3(l(\omega), \hat{\omega}, u(\hat{\omega}))u'(\hat{\omega})]d\hat{\omega} \leq 0\,.$$

The conditions (A5) and (A6) contradict each other. Now if $\omega > \omega'$, then the same logic leads us to a similar contradiction. This establishes the sufficiency of conditions (a) and (b). *Q.E.D.*

A.2.2 Proof of Lemma 1

To save on notation, I suppress the argument (λ, ω) of the contract terms whenever it does not create confusion. The incentive compatibility constraint (IC1) implicitly defines e as a function of α, which is denoted by $e(\alpha)$. Notice that

$$e'(\alpha) = \frac{\pi_e}{\psi'' - \alpha\pi_{ee}} > 0, \quad \text{and} \quad e''(\alpha) = \frac{2\pi_e\pi_{ee}}{(\psi'' - \alpha\pi_{ee})^2} \leq 0\,.$$

Substituting $e(\alpha)$ into the expressions for the expected utilities of the principal and the agent, the maximization problem $\mathcal{M}$ reduces to

(M1) $$\phi(\lambda, \omega, \bar{u}(\omega)) = \max_{\{\alpha, R\}} (1 - \alpha)\pi(\lambda, e(\alpha)) + R$$

(IR1) $$\text{subject to} \quad \alpha\pi(\lambda, e(\alpha)) - R - \psi(e(\alpha)) \geq \bar{u}(\omega)\,,$$

(LL) $$R \leq \omega\,,$$

(F) $$0 \leq \alpha \leq 1\,.$$

I ignore the feasibility constraint for the time being; it will be verified later. The Lagrangian is given by

$$\mathcal{L} = (1 - \alpha)\pi(\lambda, e(\alpha)) + R + \mu[\alpha\pi(\lambda, e(\alpha)) - R - \psi(e(\alpha)) - \bar{u}(\omega)] + \nu[\omega - R]\,,$$

where μ and ν are the Lagrange multipliers. The Karush–Kuhn–Tucker (KKT) conditions are given by

(KKT1) $$(1 - \alpha)\pi_e(\lambda, e(\alpha))e'(\alpha) - (1 - \mu)\pi(\lambda, e(\alpha)) = 0\,,$$

(KKT2) $$\nu = 1 - \mu\,,$$

(KKT3) $$\mu[\alpha\pi(\lambda, e(\alpha)) - R - \psi(e(\alpha)) - \bar{u}(\omega)] = 0\,,$$

$$\nu[\omega - R] = 0\,,$$

$$\nu \geq 0\,.$$

Notice that (KKT2) implies that both constraints cannot be nonbinding simultaneously, i.e., it cannot be the case that $\mu = \nu = 0$. Therefore, at least one constraint must bind at the optimum. Since both the principal and the agent are risk-neutral, without the limited-liability constraint the first-best effort level can always be implemented. Therefore, I consider two cases: when the limited-liability constraint binds at the optimum and when it does not.

First consider the case when the agent's limited-liability constraint binds at the optimum, i.e., $R = \omega$. Here two subcases will be considered. The first is when the individual rationality constraint does not bind. Then the principal solves the unconstrained maximization problem at $R = \omega$, whose optimality condition is derived from (KKT1) with $\mu = 0$, which is given by

$$(1-\alpha)\pi_e(\lambda, e(\alpha))e'(\alpha) - \pi(\lambda, e(\alpha)) = 0\,.$$

The above defines the optimal share $\alpha = \alpha^0(\lambda)$, which depends only on λ. The optimal effort is determined from the incentive compatibility constraint, which is given by $e = e(\alpha^0(\lambda))$. Define

$$U(\lambda, \alpha) := \alpha\pi(\lambda, e(\alpha)) - \psi(e(\alpha))\,,$$

which is the expected payoff of the agent gross of R and $\bar{u}(\omega)$. Since the individual rationality constraint does not bind, the candidate solutions $\alpha = \alpha^0(\lambda)$, $R = \omega$, and $e = e(\alpha^0(\lambda))$ are optimal only if

$$U(\lambda, \alpha^0(\lambda)) > \omega + \bar{u}(\omega) \equiv \tilde{u}(\omega) \quad \Longleftrightarrow \quad \omega < \tilde{u}^{-1}(U(\lambda, \alpha^0(\lambda))) \equiv A^0(\lambda)\,.$$

The above holds, since $\tilde{u}'(\cdot) > 0$.

Next, consider the subcase when both (LL) and (IR1) bind at the optimum. In this case $R = \omega$, and the optimal share is $\alpha = \hat{\alpha}(\lambda, \omega, \bar{u}(\omega))$, which is given by

$$\hat{\alpha}(\lambda, \omega, \bar{u}(\omega))\pi(\lambda, e(\hat{\alpha}(\lambda, \omega, \bar{u}(\omega)))) - \omega - \psi(e(\hat{\alpha}(\lambda, \omega, \bar{u}(\omega)))) - \bar{u}(\omega) = 0\,.$$

The optimal effort is determined from the incentive-compatibility constraint, which is given by $e = e(\hat{\alpha}(\lambda, \omega, \bar{u}(\omega)))$. Since $\nu = 1 - \mu$, the condition (KKT2) reduces to

$$\nu(\alpha) = \underbrace{\frac{(1-\alpha)\pi_e(\lambda, e(\alpha))}{\pi(\lambda, e(\alpha))}}_{H(\alpha)} \cdot e'(\alpha)\,.$$

Notice that

$$H'(\alpha) = \frac{(1-\alpha)e'(\alpha)[\pi\pi_{ee} - \pi_e^2] - \pi\pi_e}{\pi^2} < 0\,.$$

Since $e''(\alpha) \leq 0$, we have $\nu'(\alpha) \leq 0$. Notice that

$$\nu(\hat{\alpha}(\lambda, \omega, \bar{u}(\omega))) \leq \nu(\alpha^0(\lambda)) = 1\,.$$

Since $\nu'(\alpha) \leq 0$, one has $\hat{\alpha}(\lambda, \omega, \bar{u}(\omega)) \geq \alpha^0(\lambda)$. Next, notice that

$$U_\alpha(\lambda, \alpha) = \underbrace{[\alpha\pi_e(\lambda, e(\alpha)) - \psi'(e(\alpha))]}_{=\,0 \text{ from the incentive compatibility}} e'(\alpha) + \pi(\lambda, e(\alpha)) = \pi(\lambda, e(\alpha)) > 0\,.$$

Therefore, one has

$$U(\lambda, \alpha^0(\lambda)) \leq U(\lambda, \hat{\alpha}(\lambda, \omega, \bar{u}(\omega))) = \omega + \bar{u}(\omega) \quad \Longleftrightarrow \quad \omega \geq A^0(\lambda)\,.$$

On the other hand, since $\alpha \leq 1$, one has

$$U(\lambda, 1) \geq U(\lambda, \hat{\alpha}(\lambda, \omega, \bar{u}(\omega))) = \omega + \bar{u}(\omega) \quad \Longleftrightarrow \quad \omega \leq \tilde{u}^{-1}(U(\lambda, 1)) \equiv A^*(\lambda)\,.$$

Finally, consider the case when the individual-rationality constraint binds, but the limited-liability constraint does not bind at the optimum. In this case $\nu = 1 - \mu = 0$. Then from (KKT1) it follows that

$$(1-\alpha)\pi_e(\lambda, e(\alpha))e'(\alpha) = 0\,,$$

which implies $\alpha = 1$. Then from the incentive-compatibility constraint it follows that

$$\pi_e(\lambda, e(1)) - \psi'(e(1)) = 0 \quad \Longrightarrow \quad e(1) = e^*(\lambda)\,.$$

In other words, the optimal effort in this case reaches its first-best level. The optimal rental payment is determined from the binding participation constraint, which is given by $R^* = \pi(\lambda, e^*(\lambda)) - \psi(e^*(\lambda)) - \bar{u}(\omega) = U(\lambda, 1) - \bar{u}(\omega)$. Since the limited-liability constraint does not bind, i.e., $R^* < \omega$, it must be the case that

$$U(\lambda, 1) < \omega + \bar{u}(\omega) \equiv \tilde{u}(\omega) \quad \Longleftrightarrow \quad \omega > \tilde{u}^{-1}(U(\lambda, 1)) \equiv A^*(\lambda)\,.$$

Notice that the expected payoff of the principal at $\alpha = 1$ is equal to R^*, which must be nonnegative in order to have a viable match. This implies

$$U(\lambda, 1) \geq \bar{u}(\omega) \quad \Longleftrightarrow \quad \omega \leq \bar{u}^{-1}(U(\lambda, 1)) \equiv \bar{A}(\lambda)\,.$$

This completes the proof of Lemma 1. *Q.E.D.*

A.2.3 Proof of Lemma 2

First consider when $\omega < A^0(\lambda)$. In this case the limited-liability constraint binds, but the individual-rationality constraint does not bind. The corresponding optimal contracts are given by $\alpha^0(\lambda)$ and $R = \omega$, and optimal effort is $e^0(\lambda) = e(\alpha^0(\lambda))$. Clearly,

$$\frac{\partial \alpha^0(\lambda)}{\partial \omega} = \frac{\partial e^0(\lambda)}{\partial \omega} = 0\,.$$

Consider now the first-order condition that determines the optimal share of the agent:

$$[1 - \alpha^0(\lambda)]\pi_e(\lambda, e(\alpha^0(\lambda)))e'(\alpha^0(\lambda)) = \pi(\lambda, e(\alpha^0(\lambda)))\,.$$

Differentiating the above condition, one gets

$$\frac{\partial \alpha^0(\lambda)}{\partial \lambda} = \frac{\pi \pi_{e\lambda} - \pi_e \pi_\lambda}{(2\pi_e^2 - \pi \pi_{ee})e'(\alpha^0(\lambda)) - [1 - \alpha^0(\lambda)]\pi_e^2 e''(\alpha^0(\lambda))} \geq 0\,,$$

since $\pi(\lambda, e)$ is log-supermodular, $\pi_{ee} \leq 0$, and $e''(\alpha) \leq 0$. Since $e'(\alpha) > 0$, one has $\partial e^0(\lambda)/\partial \lambda \geq 0$. Now consider the optimal rental payment $R = \omega$. Clearly, $\partial R/\partial \omega > 0$ and $\partial R/\partial \lambda = 0$.

Next, consider the case when both the limited-liability and individual-rationality constraints bind. In this case also $R = \omega$, and hence $\partial R/\partial \omega > 0$ and $\partial R/\partial \lambda = 0$. Now, consider the following first-order conditions (KKT1) and (KKT3), which determine α and ν in this case:

(A7) $$(1 - \hat{\alpha})\pi_e(\lambda, e(\hat{\alpha}))e'(\hat{\alpha}) - \nu \pi(\lambda, e(\hat{\alpha})) = 0,$$

(A8) $$\hat{\alpha}\pi(\lambda, e(\hat{\alpha})) - \psi(e(\hat{\alpha})) = \tilde{u}(\omega)\,.$$

Equation (A7) is true, since $\nu = 1 - \mu$. Differentiating the above system with respect to ω, one gets

$$\begin{bmatrix} P & -\pi \\ \pi & 0 \end{bmatrix} \begin{bmatrix} \partial \hat{\alpha}/\partial \omega \\ \partial \nu/\partial \omega \end{bmatrix} = \begin{bmatrix} 0 \\ \tilde{u}'(\omega) \end{bmatrix},$$

where

$$P := (1-\hat{\alpha})[\pi_e e''(\hat{\alpha}) + \pi_{ee}(e'(\hat{\alpha}))^2] - (1+\nu)\pi_e e'(\hat{\alpha})\,.$$

Note that $P \leq 0$, since $e''(\alpha) \leq 0$ and $\pi_{ee} \leq 0$. Thus, from the above it follows that

$$\frac{\partial\hat{\alpha}}{\partial\omega} = \frac{\tilde{u}'(\omega)}{\pi} > 0\,.$$

Since $\hat{e} = e(\hat{\alpha})$, which is implicitly defined by the incentive-compatibility constraint, and $e'(\alpha) > 0$, we have

$$\frac{\partial\hat{e}}{\partial\omega} = e'(\hat{\alpha}) \cdot \frac{\partial\hat{\alpha}}{\partial\omega} > 0\,.$$

Next, differentiating the equations (A7) and (A8) with respect to λ, one gets

$$\text{(A9)} \qquad \begin{bmatrix} P & -\pi \\ \pi & 0 \end{bmatrix} \begin{bmatrix} \partial\hat{\alpha}/\partial\lambda \\ \partial\nu/\partial\lambda \end{bmatrix} = \begin{bmatrix} -Q \\ -\hat{\alpha}\pi_\lambda \end{bmatrix},$$

where

$$Q := \frac{1}{\pi}(1-\hat{\alpha})e'(\hat{\alpha})[\pi\pi_{e\lambda} - \pi_e\pi_\lambda].$$

Note that Q is positive, since π is log-supermodular. From the above, it follows that

$$\text{(A10)} \qquad \frac{\partial\hat{\alpha}}{\partial\lambda} = -\frac{\hat{\alpha}\pi_\lambda}{\pi} < 0\,.$$

Finally, since from the incentive constraint one has

$$\hat{\alpha}\pi_e(\lambda, e(\hat{\alpha})) = \psi'(e(\hat{\alpha}))\,,$$

it follows that

$$\frac{\partial\hat{e}}{\partial\lambda} = -\frac{\hat{\alpha}\pi_\lambda e'(\hat{\alpha})}{\pi} < 0\,.$$

Finally, consider the case when the limited-liability constraint does not bind, but the individual-rationality constraint does. In this case the optimal share is given by $\alpha^* = 1$, and hence $\partial\alpha^*/\partial\lambda = \partial\alpha^*/\partial\omega = 0$. The optimal effort and rental payment are thus determined from the incentive-compatibility constraint and the binding individual-rationality constraint, i.e.,

$$\pi_e(\lambda, e^*(\lambda)) - \psi'(e^*(\lambda)) = 0,$$
$$R^*(\lambda,\omega) = \pi(\lambda, e^*(\lambda)) - \psi(e^*(\lambda)) - \bar{u}(\omega)\,.$$

Differentiating the above two equations, one obtains

$$\frac{\partial e^*}{\partial\lambda} = \frac{\pi_{e\lambda}}{\psi'' - \pi_{ee}} > 0, \quad \text{and} \quad \frac{\partial R^*}{d\lambda} = \pi_\lambda > 0,$$
$$\frac{\partial e^*}{d\omega} = 0, \quad \text{and} \quad \frac{\partial R^*}{d\omega} = -\bar{u}'(\omega) \leq 0\,.$$

Notice that both

$$U^0(\lambda) := U(\lambda, \alpha^0(\lambda)) = \alpha^0(\lambda)\pi(\lambda, e(\alpha^0(\lambda))) - \psi(e(\alpha^0(\lambda))),$$
$$U^*(\lambda) := U(\lambda, 1) = \pi(\lambda, e^*(\lambda)) - \psi(e^*(\lambda))$$

are strictly increasing in λ. Therefore,

$$\omega < A^0(\lambda) \iff \tilde{u}(\omega) < U^0(\lambda) \iff \lambda > U^{0^{-1}}(\tilde{u}(\omega)) \equiv B^0(\omega),$$
$$\omega > A^*(\lambda) \iff \tilde{u}(\omega) > U^*(\lambda) \iff \lambda < U^{*^{-1}}(\tilde{u}(\omega)) \equiv B^*(\omega)\,.$$

Therefore, the lemma follows. *Q.E.D.*

A.2.4 Proof of Lemma 3

The expected payoff of the principal from a match (λ, ω) is given by

$$[1-\alpha(\lambda,\omega)]\pi(\lambda, e(\alpha(\lambda,\omega))) + R(\lambda,\omega)\,.$$

Substituting the values of the optimal contract, one gets the expression for $\phi(\lambda, \omega, \bar{u}(\omega))$. Now, consider the Lagrangian function associated with the maximization problem $\mathcal{M}$:

$$\mathcal{L} = (1-\alpha)\pi(\lambda, e(\alpha)) + R + \mu[\alpha\pi(\lambda, e(\alpha)) - R - \psi(e(\alpha)) - \bar{u}(\omega)] + \nu[\omega - R]\,.$$

Then, it follows from the envelope theorem that

$$\phi_1 = \frac{\partial \mathcal{L}}{\partial \lambda} = [(1-\alpha) + \mu\alpha]\pi_\lambda = (1-\nu\alpha)\pi_\lambda \geq 0,$$
$$\phi_2 = \frac{\partial \mathcal{L}}{\partial \omega} = \nu \in [0,1],$$
$$\phi_3 = \frac{\partial \mathcal{L}}{\partial \bar{u}(\omega)} = -\mu = \nu - 1 \in [-1, 0]\,.$$

Therefore, $\phi_{21} = \phi_{31} = \partial\nu/\partial\lambda$. When $\omega < A^0(\lambda)$ one has $\nu = 1$, and hence $\phi_{21} = \phi_{31} = 0$. On the other hand, when $\omega > A^*(\lambda)$ one has $\nu = 0$, and hence $\phi_{21} = \phi_{31} = 0$. Now consider the case when $A^0(\lambda) \leq \omega \leq A^*(\lambda)$. Then from (A9), it follows that $\phi_{21} = \phi_{31} > 0$. This completes the proof of the proposition. *Q.E.D.*

A.2.5 Proof of Proposition 2

From the condition (E) it follows that

$$v'(\lambda) = \phi_1(\lambda, \omega, u(\omega)) > 0\,.$$

Next, from Proposition 1 it follows that

$$u'(\omega) = -\frac{\phi_2(\lambda, \omega, u(\omega))}{\phi_3(\lambda, \omega, u(\omega))} = \frac{\nu}{1-\nu} \geq 0\,.$$

The above expression is positive, since $0 \leq \nu \leq 1$. The positivity of ν follows from the condition (KKT3). This completes the proof of the proposition. *Q.E.D.*

A.2.6 Proof of Proposition 3

In the proof of Lemma 3 we have proven that $\phi_{21} \geq 0$ and $\phi_{31} \geq 0$, and hence the single-crossing property holds. Thus, the proposition follows from Proposition 1.

A.2.7 Proof of Proposition 4

Recall that

$$A^*(\lambda) := \tilde{u}^{-1}(U(\lambda, e^*(\lambda))) .$$

Then,

$$\frac{dA^*(\lambda)}{d\lambda} = \frac{1}{\tilde{u}'\pi_e}[(1-\alpha)\pi\pi_{e\lambda} + \psi'\pi_\lambda] > 0 .$$

Since $\lambda = l(\omega)$ in equilibrium with $\lambda'(\omega) \geq 0$, and $A^*(l(\omega))$ is an increasing continuous function from the compact and convex set Ω into itself, it has a fixed point which is given by $\omega^* = A^*(l(\omega^*))$. Notice that this fixed point is unique. Suppose not, i.e., there are two fixed points ω_1^* and ω_2^* with $\omega_1^* < \omega_2^*$. Then each agent with wealth level $\omega \in (\omega_1^*, \omega_2^*]$ will implement both the first-best (because $\omega > \omega_1^*$) and the second-best (because $\omega \leq \omega_2^*$) efforts and contracts, which is not possible. This completes the proof of the proposition. *Q.E.D.*

A.3 Skewness of the Income Distribution

Consider two arbitrary levels of wealth ω_1 and ω_2 with $\omega_2 > \omega_1$. Define by

$$\Sigma(\lambda, \omega, u(\omega)) = \phi(\lambda, \omega, u(\omega)) + u(\omega)$$

the equilibrium expected aggregate surplus of a partnership (λ, ω). Now suppose that, in a Walrasian equilibrium, both types ω_1 and ω_2 choose to work in the same quality $\bar{\lambda}$ of lands. Since these assignments are optimal, one must have

$$u(\omega_2) - u(\omega_1) = \Sigma(\bar{\lambda}, \omega_2, u(\omega_2)) - \Sigma(\bar{\lambda}, \omega_1, u(\omega_1)) ,$$

since all the incremental surplus must accrue to the wealthier agent. Consider now the second-best equilibrium. Take an arbitrary wealth level $\bar{\omega}$, and let $\bar{\lambda} = l(\bar{\omega})$ be the corresponding land quality. Consider a distribution of wages defined by $u^*(\bar{\omega}) = u(\bar{\omega})$ and

$$u^*(\omega) - u^*(\bar{\omega}) = \Sigma(\bar{\lambda}, \omega, u^*(\omega)) - \Sigma(\bar{\lambda}, \bar{\omega}, u^*(\bar{\omega})) .$$

So the distributions of $u^*(\omega)$ and ω have the same shape, and both the wage functions $u(\omega)$ and $u^*(\omega)$ yield the same wage at $\bar{\omega}$. Take an wealth level $\omega_2 > \bar{\omega}$. Positive assortment implies $\lambda(\omega_2) > \bar{\lambda}$. Then

$$\begin{aligned} u(\omega_2) - u(\bar{\omega}) &= \int_{\bar{\omega}}^{\omega_2} \left. \frac{\Sigma_2(\lambda, \omega, u(\omega))}{1 - \Sigma_3(\lambda, \omega, u(\omega))} \right|_{\lambda = l(\omega)} d\omega \\ &> \int_{\bar{\omega}}^{\omega_2} \frac{\Sigma_2(\bar{\lambda}, \omega, u(\omega))}{1 - \Sigma_3(\bar{\lambda}, \omega, u(\omega))} d\omega \\ &= \Sigma(\bar{\lambda}, \omega_2, u(\omega_2)) - \Sigma(\bar{\lambda}, \bar{\omega}, u(\bar{\omega})) . \end{aligned}$$

The above together with $u(\bar{\omega}) = u^*(\bar{\omega})$ imply that

$$\begin{aligned} u(\omega_2) &> u^*(\omega_2) + [\Sigma(\bar{\lambda}, \omega_2, u(\omega_2)) - \Sigma(\bar{\lambda}, \omega_2, u^*(\omega_2))] \\ &= u^*(\omega_2) + \Sigma_3(\bar{\lambda}, \omega, u(\omega))[u(\omega_2) - u^*(\omega_2)] \\ &\iff [1 - \Sigma_3(\bar{\lambda}, \omega, u(\omega))][u(\omega_2) - u^*(\omega_2)] > 0 \\ &\iff u(\omega_2) > u^*(\omega_2)\,, \end{aligned}$$

since Σ_3 can be shown to be less that 1. Similarly, for a wealth level $\omega_1 < \bar{\omega}$ it is easy to show that $u(\omega_1) > u^*(\omega_1)$. In a Walrasian equilibrium, low-wealth and high-wealth agents choose to work in lands with low and high productivity, respectively, instead of lands of intermediate quality (equal to $\bar{\lambda}$), because their wages are higher with the principals they are matched with. This means that the density of $u(\omega)$ can be obtained from $u^*(\omega)$ by shifting a positive mass from the left tail to the right, i.e., the distribution of $u(\omega)$ is positively skewed relative to the distribution of $u(\omega)$.

References

Ackerberg, D. A., and M. Botticini (2002), "Endogenous Matching and the Empirical Determinants of Contract Form," *Journal of Political Economy*, 110(3), 564–591.

Alonso-Paulí, E., and D. Pérez-Castrillo (2012), "Codes of Best Practice in Competitive Markets for Managers," *Economic Theory*, 49(1), 113–141.

Banerjee, A. V., P. J. Gertler, and M. Ghatak (2002), "Empowerment and Efficiency: Tenancy Reform in West Bengal," *Journal of Political Economy*, 110(2), 239–280.

Basu, K. (1992), "Limited Liability and the Existence of Share Tenancy," *Journal of Development Economics*, 38(1), 203–220.

Braido, L. (2008), "Evidence on the Incentive Properties of Share Contracts," *The Journal of Law & Economics*, 51(2), 327–349.

Chakraborty, A., and A. Citanna (2005), "Occupational Choice, Incentives and Wealth Distribution," *Journal of Economic Theory*, 122(2), 206–224.

Dam, K. (2014), "Job Assignment, Market Power and Managerial Incentives," *The Quarterly Review of Economics and Finance*, published online first November 20, DOI: 10.1016/j.qref.2014.11.001.

— and D. Pérez-Castrillo (2006), "The Principal–Agent Matching Market," *The B.E. Journal of Theoretical Economics: Frontiers of Theoretical Economics*, 2(1), DOI: 10.2202/1534-5963.1257.

Edmans, A., X. Gabaix, and A. Landier (2009), "A Multiplicative Model of Optimal CEO Incentives in Market Equilibrium," *The Review of Financial Studies*, 22(12), 4881–4917.

Eswaran, M., and A. Kotwal (1985), "A Theory of Contractual Structure in Agriculture," *The American Economic Review*, 75(3), 352–367.

Fudenberg, D., B. Holmstrom, and P. Milgrom (1990), "Short-Term Contracts and Long-Term Agency Relationships," *Journal of Economic Theory*, 51(1), 1–31.

Ghatak, M., and A. Karaivanov (2014), "Contractual Structure in Agriculture with Endogenous Matching," *Journal of Development Economics*, 110, 239–249.

— and P. Pandey (2000), "Contract Choice in Agriculture with Joint Moral Hazard in Effort and Risk," *Journal of Development Economics*, 63(2), 303–326.

Grossman, S. J., and O. D. Hart (1983), "An Analysis of the Principal–Agent Problem," *Econometrica*, 51(1), 7–45.

Kremer, M. (1993), "The O-Ring Theory of Economic Development," *The Quarterly Journal of Economics*, 108(3), 551–575.

Laffont, J.-J., and M. S. Matoussi (1995), "Moral Hazard, Financial Constraints and Sharecropping in El Oulja," *The Review of Economic Studies*, 62(3), 381–399.
Legros, P., and A. F. Newman (1996), "Wealth Effects, Distribution, and the Theory of Organization," *Journal of Economic Theory*, 70(2), 312–341.
— and — (2007), "Beauty Is a Beast, Frog Is a Prince: Assortative Matching with Nontransferabilities," *Econometrica*, 75(4), 1073–1102.
Newbery, D. M. G. (1977), "Risk Sharing, Sharecropping and Uncertain Labour Markets," *The Review of Economic Studies*, 44(3), 585–594.
Ostroy, J. M. (1984), "A Reformulation of the Marginal Productivity Theory of Distribution," *Econometrica*, 52(3), 599–630.
Phelan, C., and R. M. Townsend (1991), "Computing Multi-Period, Information-Constrained Optima," *The Review of Economic Studies*, 58(5), 853–881.
Prasad, K., and T. C. Salmon (2013), "Self Selection and Market Power in Risk Sharing Contracts," *Journal of Economic Behavior & Organization*, 90, 71–86.
Quadrini, V. (2004), "Investment and Liquidation in Renegotiation-Proof Contracts with Moral Hazard," *Journal of Monetary Economics*, 51(4), 713–751.
Rao, C. H. (1971), "Uncertainty, Entrepreneurship, and Sharecropping in India," *Journal of Political Economy*, 79(3), 578–595.
Ray, T. (2005), "Sharecropping, Land Exploitation and Land-Improving Investments," *The Japanese Economic Review*, 56(2), 127–143.
— and N. Singh (2001), "Limited Liability, Contractual Choice, and the Tenancy Ladder," *Journal of Development Economics*, 66(1), 289–303.
Rosen, S. (1974), "Hedonic Prices and Implicit Markets: Product Differentiation in Pure Competition," *Journal of Political Economy*, 82(1), 34–55.
Sattinger, M. (1975), "Comparative Advantage and the Distributions of Earnings and Abilities," *Econometrica*, 43(3), 455–468.
— (1979), "Differential Rents and the Distribution of Earnings," *Oxford Economic Papers*, 31(1), 60–71.
Sengupta, K. (1997), "Limited Liability, Moral Hazard and Share Tenancy," *Journal of Development Economics*, 52(2), 393–407.
Serfes, K. (2005), "Risk Sharing vs. Incentives: Contract Design under Two-Sided Heterogeneity," *Economics Letters*, 88(3), 343–349.
Shetty, S. (1988), "Limited Liability, Wealth Differences and Tenancy Contracts in Agrarian Economies," *Journal of Development Economics*, 29(1), 1–22.
Stiglitz, J. E. (1974), "Incentives and Risk Sharing in Sharecropping," *The Review of Economic Studies*, 41(2), 219–255.
Teulings, C. N. (1995), "The Wage Distribution in a Model of the Assignment of Skills to Jobs," *Journal of Political Economy*, 103(2), 280–315.
Zhao, R. (2006), "Renegotiation-Proof Contract in Repeated Agency," *Journal of Economic Theory*, 131(1), 263–281.

Kaniṣka Dam
Centro de Investigación
y Docencia Económicas
Carretera México-Toluca 3655
Colonia Lomas de Santa Fe
01210 Mexico City
Mexico
kaniska.dam@cide.edu