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The Principal-Agent Matching Market

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Abstract

We propose an agency model based on competitive markets in order to analyse an economy with several homogeneous principals and heterogeneous agents. We model the principal-agent economy as a two-sided matching game and characterise the set of stable outcomes (equilibria) of this market. In this regard we generalise the assignment game of Shapley and Shubik (1972). Unlike in the standard principal-agent theory, equilibrium payoffs of all the individuals are endogenous, equilibrium contracts are Pareto optimal, and the incremental surplus generated in a principal-agent relationship accrues to the tenant. We design a simple non-cooperative game which implements the set of stable outcomes in subgame perfect equilibrium. We also suggest policy measures in relation to efficiency and income distribution.

KEYWORDS: principal-agent, moral hazard, matching, implementation

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1 Introduction

A large literature analyses optimal contracts in principal-agent relationships when there exist asymmetries of information. Seminal works by Pauly (1968), Mirrlees (1976), and Harris and Raviv (1978), among many others, examine contracts when only one principal and one agent interact. The *principal-agent contracts* involve the provision of incentives and typically lead to inefficiency due to the informational asymmetry.

The main goals of this paper are manifold. The above mentioned literature treats principal-agent relations as isolated entities. In a particular relationship, the principal assumes all the bargaining power, and agent's payoff is determined by his exogenously given reservation utility. In a market with several principals and agents, this is not necessarily true. The payoff to an agent in a particular relationship depends on the other principal-agent relationships being formed in the market. Hence, analysis of this situation calls for a general equilibrium-like model. One of the objectives of the current paper is to propose a useful framework to analyse the relationship between each principal-agent pair not as an isolated entity but as a part of an entire market where several principals and agents interact. In this framework, the utilities obtained by each principal and each agent are determined endogenously in the market. We consider the simultaneous determination of the identity of the parties who meet (i.e., which agent is contracted by which principal) and the contracts they sign in an environment where each relationship is subject to moral hazard. We model the principal-agent economy as a two-sided matching game. An outcome of this economy is an endogenous matching function and a set of contracts, one for each principal-agent pair under the matching. Roughly speaking, an outcome is said to be *stable* if there is no individual or no relevant pair objecting the existing outcome. The paper studies the set stable outcomes of this principal-agent matching market.[1]

In particular, we consider an economy with several identical principals and several agents differentiated only with respect to their initial wealth. A pair of individuals, one principal and one agent, can enter into a relationship by signing a contract. This contract specifies the contingent payments that are to be made by the agent. Also it sets the level of investment, which

[1]Ackerberg and Botticini (2002) provide empirical evidence for endogenous matching in determining the contract forms in tenancy relationships.

together with a non-contractible effort made by the agent, determines the probability of having a high return from the project the agent operates on. The initial wealth of the agent may not cover the amount to be invested and hence, the wealth differences imply differences in liability.

We begin by providing a complete characterisation of the set of stable outcomes of the principal-agent economy. The first simple property we prove is that all the principals earn the same profit in a stable outcome. In particular, if the principals constitute the long side of the market, their profits are zero. The second feature is that the contracts offered in a stable outcome are optimal, i.e., it is not possible to increase the utility level of the principal without making the agent strictly worse-off. More interestingly, in a stable outcome, the matching itself is efficient, in the sense that it is the one that maximises productive efficiency. For example, if the agents are in the long side of the market, only the wealthier ones, i.e., the more attractive ones are matched. Third, the productive efficiency of a contract signed in a stable outcome increases with the wealth of a matched agent. That is, the richer the agent, closer his contract to the first-best. The additional surplus generated due to this increase in efficiency accrues to the agent. Finally, the contracts signed in a stable outcome of this economy are more efficient than principal-agent contracts, i.e., the contracts signed when the principals assume all the bargaining power.

The previous characteristics of the set of stable outcomes have very relevant policy implications when applied to particular environments. For example, consider an economy where landowners (principals) contract with tenants (agents) who are subject to limited liability. Suppose that the government would like to improve the situations of the tenants by endowing the agents with some additional money. Our analysis suggests that the government will be interested in creating wealth asymmetries among tenants since otherwise, the landowners would appropriate all the incremental surplus intended to the tenants.

From the point of view of matching theory, one can see our model as a generalisation of the *assignment game* with several buyers and sellers described by Shapley and Shubik (1972).[2] In the current model, a relationship

[2]The literature on matching models distinguishes two types of situations. In the first type, first analysed by Gale and Shapley (1962), forming the matching does not involve any exchange between the parties, or equivalently, the amount exchanged is exogenously fixed. In the second type, called assignment games, proposed by Shapley and Shubik (1972)

is established through a contingent contract, rather than a price. The first distinguishing feature is that the surplus of each principal-agent pair, in our model, is determined endogenously. Next, the utility cannot be transferred between a principal and an agent on a one-to-one basis. In other words, unlike the assignment game, our model is a non-transferable utility game.

We consolidate stability as a reasonable solution concept for this principal-agent matching market by proposing a simple mechanism in which each of the agents proposes a contract and each principal chooses an agent. We show that the equilibrium outcomes of this mechanism coincide with the set of stable outcomes of the matching market.

A few other papers study agency problems with several principals and agents. Serfes (2001) analyses an economy with several agents and several principals, and shows that there can be positive, negative, or non-monotonic relationship between risk and incentives. Dam (2004) analyses a matching market with two-sided heterogeneity and characterises the stable outcomes when many venture capitalists and firm meet with each other. Besley and Ghatak (2005) analyse a principal-agent matching model in the presence of motivated agents in organisations. In a tenancy relation Shetty (1988) proposes a model where a set of principals compete for a continuum of agents in the presence of limited liability.[3] Also in an economy with a continuum of (heterogenous) participants in both sides, Legros and Newman (2003) present sufficient conditions for matching to be monotone when utility between partners is not fully transferable. Mookherjee and Ray (2002) analyse the optimal short term contracts in an infinitely repeated interaction among principals and agents who are randomly matched at each period. Finally, the work of Barros and Macho-Stadler (1998) looks into a situation where several principals compete for an agent. They also find that the competition among the principals make the incentive contracts more efficient.

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the parties involved in matching endogenously decide the amount of money to exchange. Roth and Sotomayor (1990) provide an excellent review of the literature of matching models without and with money. In the present paper we extend the previous models by considering situations where the parties involved in a matching are linked by a contract (and not only by an exchange of money). Hatfield and Milgrom (2005) also extend the above mentioned works by developing a model of matching with contracts. Their main focus in not on the particulars of the contracts, but on the analysis of worker-offering auction/matching algorithm that generalises the existing Gale-Shapley like algorithms.

[3]The role of limited liability in tenancy contracts are also analysed extensively by Basu (1992) and Sengupta (1997).

The paper is organised as follows. In Section 2 we lay out the basic model. We describe the main results in Section 3. In particular, we characterise the set of stable outcomes. In Section 4 we discuss the characteristics of contracts that are signed in a stable outcome. In Section 5 we propose a sequential mechanism that implements the set of stable outcomes. In Section 6 we put forward example of a principal-agent economy where the findings fit into. In Section 7 we conclude the paper and indicate some avenues for future research.

2 The Model

2.1 Principals and Agents

We consider an economy that consists of a (finite) set of n risk neutral *principals*, $\mathcal{P} = \{p_1, p_2, ..., p_n\}$ and a (finite) set of N risk neutral *agents*, $\mathcal{A} = \{a^1, a^2, ..., a^N\}$. A principal might be a landowner, a lender or an employer. An agent is a tenant, a borrower or a worker. Principals are of identical characteristics. Principals are denoted by p, p', etc. Agents differ with respect to their initial wealth. An agent a^j has an initial wealth w^j, which is known to the principals. Without any loss of generality, we order the wealth level as $w^1 \geq w^2 \geq .... \geq w^N \geq 0$. Agents are denoted by a, a', etc. with wealth level w, w', etc., respectively. The principals and the agents are matched in pairs and a contract is signed by each pair. We allow for the possibility that a principal or an agent can seek for an alternative partner and can sign a different contract. Hence, the matching is endogenous rather than being exogenous.

2.2 Projects

When a principal-agent pair is formed, the agent operates on a project, chooses effort level e from the set $\{0, 1\}$, and investment K is made, which is financed entirely by the principal. An agent incurs a disutility of e when he chooses the effort level e. The effort exerted is not contractible but the level of investment is.[4] Effort and investment influence the return of each project

[4] All our findings remain unaltered even if an agent with positive wealth finances part of the investment. Also the qualitative results would not change if an agent could choose

which is uncertain. Given an effort level e and investment K, let $\pi_e(K)$ be the probability of the event of success and $1-\pi_e(K)$, the probability of failure. Each project generates a return $y>0$ in case of success. In case of failure, the return is 0. We make the following assumptions:

Assumption 1:

(a) $\pi_1(K)>\pi_0(K)$, for all $K>0$,
(b) $0\leq\pi_e(K)\leq 1$, for all $K>0$ and for $e=0$, 1, and $\pi_0(0)=0$, and
(c) $\pi_e'(K)>0>\pi_e''(K)$ for all $K>0$ and $\lim_{K\to\infty}\pi_e'(K)=0$.

Assumptions (c) guarantees that the solution in K is interior. Let us denote by $\mathcal{M}\equiv\{\mathcal{P},\,\mathcal{A},\,\mathbf{w},\,\pi\}$ the market, where $\mathbf{w}\equiv(w^1,\,...,\,w^N)$ denotes the vector of initial wealth of the agents in $\mathcal{A}$ and π represents the technology.

2.3 Contracts and Payoffs

A principal-agent pair $(p,\,a)$ signs a contract, $c_{p,a}$, which is a three dimensional vector $(R_{p,a},\,r_{p,a},\,K_{p,a})$. We take the convention that the agent keeps the output. The first component of the contract $R_{p,a}$ is the transfer to the principal in the event of *success* and the second component $r_{p,a}$ is the transfer in case of *failure*. The third component $K_{p,a}$ is the level of investment. Given a contract $c_{p,a}=(R_{p,a},\,r_{p,a},\,K_{p,a})$ signed by a pair $(p,\,a)$, let $e_{c_{p,a}}$ be defined as the effort that maximises the agent's utility:[5]

$$e_{c_{p,a}}=\underset{e}{argmax}\,\{\pi_e(K_{p,a})(y-R_{p,a})-(1-\pi_e(K_{p,a}))r_{p,a}-e\}.\qquad (IC_a)$$

For a contract $c_{p,a}$, the effort chosen by the agent will be $e_{c_{p,a}}$ given that the effort is not contractible. This is the *incentive compatibility* constraint. Moreover, we normalise the per unit opportunity cost of financing a project to 1. Then the expected utilities of the principal p and the agent a when they sign the contract $c_{p,a}$ will be:

$$\begin{aligned} U_p(a,\,c_{p,a}) &= \pi_{e_{c_{p,a}}}(K_{p,a})R_{p,a}+(1-\pi_{e_{c_{p,a}}}(K_{p,a}))r_{p,a}-K_{p,a}\,, \\ u_a(p,\,c_{p,a}) &= \pi_{e_{c_{p,a}}}(K_{p,a})(y-R_{p,a})-(1-\pi_{e_{c_{p,a}}}(K_{p,a}))r_{p,a}-e_{c_{p,a}}. \end{aligned}$$

effort level from the closed interval [0, 1] (or, any compact subset of $\Re_+$).

[5] Conventionally $e_{c_{p,a}}=1$ if both 1 and 0 maximises the following expression.

Notice that we have defined the expected utility of agent a net of the wealth w. The gross expected utility of this agent would be $u_a(p,\, c_{p,a}) + w$. For future notational convenience, we denote by $c^{null} = (0,\ 0,\ 0)$, the *null contract*. Under c^{null}, $U_p(a,\, c^{null}) = u_a(p,\, c^{null}) = 0$. We assume that for an agent, signing a contract c^{null} is equivalent to the situation where he is not contracted by any principal, i.e., his reservation utility equals 0. Agent's liability is limited to his current wealth. This imposes restrictions on the set of contracts. Limited liability implies

$$R_{p,a} \leq y + w\ , \tag{LS_a}$$

$$r_{p,a} \leq w\ . \tag{LF_a}$$

The assumption of risk neutrality together with limited liability makes the incentive compatibility constraint costly and hence, it gives rise to *moral hazard* in agent's effort choice. A sensible contract for a principal-agent pair must satisfy the incentive compatibility and limited liability constraints. Furthermore, neither an agent nor a principal would accept a contract with negative expected utility. That is, a contract for a pair $(p,\, a)$ has to be acceptable to each member of the pair. We say that a contract $c_{p,a}$ is *acceptable* for $(p,\, a)$ if $U_p(a,\, c_{p,a}) \geq 0$ and $u_a(p,\, c_{p,a}) \geq 0$. We club all these natural restrictions into the following definition.[6]

Definition 1 *A contract is* **feasible for an agent** *a if it satisfies the restrictions of limited liability and acceptability.*

Denote by X^a the set of contracts feasible for agent a. From now on we will concentrate only on feasible contracts.

The incentive compatibility constraint implies that the agent may choose any of the two effort levels (high or low). In order to deal with interesting situations, we will assume, from now on, that the output y in case of success is high enough so that it is always optimal first, to establish a relationship and second, to set a contract that induces the agent to exert high effort. Hence, one can substitute the incentive compatibility constraint (IC_a) by the following:

$$(\pi_1(K_{p,a}) - \pi_0(K_{p,a}))(y - R_{p,a} + r_{p,a}) \geq 1. \tag{IC'_a}$$

[6]Notice that the limited liability constraints are agent specific.

We will denote by $Z^a \subset X^a$ the set of feasible contracts that also satisfy the incentive compatibility constraint (IC'_a). One particular class of contracts are the *principal-agent* contracts, where the principal assumes all the bargaining power. The principal-agent contract for the pair (p, a), denoted $c^*_{p,a}$, solves the following programme:

$$\max_{c_{p,a} \in Z^a} U_p(a, c_{p,a}). \tag{P1}$$

Given the limited liability constraints, the moral hazard problem is typically costly for the principal. If the limited liability constraints did not bind, then the principal would not incur any cost for the provision of incentives, and we would be at a situation similar to the symmetric information case. This is usually termed as *first best*. From the analysis of the above maximisation problem one can show that there exists a threshold level of wealth w^0 such that if agent's wealth is above this level then his limited liability constraint does not bind, and hence for $w \geq w^0$ the contracts that solve the above problems are the first-best contracts. For an agent's wealth lower than w^0 a principal earns lower profits compared to the first-best situation. Next, we show that if the principal has all the bargaining power, she strictly prefers hiring an agent with higher wealth if the *first best* has not already been reached.

PROPOSITION **1** *If $w > w'$ and $w < w^0$, then for a principal p, $U_p(a, c^*_{p,a}) > U_p(a', c^*_{p,a'})$.*

PROOF See Appendix B. ■

2.4 Matching

Principals and agents are matched in pairs and when a pair is formed, a contract is signed. The following three definitions describe a matching and a relevant outcome of this principal-agent economy.

DEFINITION **2** *A (one-to-one)* **matching** *for $\mathcal{M}$ is a mapping $\mu : \mathcal{P} \cup \mathcal{A} \to \mathcal{P} \cup \mathcal{A}$ such that (i) $\mu(p) \in \mathcal{A} \cup \{p\}$ for all $p \in \mathcal{P}$, (ii) $\mu(a) \in \mathcal{P} \cup \{a\}$ for all $a \in \mathcal{A}$ and (iii) $\mu(a) = p$ if and only if $\mu(p) = a$ for all $(p, a) \in \mathcal{P} \times \mathcal{A}$.*

The definition implies that a matching for a market $\mathcal{M}$ is a mapping which specifies that either each individual of one side of the market is assigned to another individual of the other side or, the individual remains alone. We say that the pair (p, a) is *matched* under μ if $\mu(p) = a$ (or, equivalently, $\mu(a) = p$).

DEFINITION **3** *A menu of contracts* $\mathcal{C}$ **compatible with a matching** μ *for* $\mathcal{M}$ *is a vector of contracts,* $\mathcal{C} = (c_{p_1}, ..., c_{p_n}, c_{a^1}, ..., c_{a^N})$ *such that (a)* $c_p = c_a = c_{p,a}$ *if* $\mu(p) = a$ *and* c_a *is feasible for* (p, a)*, (b)* $c_p = c^{null}$ *if* $\mu(p) = p$ *and (c)* $c_a = c^{null}$ *if* $\mu(a) = a$.

DEFINITION **4** *An* **outcome** $(\mu, \mathcal{C})$ *for the market* $\mathcal{M}$ *is a matching* μ *and a menu of contracts* $\mathcal{C}$ *compatible with* μ.

The outcomes of the market we describe here are endogenous. This endogeneity has two aspects. First, the contracts signed by the principals and the agents are endogenous. In the principal-agent theory, considerable attention has been paid in order to analyse the contracts that prevail in a given (isolated) principal-agent relationship. The second aspect is that the matching itself should be endogenous. We will approach this perspective in the same vein as the matching theory. We require that a *reasonable* outcome should be immune to the possibility of being *blocked* by any principal-agent pair (as well as by any single individual). Consider an outcome $(\mu, \mathcal{C})$. If there is a principal-agent pair which can sign a feasible contract such that both the principal and the agent are strictly better-off under the new arrangement compared to their situation in the outcome $(\mu, \mathcal{C})$, then such an outcome is not reasonable. This idea corresponds to the notion of stability.

DEFINITION **5** *An outcome* $(\mu, \mathcal{C})$ *for the market* $\mathcal{M}$ *is* **stable** *if there does not exist any pair* (p, a) *and any contract* $c' \in X^a$ *such that* $U_p(a, c') > U_p(\mu(p), c_{p,\mu(p)})$ *and* $u_a(p, c') > u_a(\mu(a), c_{\mu(a),a})$.

The above definition makes sure that there does not exist any principal-agent pair that can *block* the current outcome, signing a feasible contract c' between them. Moreover, since all the contracts in a stable outcome are feasible, this implies that a stable outcome is also *individually rational*.

3 The Set of Stable Outcomes

In this section we characterise the set of stable outcomes of the market $\mathcal{M}$. We start by stating two important properties of a stable outcome. First, all the contracts in a stable outcome are Pareto optimal. The notion of Pareto optimality is formalised in the following definition.

DEFINITION **6** *A contract $c_{p,a}$ for a principal-agent pair $(p,\, a)$ is Pareto optimal if there is no other feasible contract c' such that $U_p(a,\, c') > U_p(a,\, c_{p,a})$ and $u_a(p,\, c') \geq u_a(p,\, c_{p,a})$.*

The above definition says that if a principal-agent pair signs a Pareto optimal contract then there is no possibility of improving the utility of one individual in a principal-agent pair without making the other individual worse-off. The following lemma states the optimality property. Notice, that the notion of optimality is pairwise. Since in this set up an agent trades only with a principal (and not with any other agents), the pairwise optimality coincides with the usual notion of Pareto efficiency.

LEMMA **1** *All the contracts in a stable outcome are Pareto optimal.*

PROOF Suppose $(\mu,\, \mathcal{C})$ is stable, but the contract $c_{p,a} \in \mathcal{C}$ signed by p and a, where $\mu(a) = p$, is not optimal. Then there exists a contract c', feasible for $(p,\, a)$ such that (i) $U_p(a,\, c') > U_p(a,\, c_{p,a})$ and (ii) $u_a(p,\, c') > u_a(p,\, c_{p,a})$. In that case $(p,\, a)$ will block $(\mu,\, \mathcal{C})$ with c'. This contradicts the fact that $(\mu,\, \mathcal{C})$ is initially stable. ■

It is worth noting that the optimality of a contract between a principal and an agent in any stable outcome is guaranteed by the possibility that the same pair can block the initial outcome with a different contract. Another property of stable outcomes is that no principal can gain more than any of her counterpart does. The profits of all the principals are equal. Lemma 2 proves this assertion.

LEMMA **2** *In any stable outcome $(\mu,\, \mathcal{C})$, all principals get same utility.*

PROOF Suppose in a stable outcome $(\mu,\, \mathcal{C})$, we have $U_p(\mu(p),\, c_{p,\mu(p)}) > U_{p'}(\mu(p'),\, c_{p',\mu(p')})$. We show that there exists a contract $c' \in \mathcal{C}$ such that

$(p',\, \mu(p))$ blocks the outcome with c'. Suppose $c_{p,\mu(p)} = (R_{p,\mu(p)},\, r_{p,\mu(p)},\, K_{p,\mu(p)})$ and consider $c' = (R_{p,\mu(p)} - \varepsilon,\, r_{p,\mu(p)} - \varepsilon,\, K_{p,\mu(p)})$ with $\varepsilon > 0$.[7] It is easy to check that $e_{c_{p,\mu(p)}} = e_{c'}$. Hence, for ε small enough, we have $U_{p'}(\mu(p),\, c') = U_p(\mu(p),\, c_{p,\mu(p)}) - \varepsilon > U_{p'}(\mu(p'),\, c_{p',\mu(p')})$ and $u_{\mu(p)}(p',\, c') \geq u_{\mu(p)}(p,\, c_{p,\mu(p)}) + \varepsilon > u_{\mu(p)}(p,\, c_{p,\mu(p)})$. Therefore, $(p',\, \mu(p))$ blocks $(\mu,\, \mathcal{C})$ with c' and hence the lemma. ■

The above lemma states the intuitive property that, when the principals are identical, they must obtain the same profits in a stable outcome. This property is no longer valid if we consider some heterogeneity among the principals.

Lemma 1 implies that the contracts in a stable outcome must be optimal. Hence, a contract signed by a matched pair $(p,\, a)$ must maximise the expected utility of one party taking into account that the other gets at least a certain utility level. One particular class of optimal contracts are the *principal-agent* contracts, which have been discussed in Section 2.3.

The utility possibility frontier for any principal-agent pair is the set of utilities generated by the contracts that solve a programme similar to (P1) where the reservation utility of the agent can take value not only equal to zero as in (P1), but any number. The same set of optimal contracts results in if one maximises agent's utility subject to a participation constraint of the principal (PC_p). We will denote by $c_{p,a}(\widehat{U})$ the optimal contract that solves the following programme (as before we take agent's utility net of his wealth w):

$$\begin{cases} \max_{c_{p,a} \in Z^a} u_a(p,\, c_{p,a}) \\ s.t. \quad U_p(a,\, c_{p,a}) \geq \widehat{U}, \quad (PC_p) \end{cases} \tag{P2}$$

where $\widehat{U} \geq 0$. Notice that for $\widehat{U} = 0$, the contracts that solve the above programme are acceptable for a principal p. Also, a contract is acceptable for an agent a only if $\widehat{U}$ is not too high. More precisely, $u_a(p,\, c_{p,a}(\widehat{U})) \geq 0$ if and only if $\widehat{U} \leq U_p(a,\, c^*_{p,a})$. In the following theorems we completely characterise the set of stable outcomes. The properties that the contracts in a stable outcome are optimal and that all principals earn equal profits provide a partial characterisation. These help us complete the description of the set

[7] In some proofs we will use the notation $c - \varepsilon$ to refer to the contract $(R - \varepsilon,\, r - \varepsilon,\, K)$, when $c = (R,\, r,\, K)$.

of stable outcomes. We distinguish among different cases. In Theorem 1, we consider the situation where there are more agents than principals ($N > n$) in the economy. In Theorem 2, we analyse the situations where there are same number of principals and agents and there are more principals than agents. Notice that the two lemmas stated above hold irrespective of the cardinalities of the set of principals and the set of agents.

THEOREM **1** *If $n < N$, then an outcome $(\mu, \mathcal{C})$ is stable for the market $\mathcal{M}$ if and only if the following three conditions hold:*
(a) *$\mu(p) \in \mathcal{A}$ for all $p \in \mathcal{P}$, $\mu(a) \in \mathcal{P}$ if $w > w^{n+1}$ and $\mu(a) = a$ if $w < w^n$,*
(b) *$U_p(\mu(p), c_{p,\mu(p)}) = \widehat{U} \in \left[U_p(a^{n+1}, c^*_{p,a^{n+1}}), U_p(a^n, c^*_{p,a^n})\right]$ for all $p \in \mathcal{P}$, and*
(c) *$c_a = c_{p,a}(\widehat{U})$ if $\mu(a) = p$ and $c_a = c^{null}$ if $\mu(a) = a$.*

PROOF We first prove that **(a)-(c)** are necessary conditions for any stable outcome.

(a) Suppose first, that in a stable outcome $(\mu, \mathcal{C})$ any principal p is not matched. Then $U_p(\mu(p), c_{p,\mu(p)}) = 0$. Now consider an agent a who is initially unmatched under μ. Then the contract $c^*_{p,a} - \varepsilon \in Z^a$ yields strictly higher payoffs to both p and a. Hence, (p, a) with $c^*_{p,a} - \varepsilon$ blocks $(\mu, \mathcal{C})$. Second we show that a is matched if $w > w^{n+1}$. Suppose, on the contrary, that a is unmatched under μ and hence, $u_a(a, c_{\mu(a),a}) = 0$. Because of the previous proof, under μ there are n agents matched. Suppose, a' is a matched agent such that $w' \leq w^{n+1}$. Following Proposition 1, $U_{\mu(a')}(a, c^*_{\mu(a'),a}) > U_{\mu(a')}(a', c^*_{\mu(a'),a'})$. Given that $u_{a'}(\mu(a'), c_{\mu(a'),a'}) \geq 0$ (since, the contract is feasible), $U_{\mu(a')}(a', c_{\mu(a'),a'}) \leq U_{\mu(a')}(a', c^*_{\mu(a'),a'}) < U_{\mu(a')}(a, c^*_{\mu(a'),a})$. Take $c' = c^*_{\mu(a'),a} - \varepsilon$, with ε small enough. It is easy to see that $(\mu(a'), a)$ with the contract c' will block the outcome which is a contradiction. For the last part of (a), suppose on the contrary that a is matched under μ and $w < w^n$. Since only n agents are matched, take a' such that this agent is not matched in a stable outcome and $w' > w^n$. Applying the same argument as before, it is easy to show that $(\mu(a), a')$ with the contract $c^*_{\mu(a),a'} - \varepsilon$ will block the current outcome.

(b) We know that in all the stable outcomes the profits of the principals must be equal. Denote by $\widehat{U}$ the common profit of the principals. First we will show that in a stable outcome $(\mu, \mathcal{C})$, $\widehat{U} \geq U_p(a^{n+1}, c^*_{p,a^{n+1}})$. Suppose on the contrary, $\widehat{U} < U_p(a^{n+1}, c^*_{p,a^{n+1}})$. From part (a) of the theorem we know that any agent with less wealth than w^n cannot be matched in a stable outcome. Suppose this is a^{n+1} and consider any principal p. Then there is a contract $c' = c^*_{p,a^{n+1}}) - \varepsilon$, with ε small enough, such that (1) $U_p(a^{n+1}, c') = U_p(a^{n+1}, c^*_{p,a^{n+1}})) - \varepsilon > \widehat{U}$ and (2) $u_{a^{n+1}}(p, c') \geq \varepsilon > 0 = u_{a^{n+1}}(\mu(a^{n+1}), c_{a^{n+1},a^{n+1}})$. Hence, (p, a^{n+1}) blocks the outcome. Second, from Proposition 1 we know that $U_p(a, c^*_{p,a}) > U_p(a', c^*_{p,a'})$ if and only if $w > w'$. In a stable outcome $(\mu, \mathcal{C})$, an agent with wealth greater than w^{n+1}, say a^n is matched with some principal, say p. Then $U_p(a^n, c_{p,a^n}) = \widehat{U} > U_p(a^n, c^*_{p,a^n})$ which implies that $u_{a^n}(p, c_{p,a^n}) < u_{a^n}(p, c^*_{p,a^n})$. This is not possible in a stable outcome.

(c) Let $(\mu, \mathcal{C})$ be a stable outcome. By Lemma 1, any contract in $\mathcal{C}$ is optimal and c_a is such a contract. So, given the stability of $(\mu, \mathcal{C})$, $c_a = c_{\mu(a),a}(\widehat{U})$ if $\mu(a) \in \mathcal{P}$.

We now prove that any outcome $(\mu, \mathcal{C})$ satisfying **(a)-(c)** is indeed stable. Suppose $\mu(a) \in \mathcal{P}$ and consider any principal p who, because of part (a), is matched. Clearly, (p, a) cannot block the outcome with any contract. Indeed, there does not exist a contract such that p gets more than $\widehat{U}$ and a gets more than $u_a(\mu(a), c_{\mu(a),a})$ since $c_{\mu(a),a}(\widehat{U})$ is optimal by (c). Now suppose $\mu(a) = a$ and choose any arbitrary p (we can do so, since all principals have the same profit). By (a), we know that $w \leq w^{n+1}$. Then the maximum utility p can get by contracting a such that $u_a(..) \geq 0$ is $U_p(a, c^*_{p,a}) \leq U_p(a^{n+1}, c^*_{p,a^{n+1}})$. Given that $\widehat{U} \geq U_p(a^{n+1}, c^*_{p,a^{n+1}})$ (because of (d)), there is no room for (p, a) to block $(\mu, \mathcal{C})$. ■

We have already established that in a stable outcome all the principals get the same utility. When there are too many agents, this uniform utility cannot be less than the surplus that can be created by the *richest* unmatched agent and it cannot be more than the surplus that can be created by the *poorest* matched agent. In the following theorem, we restate Theorem 1 in cases where there are same number of principals and agents and where there are more principals than agents.

Theorem **2** ***(i)*** *If $n = N$, then an outcome $(\mu, \mathcal{C})$ is stable for the market $\mathcal{M}$ if and only if the following three conditions hold:*
(a) *All principals and agents are matched,*
(b) $U_p(\mu(p), c_p) = \widehat{U} \in [0, U_p(a^n, c^*_{p,a^n})]$ *for all $p \in \mathcal{P}$, and*

(c) *$c_a = c_{\mu(a),a}(\widehat{U})$ for any a.*

(ii) *If $n > N$, then an outcome $(\mu, \mathcal{C})$ is stable for the market $\mathcal{M}$ if and only if the following three conditions hold:*

(a) *Only N principals and all the agents are matched,*

(b) *$U_p(\mu(p), c_p) = 0$ for all $p \in \mathcal{P}$, and*

(c) *$c_a = c_{\mu(a),a}(0)$ for any a.*

We omit the proof since it is similar to that of Theorem 1. Part (i) describes the situation where there are as many principals as agents. Since all principals consume the same utility, they can obtain as low as zero utility but no more than the maximal utility that can be consumed by the principal matched with the *poorest* agent. Part (ii) concerns the situation where there is an abundance of principals. Since each principal gets the same utility level and since the unmatched ones necessarily obtain zero utility, each principal shall consume a zero utility too.

The above theorems characterise the stable outcomes for this principal-agent economy. First important thing to note is the optimality property of the contracts in the stable outcome. Optimality in this market has in fact two aspects. The contracts signed are optimal for the parties involved. This was a property already established in Lemma 1. On the other hand, part (a) in both theorems makes sure that the matching itself is optimal too. This is the case because, in a stable outcome, all the individuals in the short side of the market are matched and, when there are more agents than principals, only the *best* (wealthier) agents are the ones who get contracted.[8]

The second important property is that the profits of the principals are equal. In a stable outcome there emerges competition among the principals for the wealthier agents. In particular, when there are more principals than agents (Theorem 2(ii)), the profit of each principal is driven down to zero.

Third, in a stable outcome, all the agents whose wealth level is above the wealth of the poorest agent contracted obtain a strictly higher utility than that under a principal-agent contract. In fact, there are stable outcomes where the same is true even for the poorest agent contracted. To understand the reason for this property, notice that had the agents been symmetric, i.e., if they had equal initial wealth, and they were large in number, the principals would assume all the bargaining power. In this case, the stable outcome

[8] The optimality of both, the contracts and the matching also implies that the stable outcome is the socially best outcome.

would involve a principal-agent contract for each agent hired. The asymmetry among the agents does not let the principals appropriate all the incremental surplus generated in a principal-agent relationship, even when there are more agents than principals. Rather, the competition among principals makes the incremental surplus accrue to the agents. This competition is even more acute when there is an abundance of principals. In this case, it follows from Theorem 2 that the entire surplus generated in a relation accrues to the agent. It is worth mentioning that, instead of wealth constrained agents, one could consider a model where agents are heterogeneous with respect to risk aversion. Then a stable outcome would involve only less risk-averse agents being matched with the principals, and higher wealth levels would be a proxy for less risk aversion.

Finally, as is usual in the classical matching models, the set of stable outcomes in our economy has a nice structure. First, if $(\mu, \mathcal{C})$ is a stable outcome and μ' is an efficient matching, then $(\mu', \mathcal{C})$ is also stable. That is, the set of stable outcome is the Cartesian product of the set of efficient matching and a set of menus of optimal contracts. Second, if one stable outcome $(\mu, \mathcal{C})$ is better for an agent than another stable outcome $(\mu', \mathcal{C}')$, then $(\mu, \mathcal{C})$ is better than $(\mu', \mathcal{C}')$ for all the agents hired and worse for all the principals matched. In particular, out of all the stable outcomes there exists a stable outcome which is the best from the principals' point of view and similarly for the agents. In this economy, these two extreme points in the set of stable outcomes correspond to the outcomes in which the utilities of the principals are $\widehat{U} = U_p(a^n, c^*_{p,a^n})$ and $\widehat{U} = U_p(a^{n+1}, c^*_{p,a^{n+1}})$.[9] The first point is the principals' optimal stable outcome (we refer to this as *P-optimum*), while the second is the agents' optimal stable outcome (call this *A-optimum*).

In our framework, transactions occur via contracts. The major difference between this economy and a market where transactions go through prices (as in the assignment game analysed by Shapley and Shubik, 1972) is that the total surplus produced in a particular relation does depend on the way in which the surplus is shared between the principal and the agent and on the design of the contract. The size of the surplus that accrues to the agent influences the extent to which the limited liability constraints are binding and hence the total surplus.

[9]If $w^n = w^{n+1}$, then the common utility consumed by all principals, $\widehat{U}$, is equal to $U_p(a^n, c^*_{p,a^n}) = U_p(a^{n+1}, c^*_{p,a^{n+1}})$. Moreover, any agent obtains the same utility in all the stable outcomes.

4 Contracts in a Stable Outcome

In this section, we provide the characteristics of the contracts signed in a stable outcome. We have already shown that any such contract solves the maximisation programme (P2). Now we turn on to analyse the characteristics of the solution to this programme. We will develop the analysis under the following assumption.[10]

ASSUMPTION 2: $\pi_1(K)\pi_0'(K) - \pi_1'(K)\pi_0(K) > 0$ for all $K > 0$.

The above assumption implies that the derivative of $\frac{\pi_0}{\pi_1}$ with respect to K is positive. That is, the higher the level of investment, the lower is the difference between π_0 and π_1, and hence, the influence of making a high effort. The *first-best* level of investment, K^0 is given by the following equation:

$$\pi_1'(K^0)y = 1.$$

In the first-best contract, K^0 is the level of investment that would be chosen if there was no moral hazard problem, or equivalently, if the limited liability would not have any bite. In order to analyse the programme (P2), one can identify two disjoint ranges of values of w where the optimal solutions are different. First, for a very high level of agent's wealth both the incentive compatibility constraint and limited liability constraint (in the event of failure) are not binding.[11] This is equivalent to saying that there is no moral hazard problem. The threshold level of initial wealth, $w(\widehat{U})$, beyond which the optimal investment reaches its first-best level K^0 depends on the utility of the principal, $\widehat{U}$, and is:

$$w(\widehat{U}) \equiv -\pi_1(K^0)y + K^0 + \widehat{U} + \frac{\pi_1(K^0)}{\pi_1(K^0) - \pi_0(K^0)}.$$

For low levels of initial wealth, $w \leq w(\widehat{U})$, both the incentive constraint and the limited liability constraint bind. In this region the moral hazard problem becomes important and hence, the optimal investment is lower than

[10] In the appendix we comment on the qualitative changes if the opposite assumption holds. Appendix C provides a more complete analysis of the solution to (P2).

[11] One can easily check that the limited liability constraint in the event of success is automatically satisfied for the problem and that $R_{p,a}$ can be calculated from the (PC_p).

its first-best level. The optimal investment $\widehat{K}(w;\,\widehat{U})$ is implicitly defined by the following equation:

$$-\pi_1(K)y + K + \widehat{U} + \frac{\pi_1(K)}{\pi_1(K) - \pi_0(K)} = w.$$

Given Assumption 1, the optimal investment increases with agents' wealth. The optimal investment is summarised in the following equation:

$$K_{p,a} = \begin{cases} \widehat{K}(w;\,\widehat{U}) & \text{if } w < w(\widehat{U}) \\ K^0 & \text{if } w \geq w(\widehat{U}). \end{cases}$$

We also describe in brief the characteristics of the state contingent transfers. Notice that, for $w \geq w(\widehat{U})$, any combination of $(R_{p,a},\, r_{p,a})$ that satisfies the constraints can be candidate for the optimum. One possible optimum corresponds to $r_{p,a} = w$. In case where the constraints (IC'_a) and (LF_a) are binding (for $w \leq w(\widehat{U})$), $r_{p,a} = w$ is also an optimum. Using the participation constraint of the principal, one can then easily calculate the optimal transfer in case of success which is given by the following.

$$R_{p,a} = \begin{cases} \frac{\widehat{U} + \widehat{K}(w;\widehat{U}) - (1 - \pi_1(\widehat{K}(w;\widehat{U})))w}{\pi_1(\widehat{K}(w;\widehat{U}))} & \text{if } w < w(\widehat{U}) \\ \frac{\widehat{U} + K^0 - (1 - \pi_1(K^0))w}{\pi_1(K^0)} & \text{if } w \geq w(\widehat{U}) \end{cases}$$

Once we know the characteristics of the solutions to program (P2), we use theorems 1 and 2 to provide a description of the contracts in a stable outcome. Consider first a situation with many agents where the wealth of most of them is zero, i.e., $N > n$ and $w^n = w^{n+1} = 0$. In this economy, the contracts signed in all the stable outcomes are the same. The contract signed by the hired agents with zero wealth will be the corresponding principal-agent contract, while the contract signed by the richer agents will correspond to the solution of program (P2), for $\widehat{U} = U_p(a^n, c^*_{p,a^n})$. Figure 1 depicts the level of investments in the stable outcome.[12]

[12] For the sake of tangibility, all the figures are drawn for $\pi_1(K) = \frac{K}{1+K}$ and $\pi_0(K) = \frac{K}{2+K}$. Our results, although, hold good for a very general class of probability functions satisfying our assumptions.

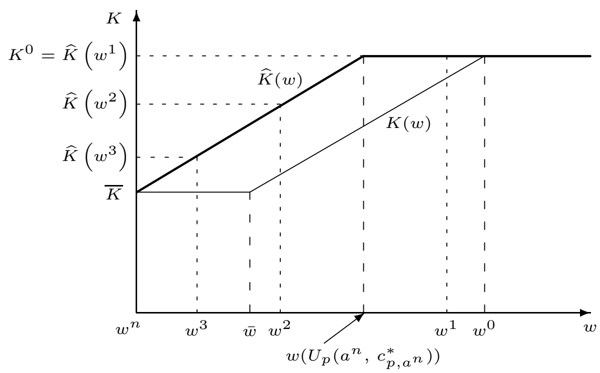

Figure 1: Optimal investment levels when $w^n = w^{n+1} = 0$

For comparison, the diagram also includes the level of investments $K(w)$ that would be made if all the agents would sign a principal-agent contract. In this figure, $\overline{K}$ is the minimum level that would be invested by the agents with very low level of wealth (say, less than $\overline{w}$). The investment level is closer to the first-best level K^0 as the wealth of an agent is higher. That is, the productive efficiency of the relationship increases with the agent's wealth. The investment level coincides with the first-best level if the agent, say agent a^1, is rich enough, i.e., $w^1 \geq w(U_p(a^n, c^*_{p,a^n}))$. It is worth noting also that these investments are always higher than those under principal-agent contracts, unless the agent's wealth is very large, $w \geq w^0$.

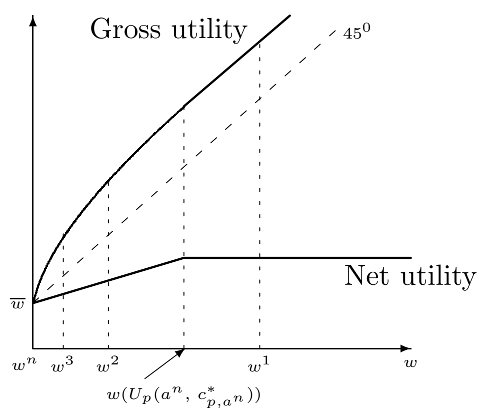

Figure 2: Gross and net utilities of an agent

For the same economy, Figure 2 depicts agents' net and gross utility levels (the common principals' utility is $U_p(a^n, c^*_{p,a^n})$). Agents' net utility increases with the wealth level (unless the level of wealth is already above $w(U_p(a^n, c^*_{p,a^n})))$. The utilities of wealthier agents are not only higher because of the initial wealth levels. They also profit from the increase in the surplus due to the more efficient (i.e., closer to the first-best) contracts.

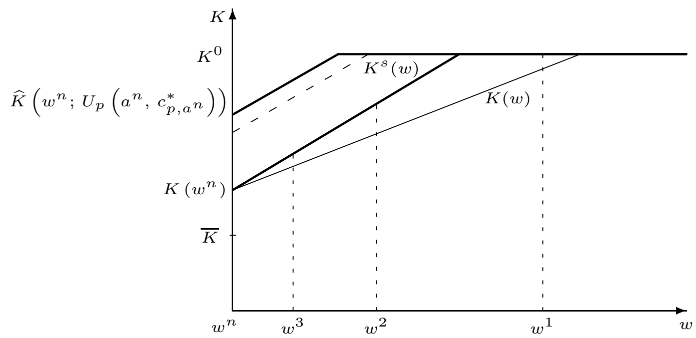

Figure 3: Optimal investment when $w^n > w^{n+1}$

For completeness, Figure 3 depicts the set of investment levels in the stable outcomes when $N > n$, $w^n > w^{n+1}$, and w^{n+1} is large. The line corresponding to the level of investments in a particular stable outcome, say $K^s(w)$ is quite similar to that in Figure 1 (although it starts from a level higher than $\overline{K}$). This line will be placed at a higher (or a lower) position depending if we are in a stable outcome closer to (or farther from) the *A-optimum*. In particular, the lowest line (that starts from $K(w^n)$) corresponds to the investment levels in the *P-optimum*.

The graphical representation of an economy with more principals than agents is very similar to that in figures 1 and 2. The levels of investment and of net and gross utilities are as in figures 1 and 2, with the only difference that they all start at a higher level than $\overline{K}$ and $\overline{w}$.

5 Implementing the Set of Stable Outcomes

In this section we further argue about stability as a reasonable solution concept for the market we analyse. We show that the set of stable outcomes that we have characterised in theorems 1 and 2 are also the equilibrium outcomes of a very simple and natural non-cooperative interaction between the principals and the agents. The simple mechanism that we propose, called Γ^A, is a two-stage game where in the first stage each agent proposes a contract. In the second stage of the game, each principal contracts an agent.[13] Formally, at the first stage of the mechanism, agents send their messages simultaneously. The message of each agent is an element of the set of feasible contracts. A message $s^a \in X^a$ of agent a should be understood as the contract he demands. At the second stage, knowing the messages of the agents, each principal p sends a message $s_p \in \mathcal{A} \cup \{p\}$. A message of a principal should be understood as the agent she wants to hire or she wants to stay unmatched. The outcome function $g(.)$ associates to each vector of messages, $s = (s_1, ..., s_n, s^1, ..., s^N)$ a matching, μ^s, and a menu of contracts, $\mathcal{C}(s)$, such that $\mu^s(a)$ is the smallest indexed principal of the set $\mathcal{P}^a = \{p \in \mathcal{P} \mid s_p = a\}$ if $\mathcal{P}^a \neq \varnothing$ and $\mu^s(a) = a$, otherwise. Moreover,

$$c_a(s) = \begin{cases} s^a & \text{if } \mu^s(a) \in \mathcal{P} \\ c^{null} & \text{, otherwise.} \end{cases}$$

The natural solution concept used here is *Subgame Perfect Equilibrium*. We will analyse the Subgame Perfect Equilibria in pure strategies (SPE).

THEOREM **3** **(i)** *When $n \neq N$, the set of SPE outcomes of the mechanism Γ^A coincides with the set of stable outcomes for the market $\mathcal{M}$.*
(ii) *When $n = N$, the set of SPE outcomes of the mechanism Γ^A coincides with the set of agents' optimal stable outcomes for the market $\mathcal{M}$*

PROOF See Appendix E. ■

From the point of view of implementation, the above theorem shows that one can propose a very simple mechanism which makes it possible to implement the set of stable outcomes (or the agents' optimal stable outcomes) of this principal-agent economy.

[13] The proposed mechanism adapts to our framework the mechanism suggested by Alcalde, Pérez-Castrillo and Romero-Medina (1998). The two main differences are that the participants now sign contracts, more complex than a salary as in the previous paper, and it is a one-to-one matching model which imposes some additional rigidities on the working of the mechanism. For implementation in matching markets, see also Kara and Sönmez (1997).

6 An Application: A Landowner-Tenant Economy

In a seminal work, Shetty (1988) shows that wealth differences among tenants play a key role in determining the credit contracts when there exists a possibility of default on the rental commitments. Difference in initial wealth implies difference in liability of the tenants. Hence, in the case where there is significant moral hazard problem due to limited liability, wealthier tenants are always preferred for a better contractual structure, since possibility of default is less with wealthier tenants. Our results can be used to analyse similar situations when a set of landowners interacts with a set of tenants through tenancy relations. One feature is to note that the kind of contracts we use can often be observed in the less developed economies. It is very common that the same person acts as landowner-cum-moneylender in the villages by leasing land and lending money to the same person (here, the tenant). The contracts described for the market $\mathcal{M}$ also capture these components. The state contingent transfers, (R, r) are the payments made to the landowners and K is the amount borrowed from the landowners that is invested eventually in land. In this economy, the tenants cannot seek loans from the formal credit sector due to lack of sufficient collateral, while the landowners can. Consequently, the landowners become the only sources of credit for the hapless borrowers. With these interpretations, our results imply:

(i) In a stable outcome, all the contracts signed among landowners and tenants are optimal and all the landowners and only the wealthier tenants are matched. All the landowners earn the same profit and the contracts maximise the expected utility of the tenants for the common profit level of the landowners. Wealthier is the tenant, the more efficient the contract he signs (closer to first-best). The above findings also conform to the findings of Shetty (1988).

(ii) The investments made in a stable outcome are, in fact, closer to first-best than those that would be implemented if the tenants would sign principal-agent contracts. As landowners compete for the wealthier tenants, they are compelled to offer these tenants *better* contracts in order to attract them. Since the tenants obtain higher utility, the limited liability constraint is less stringent and hence the investment level approaches the first best. This phenomenon is described in figures 1 and 2. This comparison is relevant because the principal-agent contracts are the contracts that would have been offered, for example, if the landowners would collude.

The property highlighted in (ii) has important implications with respect to distributive (in)equality and efficiency. It suggests that for a very low level of aggregate wealth, more is the inequality in the distribution of tenant's wealth, higher is the total investment and more efficient the relationship. Indeed, as the wealth level of the poorest agent hired decreases, the market power of the other agents increases. Consequently, these other agents take more profit from a relationship and the contract terms are more efficient (i.e., the investment level is closer to the first-best.).

From a normative point of view, the analysis suggests that if the public authority has some money to distribute which could serve as collateral in tenancy relations, it may need to induce inequality among the tenants in order to increase both the efficiency of the contracts and the utility of (some of) the tenants. Suppose all the tenants have no initial wealth. If the public authority distributes to every tenant a small amount (less than $\overline{w}$ in Figure 1), then in the stable outcome, all the tenants will sign the principal-agent contracts investing a level $\overline{K}$ which is the same they would do with zero wealth. Hence, the efficiency of the relationship will remain the same as that prior to the distribution. Moreover, the gross utility of all the tenants hired will be the same as before. That is, the landowners will appropriate the additional amount distributed, which was intended to improve the welfare of the tenants. On the other hand, if the public authority distributes the money among a few tenants (a number smaller than the number of landowners), then the contracts signed by these tenants will be more efficient than before, and their gross utility will increase by more than the additional money they receive. Hence, targeting a small group rather than all the tenants improves the welfare of this group and overall efficiency.

7 Concluding Remarks

In this paper we model a principal-agent economy as a two-sided matching market and characterise completely the set of stable outcomes of this economy. As we have mentioned earlier, our model can be seen as a generalisation of the assignment game described by Shapley and Shubik (1972). Our findings can easily be applied to various examples of principal-agent economies.

We have already mentioned two of them in the previous section. The main task of this paper lies in suggesting a general (competitive) equilibrium model of a principal-agent economy. Using the restriction of limited liability should be taken as a very simple way to tackle incentive problems. This paper also consolidates stability as a reasonable solution concept. In this regard, we show that our results are not only the outcome of a cooperative game, but can be reached through very simple non-cooperative interactions between the principals and the agents.

Our paper leaves several avenues open to further research. First, we have assumed that the principals are identical. Although some of the conclusions of our analyses can immediately be extended to apply to economies with heterogenous principals, the characteristics of the contracts signed in the stable outcomes can be quite different from those identified in the current work. On the one hand, the results that the contracts signed in a stable outcome are optimal and the matching itself is efficient (in the sense that it maximises the total surplus) hold also in a framework with heterogenous principals. On the other, there is no unique way to model the differences among the principals and the contracts will be different depending on the type of heterogeneity one would like to introduce. Second, ours is a one-to-one matching model. This can be extended to a situation where one principal can hire several agents. Two scenarios might be considered. First, the random returns from each of the several projects a principal finances are identically and independently distributed. In this case, the contracts in a stable outcome are the same as in the current model. In a stable outcome, the payoffs to the principals will be equalised across projects. The total payoff to a principal will be the sum of payoffs from all the projects she finances. Hence, the principal owning higher number of projects will earn higher total payoffs. A more interesting situation would occur if the project returns are correlated. This kind bears similarity with the agency problem in a multi-agent situation. A natural way to analyse this would be to make use of a many-to-one matching model.

Appendix

A. The Principal-Agent Contracts

We solve for the optimal principal-agent contract for a pair (p, a):

$$\begin{cases} \underset{\{R_{p,a}, r_{p,a}, K_{p,a}\}}{\text{maximise}} & U_p = \pi_1(K_{p,a})R_{p,a} + (1-\pi_1(K_{p,a}))r_{p,a} - K_{p,a} \\ \text{subject to} & (PC_a) \quad \pi_1(K_{p,a})(y - R_{p,a} + r_{p,a}) - r_{p,a} \geq 1 \\ & (IC'_a) \quad [\pi_1(K_{p,a}) - \pi_0(K_{p,a})](y - R_{p,a} + r_{p,a}) \geq 1 \\ & (LS_a) \quad R_{p,a} \leq y + w \\ & (LF_a) \quad r_{p,a} \leq w. \end{cases} \tag{P1}$$

At the optimum, (IC'_a) binds, so we write the constraint with equality.[14] Using this, one can replace $R_{p,a}$ in the objective function and the other three constraints. Moreover, if (PC_a) and (LF_a) are satisfied, (LS_a) also holds. Hence, the above programme reduces to the following:

$$\begin{cases} \underset{\{r_{p,a}, K_{p,a}\}}{\text{maximise}} & \pi_1(K_{p,a})y - \frac{\pi_1(K_{p,a})}{\pi_1(K_{p,a})-\pi_0(K_{p,a})} + r_{p,a} - K_{p,a} \\ \text{subject to} & (PC'_a) \quad \frac{\pi_1(K_{p,a})}{\pi_1(K_{p,a})-\pi_0(K_{p,a})} - r_{p,a} - 1 \geq 0 \\ & (LF_a) \quad w - r_{p,a} \geq 0. \end{cases} \tag{P1$'$}$$

We denote μ_1 and μ_2 the Lagrange multipliers of (P1$'$). Then, the Kuhn-Tucker (first-order) conditions are given by:[15]

$$y\pi'_1 - 1 + (1-\mu_1)\frac{\pi'_1\pi_0 - \pi_1\pi'_0}{(\pi_1-\pi_0)^2} = 0. \tag{1}$$

$$1 - \mu_1 - \mu_2 = 0. \tag{2}$$

$$\mu_1\left(\frac{\pi_1(K_{p,a})}{\pi_1(K_{p,a})-\pi_0(K_{p,a})} - r_{p,a} - 1\right) = 0. \tag{3}$$

[14] To be more precise, (IC'_a) does not bind if w is very high, that is in the region where the limited liability constraints do not play any role and the first best contract is signed. This corresponds to Case 2 in the analysis that follows.

[15] The hypotheses on $\pi_1(K_{p,a})$ and y make sure the optimal $K_{p,a}$ must be interior and it satisfies the first-order conditions. The corner solution for $r_{p,a}$ is explicitly taken into account.

$$\begin{aligned}
\mu_2\,(w - r_{p,a}) &= 0. && (4)\\
\frac{\pi_1(K_{p,a})}{\pi_1(K_{p,a}) - \pi_0(K_{p,a})} - r_{p,a} - 1 &\geq 0. && (5)\\
w - r_{p,a} &\geq 0. && (6)\\
\mu_1,\, \mu_2 &\geq 0. && (7)
\end{aligned}$$

Now we study different regions where the Kuhn-Tucker conditions can be satisfied. For simplicity, we develop the analysis when $\pi_1'\pi_0 - \pi_1\pi_0' < 0$.

Case 1: $\mu_1 = \mu_2 = 0$ (Both the constraints are non binding)
From (2), we can see that this case is not possible.

Case 2: $\mu_1 > 0$, $\mu_2 = 0$ ((LF_a) is non-binding and (PC_a') is binding)
From (2), $\mu_1 = 1$. Then from (1), we have $y\pi_1'(K^0) = 1$, where K^0 is the first-best level of investment. Using (PC_a') and (LF_a), one has

$$w \geq \frac{\pi_1(K^0)}{\pi_1(K^0) - \pi_0(K^0)} - 1 \equiv w^0.$$

Hence, if $w \geq w^0$ a candidate for optimal solution exists involving $K_{p,a} = K^0$. In particular, an optimal payment vector is ($R_{p,a} = y + w - \frac{1+w}{\pi_1(K^0)}$, $r_{p,a} = w$).

Case 3: $\mu_1 = 0$, $\mu_2 > 0$ ((LF_a) is binding and (PC_a') is non-binding)
From (2), $\mu_2 = 1$. Then (1) implicitly defines the level of optimum investment $\overline{K}$,

$$y\pi_1'(\overline{K}) - 1 + \frac{\pi_1'(\overline{K})\pi_0(\overline{K}) - \pi_1(\overline{K})\pi_0'(\overline{K})}{(\pi_1(\overline{K}) - \pi_0(\overline{K}))^2} = 0.$$

From (LF_a), we also have $r_{p,a} = w$. Moreover, $R_{p,a}$ is determined by (IC_a') as $R_{p,a} = y + w - \frac{1}{\pi_1(\overline{K}) - \pi_0(\overline{K})}$. And from the non-binding (PC$'$) we have

$$w \leq \frac{\pi_1(\overline{K})}{\pi_1(\overline{K}) - \pi_0(\overline{K})} - 1 \equiv \overline{w}.$$

That is, the previous contract can only be a candidate if $w \leq \overline{w}$.

Case 4: $\mu_1 > 0$, $\mu_2 > 0$ (Both the constraints are binding)
From (LF_a), $r_{p,a} = w$. Then (PC_a') defines the optimal $K_{p,a}$ as an implicit

function of w. Denote this by $K(w)$, which must satisfy the following condition

$$\frac{\pi_1(K(w))}{\pi_1(K(w)) - \pi_0(K(w))} = w + 1. \tag{8}$$

Finally, $R_{p,a}$ is determined by (IC'_a). Previously found $R_{p,a}$, $r_{p,a}$ and $K(w)$ are indeed the candidates for optimum if the Lagrange multiplier, μ_1, implicitly defined by (1) lies in the interval [0, 1] (so that constraints (2) and (7) are satisfied). Given that $\pi'_1\pi_0 - \pi_1\pi'_0 < 0$, $\mu_1 < 1$ if and only if

$$y\pi'_1(K(w)) - 1 > 0 \Rightarrow K(w) < K^0.$$

Again using $\pi'_1\pi_0 - \pi_1\pi'_0 < 0$, $K(w) < K^0$ is optimal if

$$\frac{\pi_1(K^0)}{\pi_1(K^0) - \pi_0(K^0)} < w + 1 \Rightarrow w < w^0.$$

Similarly, $\mu_1 > 0$ if and only if

$$y\pi'_1(K(w)) - 1 + \frac{\pi'_1(K(w))\pi_0(K(w)) - \pi_1(K(w))\pi'_0(K(w))}{(\pi_1(K(w)) - \pi_0(K(w)))^2} < 0.$$

The above inequality implies $K(w) > \overline{K} \Rightarrow \frac{\pi_1(\overline{K})}{\pi_1(\overline{K}) - \pi_0(\overline{K})} < 1 + w \Rightarrow w > \overline{w}$. Hence, the optimal contract corresponds to the solution found in Case 3 when $w < \overline{w}$, is the candidate found in Case 4 when $\overline{w} < w < w^0$, and it is the first-best contract of Case 2 when $w^0 \leq w$.

B. Proof of Proposition 1

We are to show that if $w > w'$ in the region $w < w^0$, then $U_p(a, c^*_{p,a}) > U_p(a', c^*_{p,a'})$. From the previous section one can write the value function $v(w) = U_p(a, c^*_{p,a})$. Using the Envelope theorem, we get $v'(w) = \mu_2 > 0$ and hence the proposition.

C. Contracts in a Stable Outcome

Let us rewrite (P2):

$$\begin{cases} \underset{\{R_{p,a}, r_{p,a}, K_{p,a}\}}{\text{maximise}} & u_a = \pi_1(K_{p,a})(y - R_{p,a}) - (1 - \pi_1(K_{p,a}))r_{p,a} - 1 \\ \text{subject to} & (PC_p) \quad \pi_1(K_{p,a})R_{p,a} + (1 - \pi_1(K_{p,a}))r_{p,a} - K_{p,a} \geq \widehat{U} \\ & (IC'_a) \quad [\pi_1(K_{p,a}) - \pi_0(K_{p,a})](y - R_{p,a} + r_{p,a}) \geq 1 \\ & (LS_a) \quad R_{p,a} \leq y + w \\ & (LF_a) \quad r_{p,a} \leq w. \end{cases} \tag{P2}$$

As we have pointed out in the paper, this programme is individually rational for the agent only if $\widehat{U} \leq U_p(a, c^*_{p,a})$. Denote by $w^{min}(\widehat{U})$ the level of wealth such that $\widehat{U}$ is the utility of a principal that hires an agent with this wealth under a principal-agent contract. Programme $(P2)$ is only well defined for $w \geq w^{min}(\widehat{U})$. At the optimum, (PC_p) binds. Hence, one can substitute for $R_{p,a}$ in the objective function and the rest of the constraints. Also, if both (IC'_a) and (LF_a) hold, then (LS_a) becomes redundant. Then one has the above programme reduced as the following:

$$\begin{cases} \underset{\{r_{p,a}, K_{p,a}\}}{\text{maximise}} & \pi_1(K_{p,a})y - \widehat{U} - K_{p,a} - 1 \\ \text{subject to} & (IC''_a) \quad \pi_1(K_{p,a})y - \frac{\pi_1(K_{p,a})}{\pi_1(K_{p,a}) - \pi_0(K_{p,a})} + r_{p,a} - K_{p,a} - \widehat{U} \geq 0 \\ & (LF_a) \quad r_{p,a} \leq w. \end{cases} \tag{P2$'$}$$

Let ν_1 and ν_2 be the Lagrange multipliers for (IC''_a) and (LF_a), respectively. The Kuhn-Tucker (first-order) conditions of the above maximisation problem are given by:

$$y\pi'_1 - 1 + \nu_1 \left(y\pi'_1(K_{p,a}) - 1 + \frac{\pi'_1(K_{p,a})\pi_0(K_{p,a}) - \pi_1(K_{p,a})\pi'_0(K_{p,})}{(\pi_1(K_{p,a}) - \pi_0(K_{p,a}))^2} \right) = 0. \tag{9}$$

$$\begin{aligned}
\nu_1\left(\frac{\pi_1(K_{p,a})-\pi_0(K_{p,a})}{\pi_1(K_{p,a})}\right)-\nu_2 &= 0. && (10)\\
\nu_1\left((\pi_1(K_{p,a})-\pi_0(K_{p,a}))\left(y-\frac{\widehat{U}-r_{p,a}+K_{p,a}}{\pi_1(K_{p,a})}\right)-1\right) &= 0 && (11)\\
\nu_2(w-r_{p,a}) &= 0. && (12)\\
\left((\pi_1(K_{p,a})-\pi_0(K_{p,a}))\left(y-\frac{\widehat{U}-r_{p,a}+K_{p,a}}{\pi_1(K_{p,a})}\right)-1\right) &\geq 0. && (13)\\
w-r_{p,a} &\geq 0. && (14)\\
\nu_1,\ \nu_2 &\geq 0. && (15)
\end{aligned}$$

Now we study different regions for the Kuhn-Tucker conditions to be satisfied.

Case 1: $\nu_1 = 0$, $\nu_2 > 0$ ((LF_a) is binding and (IC''_a), non-binding)
Using (10), one can see that this case is not possible.

Case 2: $\nu_1 > 0$, $\nu_2 = 0$ ((LF_a) is non-binding and (IC''_a), binding)
From (10), it is clear that this case is not possible either.

Case 3: $\nu_1 = \nu_2 = 0$ (Both the constraints are non-binding)
From (9), $K_{p,a} = K^0$, the first best level of investment.The payment made to the principal in case of failure, $r_{p,a}$ is calculated from (PC_p). For example, $r_{p,a} = w$ and $R_{p,a} = \frac{\widehat{U}+K^0-(1-\pi_1(K^0))w}{\pi_1(K^0)}$ are optimal. From (IC''_a) and (LF_a), the above is only possible if

$$w \geq -\pi_1(K^0)y + K^0 + \widehat{U} + \frac{\pi_1(K^0)}{\pi_1(K^0)-\pi_0(K^0)} \equiv w(\widehat{U}).$$

Case 4: $\nu_1 > 0$, $\nu_2 > 0$ (Both the constraints are binding)
In this case, $r_{p,a} = w$ and optimal investment is a function of w, $\widehat{K}(w;\widehat{U})$, that is implicitly defined by the condition

$$-\pi_1(K(w;\widehat{U}))y + K(w;\widehat{U}) + \widehat{U} + \frac{\pi_1(K(w;\widehat{U}))}{\pi_1(K(w;\widehat{U}))-\pi_0(K(w;\widehat{U}))} = w. \quad (16)$$

Notice that, from (9), for $K(w;\widehat{U}) \leq K^0$, we have

$$y\pi_1'(K(w;\widehat{U}))-1+\frac{\pi_1'(K(w;\widehat{U}))\pi_0(K(w;\widehat{U}))-\pi_1(K(w;\widehat{U}))\pi_0'(K(w;\widehat{U}))}{(\pi_1(K(w;\widehat{U}))-\pi_0(K(w;\widehat{U})))^2} \geq 0.$$

From the above expression, this immediately implies that $\widehat{K}(.)$ is increasing in w. The previous values of $r_{p,a}$, $R_{p,a}$ and $K(w;\widehat{U})$ are optimal solutions to the above programme if the multipliers ν_1 and ν_2 defined in equations (9) and (10) satisfy (15), i.e., they are non-negative. Notice that (9) implies $\nu_2 > 0$ if and only if $\nu_1 > 0$. To check when $\nu_1 > 0$, notice that if $w > w(\widehat{U})$, then it is necessary that

$$\begin{aligned} w &= -\pi_1(\widehat{K}(w;\widehat{U}))y + \widehat{K}(w;\widehat{U}) + \widehat{U} + \frac{\pi_1(\widehat{K}(w;\widehat{U}))}{\pi_1(\widehat{K}(w;\widehat{U})) - \pi_0(\widehat{K}(w;\widehat{U}))} \\ &> -\pi_1(K^0)y + K^0 + \widehat{U} + \frac{\pi_1(K^0)}{\pi_1(K^0) - \ pi_0(K^0)} \equiv w(\widehat{U}). \end{aligned}$$

Now we can characterise the optimal contract as follows.

$$K_{p,a} = \begin{cases} \widehat{K}(w;\ \widehat{U}) & \text{if } w < w(\widehat{U}) \\ K^0 & \text{if } w \geq w(\widehat{U}). \end{cases}$$

$$R_{p,a} = \begin{cases} \frac{\widehat{U}+\widehat{K}(w;\widehat{U})-(1-\pi_1(\widehat{K}(w;\widehat{U})))w}{\pi_1(\widehat{K}(w;\widehat{U}))} & \text{if } w < w(\widehat{U}) \\ \frac{\widehat{U}+K^0-(1-\pi_1(K^0))w}{\pi_1(K^0)} & \text{if} w \geq w(\widehat{U}) \end{cases}$$

$$\text{and} \quad r_{p,a} = w$$

We also want to prove that for any level of $w \geq w^{min}(\widehat{U})$, $\widehat{K}(w;.) \geq K(w)$, where $\widehat{K}(w;.)$ is the optimal investment under moral hazard in the programme (P2) and $K(w)$ is the optimal investment in the principal-agent contract. First of all we know that, $\widehat{K}(w) > \overline{K}$. Comparing (9) and (17), it is clear that proving $\widehat{K}((w;\widehat{U})) \geq K(w)$ is equivalent to showing that $\pi_1(\widehat{K}(w;\widehat{U}))y - \widehat{K}(w;\widehat{U}) - \widehat{U} \geq 1$. Suppose that $w^{min}(\widehat{U}) \leq \overline{w}$. Then $\widehat{U}$ is given by

$$\widehat{U} = \pi_1(\overline{K})y - \frac{\pi_1(\overline{K})}{\pi_1(\overline{K}) - \pi_0(\overline{K})} + w^{min}(\widehat{U}) - \overline{K}.$$

Using the above together with (5), it is easy to see that $\pi_1(\widehat{K}(w;\widehat{U}))y - \widehat{K}(w;\widehat{U}) - \widehat{U} > 1$. This also proves that $w(\widehat{U}) \leq w^0$. We now do the same considering $w^{min}(\widehat{U}) > \overline{w}$. Notice that, in this case $\widehat{U} = \pi_1(K(w^{min}(\widehat{U})))y - K(w^{min}(\widehat{U}))$. Since investment is increasing in wealth, we must have that $[\pi_1(\widehat{K}(w;\widehat{U}))y - \widehat{K}(w;\widehat{U})] - [\pi_1(K(w^{min}(\widehat{U})))y - K(w^{min}(\widehat{U}))] > 0$. These previous two facts imply the above assertion that $\widehat{K}(w; \widehat{U}) \geq K(w)$ for all $w \geq w^{min}(\widehat{U})$.

D. The Case when $\pi_1(K)\pi'_0(K) < \pi'_1(K)\pi_0(K)$

In the paper we have analysed our model under the assumption that $\pi_1\pi'_0 > \pi'_1\pi_0$. We also asserted that, all the qualitative results of our model would hold good under the assumption that $\pi_1\pi'_0 < \pi'_1\pi_0$. Under this assumption, the findings in Appendix A imply $\overline{K} > K(w) > K^0$ and $K(w)$ is decreasing for $w \in (\overline{w},\ w^0)$. The reason behind this is the following. When $\pi_1(K)$ is increasing relative to $\pi_0(K)$, for a high level of initial investment, giving incentives is much easier. Because of this, for low level of wealth, the principal gives over incentives to the agent by lending more money (equivalently, the optimal investment is higher). Similarly, under this assumption, the findings of Appendix C imply that $\widehat{K}(w; \widehat{U}) > K^0$ for $w > w(\widehat{U})$.

E. Proof of Theorem 4

Consider $N > n$. First we prove that each SPE outcome is stable. We do that through several claims. **(a)** At any SPE, the contracts accepted are (among) the ones yielding the highest utility to the principals. Otherwise, a principal accepting a contract that yields lower utility would have incentives to switch to a better contract that has not already been taken. **(b)** At any SPE, all the contracts that are accepted provide the same utility to all the principals. Otherwise, on the contrary, consider one of the (at most $n-1$) contracts that gives the maximum utility to the principals. If one of the agents slightly decreases the payments offered at the first stage, his contract will still be accepted at any Nash equilibrium (NE) of the second-stage game for the new set of offers (because of **(a)**). **(c)** At any SPE, precisely n contracts are accepted. To see this, suppose on the contrary that at most $n-1$ contracts are accepted. Then there is a (unmatched) principal with zero utility. This is not possible since **(b)** holds. **(d)** The contracts that are finally accepted are those offered by the wealthiest agents. Suppose $w' > w$ and the contract offered by a is accepted, but not the one by a'. Then a' can offer a slightly better (for the principals) contract than s^a. Given **(a)**-**(c)**, this new contract will be accepted at any NE of the second-stage game. This is a contradiction. **(e)** All the contracts signed are optimal. Otherwise, an agent offering a non-optimal contract could improve it for both (any principal and himself). This new contract will certainly be among the n best contracts for the principals (since the previous contract was) and hence, will be accepted at any SPE outcome. **(f)** Finally, any SPE outcome is stable. It only remains to prove that the common utility level of the principals at an SPE, denoted by $\widehat{U}$, lies in $[U_p(a^{n+1}, c^*_{p,a^{n+1}}), U_p(a^n, c^*_{p,a^n})]$. First, $\widehat{U} \leq U_p(a^n, c^*_{p,a^n})$, because otherwise, some agents would be better-off by not offering any contract. Secondly, we must have $\widehat{U} \geq U_p(a^{n+1}, c^*_{p,a^{n+1}})$ such that the agent a^{n+1} (unmatched) does not to have incentives to propose a contract that would have been accepted by a principal.

We now prove that any stable outcome can be supported by an SPE strategy. Let $(\mu, \mathcal{C})$ be a stable allocation where each principal gets utility $\widehat{U}$. Consider the following strategies of each agent a for all j and of each principal p for all i:

$$\widehat{s}^a = \begin{cases} c_{\mu(a)} & \text{if } \mu(a) \in \mathcal{P} \\ \widehat{c} \quad \text{s.t.} \quad U_p(a, \widehat{c}) = \widehat{U} \ \ ; \text{ for any } \ p \in \mathcal{P}, & \text{otherwise.} \end{cases}$$

And $\widehat{s}_p = \mu(p)$ if $\widehat{s}$ is played in the first stage. Otherwise, principals select any agent compatible with an NE in pure strategies given any other message s sent in the first period. These strategies constitute an SPE yielding the stable outcome $(\mu, \mathcal{C})$. To see this, notice that given any message $s^a \neq \widehat{s}^a$, principals play their NE strategies. Given that $\widehat{s}$ is played in the first stage, by deviating any principal p she cannot gain more than $\widehat{U}$. This is true because any contract offered in the first stage yields the same utility $\widehat{U}$ to any principal. Now consider deviations by the agents. Given that $\widehat{U} \geq U_p(a^{n+1}, c^*_{p,a^{n+1}})$, by stability, there does not exist any contract that would be offered by an unmatched agent that guarantees him a positive utility while yielding at least $\widehat{U}$ to a principal. Hence, unmatched agents do not have incentives to deviate. Also, given the efficiency of the contracts in a stable allocation, there does not exist a different contract that a matched agent could offer at which he could have been strictly improved while still guaranteeing at least $\widehat{U}$ to the principals. If there is a plethora of contracts that yields utility $\widehat{U}$ to the principals, it is easy to check that there is no NE of the game at which a contract providing utility lower than $\widehat{U}$ is accepted by a principal. Hence, the matched agents do not also have any incentive to deviate from $\widehat{s}$.

The proof when $N \leq n$ is easier than before and follows similar arguments. To prove that each SPE yields stable outcomes where principals obtain zero profits, it is sufficient to check that the following three claims hold. (a) At any SPE, the contracts accepted are (among) the ones yielding the highest utility to the principals. (b) At any SPE, precisely N contracts are accepted and they provide zero utility to all the principals. (c) All the contracts signed are optimal.

To prove that the stable outcomes (the agents' optimal, if $N \leq n$) can be supported by an SPE strategy, let $(\mu, \mathcal{C})$ be a stable allocation where each principal gets utility $\widehat{U}$. Consider the following strategies of each agent a for all j and of each principal p for all i:

$$\widehat{s}^a = c^a(0) \text{ for any } a$$

And $\widehat{s}_p = \mu(p)$ if $\widehat{s}$ is played in the first stage. Otherwise, principals select any agent compatible with an NE in pure strategies given any other message s sent in the first period.

References

Ackerberg, D. A., and M. Botticini, (2002), "Endogenous Matching and the Empirical Detarminants of Contract Form," *Journal of Political Economy*, 110, 564–591.

Alcalde, J., D. Pérez-Castrillo, and A. Romero-Medina, (1998), "Hiring Procedures to Implement Stable Allocations," *Journal of Economic Theory*, 82, 469–480.

Barros, F., and I. Macho-Stadler, (1998), "Competition for Managers and Market Efficiency," *Journal of Economics and Management Strategy*, 7, 89–103.

Basu, K., (1992), "Limited Liability and the Existence of Share Tenancy," *Journal of Development Economics*, 38, 203–220.

Besley, T., and M. Ghatak, (2005), "Competition and Incentives with Motivated Agents," *American Economic Review*, 95, 616–636.

Dam, K., (2004), "A Two-Sided Matching Model of Monitored Finance," Mimeo, Center for Operations Research and Econometrics, Université Catholique de Louvain.

Gale, D., and L. Shapley, (1962), "College Admissions and the Stability of Marriage," *American Mathematical Monthly*, 69, 9–15.

Harris, M., and A. Raviv, (1978), "Some Results on Incentive Contracts with Applications to Education and Employment, Health Insurance, and Law Enforcement," *American Economic review*, 68, 20–30.

Hatfield, J. W., and P. Milgrom, (2005), "Matching with Contracts," *American Economic Review*, 95, 913–935.

Kara, T., and T. Sönmez, (1997), "Implementation of College Admission Rules," *Economic Theory*, 9, 197–218.

Legros, P., and A. F. Newman, (2003), "Beauty is a Beast, Frog is a Prince: Assortative Matching with Nontransferabilities," Institute of Advanced Study Working Paper No. 30.

Mirlees, J. A., (1976), "The Optimal Structure of Incentives and Authority within an Organisation," *Bell Journal of Economics*, 7, 105–131.

Mookherjee, D., and D. Ray, (2002), "Contractual Structure and Wealth Accumulation," *American Economic Review*, 92, 818–849.

Pauly, M. V., (1968), "The Economics of Moral Hazard," *American Economic Review*, 58, 531–537.

Roth, A. E., and M. Sotomayor, (1990), *Two-sided Matching: A Study in Game-Theoretic Modeling and Analysis.*, Cambridge University Press, New York and Melbourne, Econometric Society Monographs, no. 18.

Sengupta, K., (1997), "Limited Liability, Moral Hazard and Share Tenancy," *Journal of Development Economics*, 52, 393–407.

Serfes, K., (2001), "Risk Sharing versus Incentives: Contract Design under Two-Sided Heterogeneity," Mimeo, SUNY Stoneybrook.

Shapley, L., and M. Shubik, (1972), "The Assignment Game I: The Core," *International Journal of Game Theory*, 1, 111–130.

Shetty, S., (1988), "Limited Liability, Wealth Differences and Tenancy Contract in Agrarian Economies," *Journal of Development Economics*, 29, 1–22.