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A Two-sided Matching Model of Monitored Finance

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We analyse an incentive contracting model of partnership formation between heterogeneous investors and entrepreneurs. Partnerships are subject to double-sided moral hazard problems in entrepreneurial action and monitoring by investors. Greater monitoring ability implies stronger incentives to monitor. On the other hand, low-collateral borrowers have lower inside equity participation. Hence the incentive problem is best mitigated by assigning low-collateralized entrepreneurs to high-ability investors following a negative assortative matching pattern. Moreover, negative assortative matching implies that the equilibrium loan rate is in general non-monotonic in borrower collateral. Finally, our model sheds light on how changes in the inequality of collateral distribution affect the cost of external borrowing.

INTRODUCTION

It is well-known that borrower collateral plays an important role in ameliorating incentive problems in a borrower–lender relationship. Notwithstanding this, there is a lack of consensus on the role of collateral in determining the essential characteristics of a loan contract such as the loan rate. *Ex post* theories of collateral assert that observably riskier borrowers, who pay a higher loan rate, are often required to pledge higher collateral to reduce agency costs (hidden action), hence there is a positive association between the loan rate and collateral (e.g. Boot and Thakor 1994). *Ex ante* theories of collateral, by contrast, postulate that when borrower quality is unobservable (hidden information), safer borrowers tend to pledge higher collateral to signal quality, hence there is a negative association (e.g. Besanko and Thakor 1987). Empirical findings endorse both views.[1] Berger *et al.* (2016) find empirical evidence of a non-monotonic relation between loan rate and collateral.

Given the aforementioned dissent on the theoretical predictions regarding the optimal association between loan rate and collateral, our paper aims to provide a unified framework that is amenable to rationalizing a possible non-monotonicity of loan rate with respect to borrower collateral. For that purpose, we analyse a competitive credit market in which entrepreneurs form partnerships with investors. Risk-neutral entrepreneurs are heterogeneous with respect to initial wealth that can be fully pledged as collateral. Borrower wealth is not sufficient to cover the fixed project cost, hence all entrepreneurs must rely on external borrowing. A loan contract, which specifies the loan rate to be paid to the lender, is subject to limited liability. Borrower moral hazard stems from the fact that an entrepreneur, in the absence of any monitoring, may deliberately decrease the probability of obtaining a high cash flow for his project in order to consume private benefits (as in Hölmstrom and Tirole 1997). Lenders can potentially mitigate the borrower moral hazard problem by costly monitoring. Risk-neutral investors are heterogeneous with respect to their monitoring ability. Lenders with a greater monitoring ability are the ones who entail a lower marginal cost of monitoring, and hence are more efficient. However, an investor cannot credibly commit to a predetermined level of monitoring, which gives rise to a lender moral hazard problem. Therefore endogenous partnership formation is subject to a double-sided moral hazard problem that hinders the implementation of efficient outcomes.

Differences in monitoring ability imply differences in the lender moral hazard problem as more efficient investors have stronger incentives to monitor. Because monitoring enhances firm value, competition for more efficient lenders naturally emerges in such a market. To capture this idea, we model the lender–borrower market as a two-sided assignment game (e.g. Shapley and Shubik 1971). Investors with greater monitoring ability have comparative advantage in lending to low-collateral firms. Thus, to maximize efficiency in each partnership, it is optimal to assign low-collateral firms to high-ability investors following a *negative assortative matching* (NAM) pattern. In other words, monitoring ability and collateral are substitutes in mitigating the associated double-sided moral hazard problem.

One main result of our model is a potential non-monotonic association between loan rate and collateral. The intuition is as follows. The optimal loan rate associated with an isolated lender–borrower partnership, which serves as the instrument that balances double-sided incentive problems, is in general a function of monitoring ability (lender type), collateral (borrower type), and the exogenous outside option of the borrower. An investor with lower monitoring ability (i.e. higher marginal cost of monitoring) has weaker incentives to monitor, and hence requires higher marginal compensation (in the form of loan rates) to exert an additional unit of monitoring effort. By contrast, a borrower with high collateral requires less intense monitoring, hence the monitor must retain a smaller portion of the realized cash flow. Finally, if the exogenous outside option of an entrepreneur increases, then he must pay a lower loan rate due to the increased bargaining power. To understand why under endogenous sorting the equilibrium loan rate may be non-monotonic in borrower collateral, consider two investor–entrepreneur partnerships with two distinct levels of collateral and monitoring ability. First, note that in an assignment model, each borrower's outside option is endogenous, and is increasing in borrower type. In equilibrium, the borrower with high collateral is matched with a less efficient monitor following NAM. Thus this entrepreneur must pay a lower loan rate than the other borrower because both his collateral and his outside option are higher. However, NAM implies that the matched partner of this high-collateral entrepreneur must receive a higher loan rate because she has weaker incentives to monitor. Because of these countervailing effects, the equilibrium loan rate may be non-monotonic in collateral.

As is typical with assignment models, the nature of equilibrium matching (NAM in this particular context) between investors and entrepreneurs is independent of the type distributions. However, the shapes of the equilibrium matching and payoff functions may change following a change in the type distributions. Terviö (2008) shows how changes in type distributions induce the equilibrium matching function to change, and has positive spillover effects on the upper tail of the type distributions in terms of equilibrium payoffs. In our model, changes in type distributions have important comparative statics implications for the equilibrium loan rate as a function of collateral. If, at a given level of collateral, the number of borrowers increases relative to the number of lenders with the corresponding monitoring ability, then these lenders gain higher bargaining power because the borrowers are now relatively abundant. As a result, the same type of borrowers must pay a higher loan rate. By exploiting this simple intuition, we carry out a numerical exercise to show the effect of a more unequal distribution of collateral on the equilibrium matching function and the relative bargaining power of the borrowers. In particular, we consider two distributions of collateral based on yearly data of Italian firms, where one distribution is more unequal than the other. Such a cross-sectional variation in the collateral distribution affects the equilibrium loan rate and monitoring functions via the shifts in the matching function and bargaining power of the borrowers. The resultant shifts in the matching and borrower utility functions may not point in the same direction, and consequently, following a change in the collateral distribution, some borrowers pay higher loan rates, whereas others end up paying lower rates. The effect of such cross-sectional change in the distribution of collateral is also asymmetric with respect to the equilibrium monitoring intensity.

Our assumption about the differences in monitoring abilities across lenders is motivated by the works of Stein (2002) and Berger *et al*. (2005). Stein (2002) studies how different organizational structures can generate and process information on investment projects. In the context of banking, the Stein (2002) framework postulates that large banks have a disadvantage—relative to small banks—in collecting ‘soft’ information that cannot be credibly transmitted. The prediction that small banks have a comparative advantage over large ones in processing soft information is confirmed empirically by Berger *et al*. (2005). Thus the differences in monitoring ability in our model may be interpreted in terms of the differences across lenders in processing soft information.

The contribution of the present paper to the literature on incentive contracting and partnership formation is twofold. First, when individuals seek alternative partners, the model helps to endogenize the outside option of each borrower as opposed to models in which the outside options of individuals on one side of the market are exogenously given (e.g. Besanko and Kanatas 1993; Hölmstrom and Tirole 1997; Repullo and Suárez 2000). The aforementioned models are amenable to determining the optimal incentive structure in an organization in the sense that they determine the way a fixed surplus must be divided between the lender and borrower in a given lending relationship. A fixed outside option of a borrower also pins down the payoff achievable by his matched partner. In an assignment model such as ours, the endogenous outside option determines not only the structure of incentive pay, but also its level in each partnership, hence the bargaining power of each individual is endogenous. In Section IV, we show how changes in type distributions alter the endogenous bargaining power of the market participants. Second, we contribute to the literature on partnership formation (e.g. Farrell and Sctochmer 1988), which argues that economic agents who differ in abilities will form partnerships by equally sharing the surplus if abilities are complementary. In the context of corporate lending, formation of partnerships is often subject to several market imperfections, among which informational constraints play an important role. When partnerships are subject to moral hazard, incentive contract for a particular match gives rise to a non-linear Pareto frontier, implying that match surplus cannot be transferred between the principal and agent on a one-to-one basis, and an equal sharing of surplus cannot be implemented. Thus substitutability rather than complementarity explains why heterogeneous partnerships are formed, and individuals share the match output according to endogenously determined sharing rules.

I. Related Literature

Legros and Newman (2007) extend the assignment game (e.g. Shapley and Shubik 1971) to an environment with imperfect transferability where the Pareto frontier associated with each match is non-linear. They propose the *generalized decreasing differences* (GDD) condition, which is a necessary and sufficient condition for NAM in markets with two-sided heterogeneity. Our model contributes to this strand of literature by showing that the double-sided moral hazard problem is a way to induce imperfect transferability in an assignment game. Moreover, we show that GDD holds in our model, and hence the equilibrium allocation exhibits NAM. The present model resembles that of Chakraborty and Citanna (2005), which also analyses partnerships under double-sided moral hazard. Because wealthier individuals are less wealth-constrained, they accept occupations with more severe incentive problems. Partnerships are assortative, and an increase in median wealth improves the welfare of poorer agents.

In the context of (corporate) finance, two classes of paper analyse the effect of endogenous matching on incentive contracts. The first type studies the effects of endogenous investor–entrepreneur matching on optimal financial contracts.[2] Besley *et al.* (2012) analyse the effect of competition between lenders that are heterogeneous with respect to the cost of capital, and consider variations in property rights on optimal loan contracts. If competition is sufficiently intense (more similar lenders), then the borrowers receive their outside option. Improved property rights, which allow the borrower to pledge a larger proportion of wealth as collateral, relax the borrower incentive problem and reduce the loan rate. However, Besley *et al.* (2012) consider matching markets with one-sided heterogeneity, and hence do not take into account the effect of assortative matching.[3] Thus any changes in market fundamentals affect equilibrium contracts only through the endogenous outside option. By contrast, we show that changes in type distributions may have asymmetric effects on the equilibrium allocations. One paper that considers the effects of lender–borrower sorting is that of Cabolis *et al.* (2015), who analyse a venture capital (VC) market. They show positive assortative matching (PAM) between VC rank (given by their stage-specific expertise) and firm quality (proxied by the probability of successful exit) at each stage. Moreover, those authors establish a non-monotonic relationship between specialization and competition.

II. The Model

Lender–borrower matching

The economy, which spans three dates $t = 0,1,2$, consists of two classes of agents—a continuum $I = [0,1]$ of risk-neutral investors (lenders) and a continuum $J = [0,1]$ of risk-neutral entrepreneurs (borrowers). Entrepreneurs are heterogeneous with respect to their initial wealth. In particular, a type w entrepreneur has initial wealth that can be pledged entirely as collateral whose market value is $w \in W = [w_{\min}, w_{\max}]$, with $w_{\min} > 0$. Borrower types are publicly observable. Let $F(w)$ denote the fraction of borrowers with collateral lower than or equal to w. In other words, $F(w)$ is the cumulative distribution function of collateral, and let $f(w)$ be the corresponding density function with $f(w) > 0$ for all $w \in W$. Further, each entrepreneur has a startup project whose initial outlay is \$1. A dollar invested in the project at date 1 yields a stochastic but verifiable cash flow $Q > 1$ (success) or 0 (failure) at $t = 2$.

A borrower can choose between two non-verifiable actions—'behave' (H) and 'shirk' or 'misbehave' (L), which determine the probability of success of the project. Moreover, if a borrower misbehaves, then she enjoys a private benefit $B > 0$, whereas the projects yield no private benefit if a borrower chooses H. Thus B represents the severity of borrower moral hazard problem. The structure of cash flow and private benefit is described in Table 1.

TABLE 1
CASH FLOW AND PRIVATE BENEFIT UNDER TWO ACTIONS OF A BORROWER

	Behave (H)	Shirk (L)
Probability of success	$p_H = p \in (0,1)$	$p_L = 0$
Private benefit	$B_H = 0$	$B_L = B > 0$

We assume $pQ > B$ so that the project is economically viable at least at the first-best level. By monitoring at an intensity $m \in [0,1]$, a lender can oblige an entrepreneur to behave with probability m, for which he has to incur a cost that takes the following functional form:[4]

$$D(m; c) = \frac{m^2}{2c}.$$

The parameter c represents the 'ability' or 'efficiency' of a lender or the lender 'type'. The higher c, the greater the ability, as a lender with higher c entails a lower cost for each additional unit of monitoring.[5] No lender can pre-commit to such actions, hence costly monitoring gives rise to a *lender moral hazard* problem. We assume that lenders are heterogeneous with respect to monitoring ability. Let $G(c)$ be the cumulative distribution function of monitoring ability, and let $g(c)$ be the corresponding density function, with $g(c) > 0$ for all $c \in C = [c_{\min}, c_{\max}]$, and $c_{\min} > 0$. Lender types are also publicly observable, and the type distributions are taken as the primitives of our economy under consideration. Let $\xi = (G,F)$ denote a generic lender–borrower economy or market with two-sided heterogeneity.

On date 0, if a lender agrees to finance an entrepreneur's project, then a lender–borrower partnership or match forms. As types, not individual names, matter, a typical partnership will be denoted by (c,w). We treat partnership formation as an endogenous matching problem in which a lender with a given ability c is assigned to a borrower with a given level of collateral w. To this end, we extend the Sattinger (1979) 'differential rents' model to an environment in which utility is not perfectly transferable. Formally, each partnership (c,w) forms via a one-to-one matching rule $\lambda: W \to C$, which assigns to each collateral level $w \in W$ a lender ability $\lambda(w) \in C$. One of our main objectives is to determine the equilibrium matching pattern, that is, which types of lenders and borrowers will form partnerships. The following definition describes a negative assortative matching (NAM) pattern.

Definition 1 (Negative assortative matching) Lender–borrower matching is negatively assortative if, given any two levels of collateral, w' and w'' with $w'' > w'$, we have $\lambda(w'') \leq \lambda(w')$, that is, a lender with higher ability forms a partnership with an entrepreneur with lower collateral.

Each partnership (c,w) writes a binding *loan contract* that specifies state-contingent transfers $r(0)$ and $r(Q)$ to the investor at $t = 1$. We assume *limited liability* such that in the event of failure, no agent is paid, that is, $r(0) = 0$. Let $R = r(Q)$ denote the 'loan rate'.

The timing of events

At date 0, investors and entrepreneurs form partnerships via a one-to-one matching rule. At $t = 1$, each lender makes a take-it-or-leave-it contract offer to a borrower that specifies the loan rate, and decides how much monitoring effort to exert. Finally, at $t = 2$, the true cash flow is realized and the agreed payments are made. We solve the model by backward induction.

Equilibrium

An allocation (λ,v,u) for the lender–borrower market ξ consists of matches (c,w) formed through feasible contracts, and payoff allocations (v,u) such that $v : C \longrightarrow \mathbb{R}_+$ and $u : W \longrightarrow \mathbb{R}_+$ are the utilities of the lenders and the borrowers, respectively. Further, let $\phi(c,w,u)$ be the maximum utility achievable by any type c investor in a given partnership (c,w) when the entrepreneur consumes u. In other words, $\phi(c,w,u)$, which will be the object of our interest, represents the *Pareto frontier* associated with a given partnership (c,w).

Definition 2 (Equilibrium allocation) An allocation (λ,v,u) is an equilibrium allocation for the investor–entrepreneur economy ξ if the following conditions are satisfied:

(a) *Feasibility*: Given a matching rule λ, for all $w \in W$ the payoff vector $(v(\lambda(w)),u(w))$ must be feasible for the pair $(\lambda(w),w)$, that is, $u_0 \le u(w) \le u_{\max}(\lambda(w),w)$ and $v(\lambda(w)) \le \phi(\lambda(w),w,u(w))$. The constant $u_0 \ge 0$ represents the reservation utility of each entrepreneur, that is, the utility obtained by any entrepreneur if his project is not financed, and $u_{\max}(\lambda(w),w)$ solves $\phi(\lambda(w),w,u) = 0$.
(b) *Stability*: Given the payoff vectors (v,u), there do not exist pairs (c,w) and $\hat{u} > u(w)$ such that $v(c) < \phi(c, w, \hat{u})$.
(c) *Measure consistency*: For any subinterval $[i_0,i_1] \subseteq I$, let $i_k = G(c_k)$ for $k = 0,1$, that is, c_k is the ability of the investor at the i_kth quantile. Similarly, for any subinterval $[j_0,j_1] \subseteq J$, let $j_h = F(w_h)$ for $h = 0,1$. If $[c_0,c_1] = \lambda([w_0,w_1])$, then it must be the case that $G(c_1)-G(c_0) = F(w_1)-F(w_0)$.

Feasibility requires that no utility allocation can be outside the Pareto set of a matched pair of investor and entrepreneur. Part (b) of Definition 2 asserts that if any investor–entrepreneur pair can make an alternative contractual arrangement that would make both of them strictly better off, then they would 'block' the current allocation. Thus in an equilibrium allocation, such contracts cannot exist. It is immediate to see that the first two parts of Definition 2 are equivalent to the fact that, given any feasible $u(w)$, each type c lender solves the maximization problem

$$(\mathcal{P}) \qquad v(c) = \max_{w} \phi(c, w, u(w)).$$

Note also that the notion of outside option differs from that of reservation utility, which is an exogenous object. Part (c) of Definition 2, the measure consistency requirement, is the standard 'demand–supply equality' condition of a continuum economy. Note that the Lebesgue measure of a subinterval $[i_0,i_1]$ of investors is $i_1-i_0 = G(c_1)-G(c_0)$, and that of a subinterval $[j_0,j_1]$ of entrepreneurs is $j_1-j_0 = F(w_1)-F(w_0)$. Thus measure consistency requires that if $[j_0,j_1]$ is matched to $[i_0,i_1]$, then these two subintervals cannot have different measures.

III. Equilibrium Sorting and Loan Contracts

We proceed as follows. We first analyse the optimal loan contract for an arbitrary match (c,w). In the following subsection, we present an illustrative example with two heterogeneous lenders and two heterogeneous borrowers. Next, we derive that the equilibrium matching is negatively assortative. Finally, we analyse the behaviour of the equilibrium loan rate as a function of borrower collateral.

Optimal loan contract and the Pareto frontier of an arbitrary match

Any optimal contract for an arbitrary match (c,w) will depend on lender and borrower types, but to save on notation, we suppress the argument (c,w) from the contract terms. An optimal loan contract solves the following maximization problem:

$$(\mathcal{M}) \qquad \max_{\{R,m\}} V(R,m) \equiv mpR + (1-mp)w - \frac{m^2}{2c} - 1,$$

subject to

$$(\text{PCB}) \qquad U(R,m) \equiv mp(Q+w-R) + (1-m)B \geq u,$$

$$(\text{ICL}) \qquad m = \underset{\hat{m}}{\operatorname{argmax}} \left\{ \hat{m}pR + (1-\hat{m}p)w - \frac{\hat{m}^2}{2c} - 1 \right\} = cp(R-w),$$

$$(\text{LL}) \qquad 0 \leq R \leq Q + w.$$

The expected payoffs of the lender and the borrower are respectively denoted by $V(R,m)$ and $U(R,m)$. Under limited liability, the lender collects the collateral w of the entrepreneur in the event of failure, which occurs with probability $1-mp$, whereas he receives the loan obligation R with probability mp in the event of success. As far as the borrower is concerned, her net income in the event of success is $Q+w-R$, as w remains with her; however, in the event of failure she receives nothing, as w is paid to the investor. Moreover, if monitoring is not successful in detecting borrower misbehaviour, then she enjoys the private benefit B with probability $1-m$. Constraint (PCB) is the borrower's participation constraint, where $u \geq 0$ is his outside option. Constraint (ICL) is the lender's incentive compatibility constraint, and (LL) represents two-sided limited liability in the event of success, that is, neither the borrower nor the lender has a negative income. We make the following assumptions.

Assumption 1 $u < B$.

Assumption 2 $\frac{1}{pQ}\left(1-\frac{u}{B}\right) \leq c \leq \frac{1}{pQ-u}$.

Assumption 1 ensures that the borrower moral hazard problem has a bite in a lending relationship, hence monitoring has a non-trivial role. The first inequality in Assumption 2 guarantees that the optimal monitoring intensity is less than 1, while the second inequality implies that constraint (LL) holds. The following lemma characterizes the optimal contract for an arbitrary partnership (c,w).

Lemma 1 In an arbitrary partnership (c,w), the participation constraint of the borrower binds. Let $m(c,w,u)$ and $R(c,w,u)$ denote respectively the optimal monitoring intensity and loan rate.

(a) The optimal monitoring intensity is given by

$$m(c,w,u) = \frac{1}{2}\left\{c(pQ-B) + \sqrt{c^2(pQ-B)^2 + 4c(B-u)}\right\}. \tag{1}$$

At the optimal contract, there is always overmonitoring, that is, $m(c,w,u) > m^{FB}$, where $m^{FB} = c(pQ-B)$ is the first-best level of monitoring. Moreover, the optimal monitoring is monotonically increasing in investor ability c, and monotonically decreasing in entrepreneur's outside option u, but is constant with respect to borrower collateral w.

(b) The optimal loan rate is given by

$$R(c,w,u) = w + \frac{1}{2cp}\left\{c(pQ-B) + \sqrt{c^2(pQ-B)^2 + 4c(B-u)}\right\}, \tag{2}$$

which is monotonically increasing in borrower collateral w, and monotonically decreasing in investor ability c and entrepreneur's outside option u.

It is natural in such an optimal contracting problem that binding limited liability at the event of failure coupled with the non-verifiability of monitoring effort prevents the implementation of the first-best outcome. To understand why the optimal contract implies overmonitoring, consider the following. Note from the binding (PCB) that $B > u$ is equivalent to $B > p(Q+w-R)$. The right-hand side of this inequality is the expected income of the borrower if she behaves, whereas the left-hand side is the benefit from shirking. Thus if $B \leq u$, then the borrower has no incentive to misbehave, hence the lender chooses not to monitor. Thus a strictly positive monitoring intensity $m < m^{FB}$ is not optimal.

First, consider the effect of an increase in c. For a given loan rate, a higher c implies a lower marginal cost of effort, hence an increased monitoring intensity. On the other hand, (ICL) dictates that the lender's expected marginal income $p(R-w)$ must be equal to the marginal cost of monitoring m/c. Because the marginal cost is monotonically decreasing in c, other things being equal, she requires a lower marginal compensation R, hence the optimal loan rate decreases with c.

Next, consider an increase in w, which implies that the investor faces a trade-off between incentive provision and rent extraction. First, the lender can increase R by the same amount so as to keep incentives constant. Moreover, at this increase, the participation constraint of the borrower remains binding, that is, no additional rent is left to him. By contrast, the lender may keep R fixed, which would cause a decrease in incentives, and hence a decrease in m. But in order to have (PCB) binding, m must increase following an increase in w.[6] Thus the first option is the only feasible one, and hence, following an increase in w, R increases proportionally, and m remains constant as there are no changes in incentives.

Finally, consider the effect of an increase in the entrepreneur's outside option u. When u increases, the entrepreneur gains greater bargaining power, hence the investor is forced to pay him more. Because the participation constraint binds, this extra payment cannot be given in the form of additional rent; rather, it must be given by weakening the incentives to monitor, that is, lowering R. Further, a decrease in R implies a lower monitoring at the optimum.

In the following lemma we state a useful property of the Pareto frontier. (Proofs of all results are in the Appendix.)

Lemma 2 The Pareto frontier of an arbitrary match (c,w), which is given by

$$\text{(PF)} \qquad \phi(c,w,u) = \frac{1}{8c}\left\{c(pQ-B) + \sqrt{c^2(pQ-B)^2 + 4c(B-u)}\right\}^2 + w - 1,$$

satisfies

(a) $\phi_1(c,w,u) > 0$, $\phi_2(c,w,u) > 0$ and $\phi_3(c,w,u) < 0$;
(b) the single-crossing condition

$$\text{(SC)} \qquad \frac{\partial}{\partial c}\left[-\frac{\phi_2(c,w,u)}{\phi_3(c,w,u)}\right] < 0.$$

Both investor ability and collateral contribute favourably to the surplus of an arbitrary match, as higher c implies lower marginal cost of monitoring, and higher w implies more rent for the investor in the event of failure. As a result, the Pareto frontier expands following an increase in either c or w. Clearly, the frontier is downward sloping because higher u implies lower utility left for the lender to consume.

The single-crossing condition is depicted in Figure 1. The slope of the indifference curve of any investor in the (w,u) space is given by $-\phi_2/\phi_3$. Condition (SC) asserts that the slope of the indifference curve of any lender is decreasing in lender type, hence the indifference curves of any two investors with different types can cross only once.

Equilibrium with two investors and two entrepreneurs: an illustrative example

Prior to characterizing the equilibrium allocation under a continuum of types, we consider a simpler version of our model with two lenders and two borrowers to explain the intuition behind our main results (Propositions 1 and 2 below). The monitoring abilities of the two investors are c' and c'', with $c'' > c'$, and two entrepreneurs have collateral w' and w'', with $w'' > w'$. The market equilibrium (Definition 2) is worked out under the parameter values $p = 0.4$, $Q = 5$, $B = 1.5$ and $u_0 = 0.3$. As the benchmark economy, we take $c' = 0.45$, $c'' = 0.53$, $w' = 0.4$ and $w'' = 0.7$. The parameter values satisfy Assumptions 1 and 2. We first establish the following claim.

Claim 1 In the equilibrium allocation of the lender–borrower market with $c' = 0.45$, $c'' = 0.53$, $w' = 0.4$ and $w'' = 0.7$, the utilities are given by $u(w') = 0.3$, $u(w'') \in [0.807, 0.812]$, $v(c') \in [0.214, 0.217]$ and $v(c'') = 0.235$, and the equilibrium exhibits NAM, that is, $c' = \lambda(w'')$ and $c'' = \lambda(w')$.

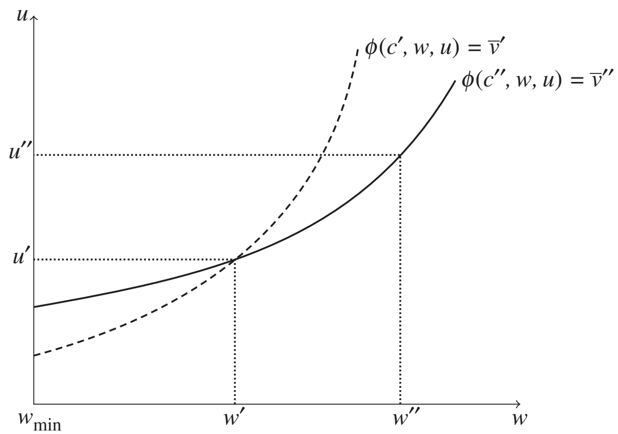

FIGURE 1. Single-crossing condition: the indifference curve of lender type c'' is flatter everywhere than that of c', where $c'' > c'$.

We first show that in any equilibrium allocation, $u(w') = u_0$=0.3. Consider any lender c who is matched with w' and $u(w') > u_0$. Because $\phi(c,w,u)$ is strictly decreasing in u (cf. Lemma 2(a)), no lender would be willing to pay him strictly higher than u_0 as she would consume strictly less than $\phi(c,w',u_0)$. Next, we show that $u(w'') = u_0$ cannot be part of an equilibrium allocation. Suppose that this were the case. Because $\phi(c,w,u)$ is strictly increasing in w (cf. Lemma 2(a)), we have $\phi(\lambda(w'),w'',u_0) > \phi(\lambda(w'),w',u_0)$, hence lender $\lambda(w')$—i.e. the one matched with w'—can offer $u_0+\varepsilon$, where $\varepsilon > 0$ is very small. At this arrangement, w'' would get $u_0+\varepsilon > u_0$, and $\lambda(w')$ would obtain $\phi(\lambda(w'),w'',u_0+\varepsilon) > \phi(\lambda(w'),w',u_0)$. This contradicts the notion of stability in Definition 2(b). Thus in any equilibrium allocation, we have $u(w'') > u_0$.

Next, we show that under the aforementioned utility allocation, the equilibrium matching is NAM. Note that in this benchmark economy, there are only two possible matchings—namely, a NAM, that is, $c' = \lambda(w'')$ and $c'' = \lambda(w')$, and a PAM, that is, $c' = \lambda(w')$ and $c'' = \lambda(w'')$. It is easy to check that NAM yields a higher aggregate surplus than PAM for any $u(w'') > 0.3$, that is,

$$\begin{aligned} & \phi(c', w'', u(w'')) + \phi(c'', w', u_0) > \phi(c', w', u_0) + \phi(c'', w'', u(w'')) \\ \iff & \phi(c', w'', u(w'')) + 0.235 > 0.214 + \phi(c'', w'', u(w'')), \end{aligned}$$

and hence, at $u(w') = 0.3$ and $u(w'') > 0.3$, PAM cannot be part of an equilibrium allocation. The values of $\phi(c,w,u)$ in the above inequalities are computed from the expression (PF). Now, the maximization problem ($\mathcal{P}$) implies that

$$\begin{aligned} \phi(c', w'', u(w'')) \geq \phi(c', w', u_0) = 0.214 \quad &\Longrightarrow \quad u(w'') \leq 0.812, \\ \phi(c'', w', u_0) = 0.235 \geq \phi(c'', w'', u(w'')) \quad &\Longrightarrow \quad u(w'') \geq 0.807. \end{aligned}$$

Given that the equilibrium matching is NAM, we have $v(c'') = \phi(c'',w',u_0) = 0.235$. On the other hand, $\phi(c',w'',0.807) = 0.217$ and $\phi(c',w'',0.812) = 0.214$ imply that $0.214 \leq v(c') \leq 0.217$.

Recent empirical literature on principal–agent matching (e.g. Ackerberg and Botticini 2002) claims that optimal incentive contracts under endogenous matching can be very different from those predicted by the standard agency theory that treats a principal–agent partnership in isolation. To understand this point, consider our example of the market with two lenders and two borrowers. Standard agency models would predict, as Lemma 1(b) does, that a higher loan rate must be associated with lower collateral, that is, the loan rate associated with the match (c'',w') must be higher. But this is purely a partial equilibrium phenomenon where the difference in loan rate is implied only by the difference in collateral values. On the other hand, Lemma 1(b) also predicts that the loan rate for (c'',w') should be lower because this match involves greater monitoring efficiency. Thus the (market) equilibrium loan rate for (c'',w') may be higher or lower than that associated with (c',w'') depending on which of the two aforementioned countervailing forces is stronger. Therefore the outcome of an assignment model offers predictions about the equilibrium loan rate with respect to borrower collateral that may be exactly the opposite of what would have been predicted by standard agency theory.

Note first that in the benchmark economy, $R(c'',w',u_0) > R(c',w'',u(w''))$ occurs for all $u(w'') \in (0.639, 1.528]$.[7] Because $u(w'') \in [0.807, 0.812] \subset (0.639, 1.528]$ in the equilibrium allocation, the high-type entrepreneur w'' pays a lower loan rate. This is in line with the prediction of a partial equilibrium contracting model that a high-type borrower should pay a lower loan rate than his low-type counterpart in order to finance his project. But this may always not be the case. In what follows, we construct an example where the equilibrium loan rate associated with the match (c',w'') is higher than that for (c'',w'), that is, the equilibrium loan rate is increasing in collateral value. We keep all the parameter values the same as in the benchmark economy except that we take $c' = 0.4$ and $c'' = 0.58$. Following the same procedure as above, it is easy to establish that $u(w') = 0.3$, $u(w'') \in [0.803, 0.816]$, $v(c') \in [0.2, 0.208]$ and $v(c'') = 0.248$, and that the equilibrium matching is NAM. As far as the equilibrium loan rates associated with the two matches are concerned, note that $R(c'',w',u_0) < R(c',w'',u(w''))$ if and only if $u(w'') \in [0.803, 0.807)$. Therefore we make the following claim.

Claim 2 In the equilibrium allocation of the lender–borrower market with $c' = 0.45$, $c'' = 0.53$, $w' = 0.4$ and $w'' = 0.7$, the high-type borrower pays a lower loan rate. By contrast, in the market with $c' = 0.4$, $c'' = 0.58$, $w' = 0.4$ and $w'' = 0.7$, the high-type borrower may end up paying a higher loan rate.

The intuition behind Claim 2 is best understood in terms of Figure 2. First, consider the match (c'',w'). In the left-hand panel, an increase in c'' from 0.53 to 0.58, while w' remains the same, shifts the Pareto frontier associated with this match (represented by the dashed frontier) because $\phi_1(c,w,u) > 0$ by Lemma 2(a). Because there are no changes in w' and $u(w')$ (equal to u_0 in both equilibria), from Lemma 1(b) it follows that the loan rate for the match (c'',w') decreases relative to its initial equilibrium value, that is, $R(0.58,0.4,0.3) < R(0.53,0.4,0.3)$.

In the other equilibrium match, (c',w''), there are two effects of a decrease in the value of c' from 0.45 to 0.4. The first is a direct effect that increases the equilibrium loan rate (cf. Lemma 1(b)). The second is an indirect effect. A decrease in c' shifts the Pareto frontier associated with this match (as shown in the right-hand panel of Figure 2), hence there is a reduction in the match surplus, which implies a reallocation of utilities between the lender and the borrower. If $u(w'')$ goes up, then the new equilibrium loan rate will be lower because it is strictly decreasing in u. By contrast, if $u(w'')$ decreases in the new

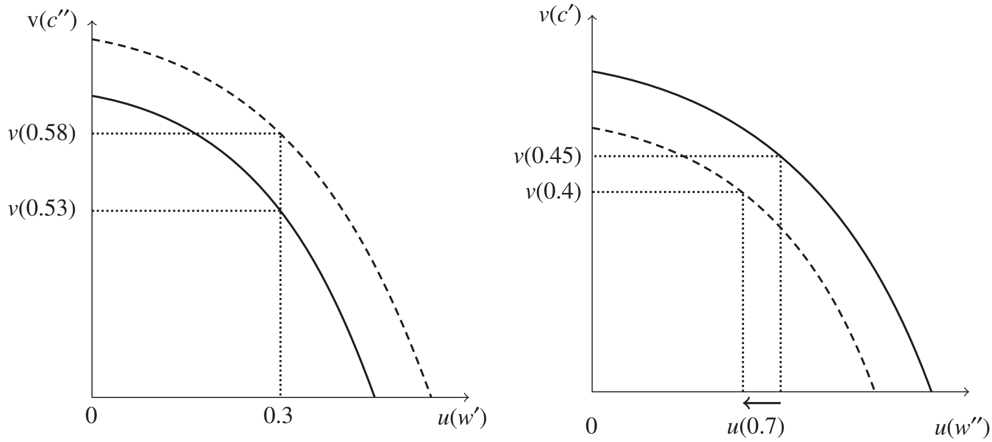

FIGURE 2. A change in lender types may imply a positive association between loan rate and collateral.

equilibrium allocation, then the loan rate must go up. It turns out that if the new utility allocation is such that the high-type borrower obtains $u(w'') < 0.807$, then the net effect of a decrease in c' is that the loan rate paid by w'' is higher than that is paid by w', hence there is a reversal of the association between loan rate and borrower collateral relative to the equilibrium of the benchmark economy.

Note that a decrease in the value of c' from 0.45 to 0.4 and an increase in c'' from 0.53 to 0.58 is nothing but a mean-preserving spread of the lender types. A result similar to that in Claim 2 can be obtained by a mean-preserving contraction of the borrower types, keeping the values of c' and c'' fixed, in which case the shifts in the Pareto frontiers are similar to those in Figure 2, or by a combination of such changes in both lender and borrower types. The crux of the argument lies in the fact that the surplus associated with one match increases, while that of the other match decreases, implying a reallocation of the bargaining power between the high-type lender and the high-type borrower.

Equilibrium payoffs and matching

Now we characterize the equilibrium allocation of the lender–borrower market with a continuum of types. The first-order condition of the maximization problem $(\mathcal{P})$ is given by the ordinary differential equation

$$\text{(U)} \qquad u'(w) = -\frac{\phi_2(c, w, u(w))}{\phi_3(c, w, u(w))} \text{ for } c = \lambda(w).$$

It follows from the Envelope Theorem that

$$\text{(V)} \qquad v'(c) = \phi_1(c, w, u(w)) \text{ for } c = \lambda(w).$$

Because $\phi_1, \phi_2 > 0$ and $\phi_3 < 0$ by Lemma 2, we have $u'(w) > 0$ and $v'(c) > 0$. In an equilibrium allocation, we must have $u(w) \geq u_0$ for all $w \in W$. This is the 'individual rationality' constraint associated with the maximization problem $(\mathcal{P})$. As $u(w_{\min})$ is bounded above by $u(w)$ for $w > w_{\min}$, any entrepreneur with collateral $w_{\min}$ must be pushed to his reservation utility, that is, $u(w_{\min}) = u_0$ in the equilibrium allocation. Therefore it follows from (U) that

$$(\mathrm{U}') \qquad u(w) = u_0 + \int_{w_{\min}}^{w} u'(x)\mathrm{d}x.$$

Thus all borrower types except the one with the lowest collateral earn type-specific rents. At this juncture, it is worth noting the difference between the model with discrete types described in the previous subsection and that with a continuum of types. Recall that with only two types, the equilibrium utility that accrues to the high-type borrower may take values in a closed interval. In the equilibrium of the two-sided market with a continuum of types, the utility of each borrower type w is just a positive real number, which is pinned down by the expression (U′). This is because in a continuum model, unlike the market with discrete types, the types are too close to each other, and hence in an equilibrium there is no extra surplus for each type to bargain over. Next, we show that in any equilibrium allocation, the matching is negative assortative, which is an immediate consequence of the single-crossing condition (SC).

Proposition 1 In any equilibrium allocation (λ, v, u), the lender-borrower matching is negative assortative; that is, more efficient lenders (monitors) invest in firms with lower collateral following a negative assortative matching pattern.

Condition (SC) implies that if a lender with higher type c'' is indifferent between the firm-type–payoff combinations (w', u') and (w'', u''), with $w'' > w'$ and $u'' > u'$, then a lower type c' prefers to pay more (than u'') to a borrower of type w''; that is, for any $c'' > c'$,

$$(\mathrm{GDD}) \qquad \phi(c'', w', u') = \phi(c'', w'', u'') \quad \Longrightarrow \quad \phi(c', w'', u'') \geq \phi(c', w', u').$$

The above is the *generalized decreasing difference* (GDD) condition in Legros and Newman (2007), which implies that lender and borrower types are substitutes, and is a necessary and sufficient condition for NAM. Condition (GDD) is weaker than the usual substitutability implied by the *decreasing differences* or *submodularity* property of the Pareto frontier.[8] A consequence of NAM and the measure consistency condition (cf. Definition 2(c)) is that $G(\lambda(w)) = 1 - F(w)$, which implies that

$$\lambda'(w) = -\frac{f(w)}{g(\lambda(w))}.$$

Thus, given the assumption of strictly positive type densities, $\lambda'(w) < 0$, that is, the equilibrium matching function is strictly decreasing on W, hence it is differentiable almost everywhere.

The phenomenon of endogenous lender–borrower matching is ubiquitous in financial markets (e.g. Chen 2013). However, empirical evidence of negative sorting in such markets has been scarce until recently. Schwert (2018) finds evidence of NAM in a syndicated corporate loan market—bank-dependent borrowers (low equity capital or collateral) tend to secure funding from well-capitalized banks who, according to the 'equity monitoring hypothesis' (e.g. Hölmstrom and Tirole 1997; Mehran and Thakor 2011), have stronger incentives to monitor borrowers. Therefore the Schwert (2018) finding supports our theoretical prediction that lenders with stronger incentives to monitor sort themselves into borrowers with lower collateral.

Equilibrium loan rates

An increase in the borrower collateral in our general equilibrium framework affects the equilibrium loan rate through three channels (one direct and two indirect), as follows from Lemma 1(b). First, a direct channel; that is, greater collateral implies a higher optimal loan rate to be paid to the lender. Second, collateral affects the loan rate through the equilibrium matching function; that is, a higher value of collateral in a given partnership implies a lower monitoring ability because of NAM, and hence must be associated with a higher loan rate as less efficient monitors require greater marginal compensation. Third, a change in collateral value alters a borrower's bargaining power via the equilibrium utility function $u(w)$, and a greater bargaining power caused by higher collateral implies lower loan rates. Clearly, the three aforementioned effects do not point in the same direction, yielding a potential non-monotonic behaviour of the equilibrium loan rate with respect to borrower collateral. To see this formally, let the equilibrium loan rate function be given by

$$R(w) \equiv R(\lambda(w), w, u(w)).$$

The slope of the equilibrium loan rate function at a given collateral value $w = w^0$ is given by

$$\begin{aligned} R'(w^0) = {} & \underbrace{R_1(\lambda(w^0), w^0, u(w^0)) \cdot \lambda'(w^0)}_{(+)} + \underbrace{R_2(\lambda(w^0), w^0, u(w^0))}_{(+)} \\ & + \underbrace{R_3(\lambda(w^0), w^0, u(w^0)) \cdot u'(w^0)}_{(-)}. \end{aligned}$$

The first two terms are strictly positive because $R_1 \equiv \partial R/\partial c < 0$, $\lambda'(w) < 0$ and $R_2 \equiv \partial R/\partial w > 0$. The third term, however, is strictly negative because $R_3 \equiv \partial R/\partial u < 0$ and $u'(w) > 0$. Thus at w^0, whether $R'(w)$ is positive or negative depends on which of the countervailing forces dominates. The following proposition is stated without proof.

Proposition 2 The equilibrium loan rate $R(w)$ is in general non-monotonic with respect to borrower collateral.

Note that the type densities $f(w)$ and $g(c)$ are local measures of dispersion that determine the slope of the equilibrium matching and borrower utility functions at a given level of borrower collateral, and hence the relative strength of the aforementioned countervailing forces. To fix ideas, consider the match (c^0, w^0) in an equilibrium allocation, that is, $c^0 = \lambda(w^0)$. In a small neighbourhood of (c^0, w^0), if the lenders are sufficiently heterogeneous relative to the borrowers, then there would be a greater measure of borrowers concentrated around w^0 relative to the measure of lenders around c^0;

that is, $f(w^0)$ would be sufficiently higher relative to $g(\lambda(w^0))$, and hence the equilibrium matching function would be sufficiently steep at w^0 because $|\lambda'(w^0)| = f(w^0)/g(\lambda(w^0))$, and the equilibrium borrower utility function $u(w)$, by contrast, would be sufficiently flat around w^0. In this case, around w^0, the positive effect is more likely to dominate the negative one, and consequently, $R(w)$ would be increasing at w^0. If, on the other hand, the borrowers are sufficiently heterogeneous relative to the lenders around a given match (c^0, w^0), then the negative effect is more likely to be dominant and the equilibrium loan rate function would be decreasing at w^0. In general, the condition of local relative heterogeneity would change over $[c_{\min}, c_{\max}]$ and $[w_{\min}, w_{\max}]$, and consequently, $R'(w)$ may change sign several times in the interval $[w_{\min}, w_{\max}]$, that is, $R(w)$ may be non-monotonic in w, as illustrated in Figure 4 below. The illustrative examples analysed earlier also hint towards such non-monotonic behaviour in the sense that when one performs a mean-preserving spread of lender types or a mean-preserving contraction of borrower types, the equilibrium loan rate may become increasing in w as compared with being decreasing in the benchmark economy.

It is worth noting that the above local condition that is sufficient to determine the sign of the slope of the equilibrium loan rate function is not necessarily a global sufficient condition since for any arbitrary type densities $f(w)$ and $g(c)$, the ratio of $f(w)$ to $g(\lambda(w))$ may be very high at one value of borrower collateral, and very low at some other collateral value. Moreover, the expression of $R'(w)$ involves $u(w)$, whose exact functional form is unknown because the ordinary differential equation (U) does not have an analytical solution. Therefore it is difficult to provide a sufficient condition for an equilibrium loan rate function $R(w)$ that is monotonic with respect to w. Thus if the aforementioned local condition holds for all $w^0 \in W$, then the equilibrium $R(w)$ will be either monotonically increasing or monotonically decreasing with respect to borrower collateral.

Berger *et al.* (2016) find empirical evidence of a non-monotonic relation between loan rate and collateral. They argue that collateral may have different desirable economic characteristics, such as liquidity and outside ownership status, each of which may influence loan risk in a different way; hence the empirical relation between the loan rate (or equivalently, loan risk premium) and collateral may not be monotonic. Our alternative explanation for the non-monotonicity of loan rate is based on two-sided heterogeneity and endogenous matching in which the equilibrium matching pattern and endogenous payoffs play crucial roles.

IV. Effect of Cross-sectional Variations in the Type Distribution

The property that any equilibrium allocation of the lender–borrower market exhibits NAM is implied by the single-crossing condition (GDD) of the Pareto frontier, and not on the distributions of types, $G(c)$ and $F(w)$. However, the measure consistency condition together with the cumulative distributions of types pin down the equilibrium matching function. Therefore although the pattern of equilibrium matching is distribution-free, the shape of the matching function is not. Such dependence of the equilibrium assignment on the type distributions allows us to conduct meaningful comparative statics exercises, namely, the effects of changes in the type distributions on the equilibrium lender–borrower contracts.

Effect on equilibrium matching and borrower utility

We first analyse the effects of changes in type distributions on the equilibrium matching function $\lambda(w)$ and the borrower payoff function $u(w)$. Because only the relative density $f(w)/g(c)$ is relevant for the shape of the matching function, we henceforth fix the distribution $G(c)$ of c, varying only $F(w)$ in order to analyse the effects of changes in the distribution of collateral on the equilibrium variables. A change in $F(w)$ would affect both $\lambda(w)$ and $u(w)$ via shifts in the matching function. Consider two lender–borrower economies $\xi^s = (F_s,G)$ and $\xi^t = (F_t,G)$ with $s \neq t$, which differ only in the distributions of collateral. For our purpose, let us interpret s and t as two distinct time points (years). Recall the following definition.

Definition 3 (Increasing collateral inequality) The distribution of collateral in economy ξ^t is more unequal than that in ξ^s if F_s dominates F_t in the sense of rotation order, written as $F_s \succ F_t$, that is, if there is a unique $w^* \in [w_{\min}, w_{\max}]$ such that $F_s(w) < (>) F_t(w)$ for all $w < (>) w^*$.

The above definition, which follows from Johnson and Myatt (2006), implies that the two distribution functions F_s and F_t cross each other only once at $w = w^*$, that is, $F_t(w)$ is obtained by rotating $F_s(w)$ clockwise around $w = w^*$. It is worth noting that Definition 3 accommodates the concepts of both a 'first-order stochastic dominance', in which case either $w^* = w_{\min}$ or $w^* = w_{\max}$, and a 'spread', where the density functions $f_s(w)$ and $f_t(w)$ cross only twice. In the following proposition, we state our main comparative statics result.

Proposition 3 Consider two distinct lender–borrower economies ξ^s and ξ^t that differ only in the distributions F_s and F_t of collateral. Furthermore, let $\lambda_\theta(w)$ be the equilibrium matching function, and let $u_\theta(w)$ be the equilibrium payoff of each type w entrepreneur in economy ξ^θ for $\theta = s,t$. If the distribution of collateral in ξ^t is more unequal than that in ξ^s (i.e. $F_s \succ F_t$), then $\lambda_s(w) > (<) \lambda_t(w)$ for $w < (>) w^*$, and either (i) $u_s(w) < u_t(w)$ for all $w \in (w_{\min}, w_{\max}]$, or (ii) there is a unique $\hat{w} \in (w^*, w_{\max})$ such that $u_s(w) < (>) u_t(w)$ for $w < (>) \hat{w}$.

The nature of the shift in the equilibrium matching function resulting from a change in the distribution of collateral is somewhat trivial. Recall that NAM implies that $G(\lambda_\theta(w)) = 1 - F_\theta(w)$ for $\theta = s,t$. Because $G'(c) = g(c) > 0$ for all $c \in C$, it must be the case that $\lambda_s(w) > \lambda_t(w)$ if and only if $F_s(w) < F_t(w)$. Therefore if for any given $w \in W$ we have $F_s(w) < (>) F_t(w)$, then any entrepreneur of type w must be matched with an investor with lower (higher) monitoring ability in the equilibrium of ξ^t.

The nature of the shift in the equilibrium borrower utility function following a change in the type distribution function is, however, less trivial. The single-crossing condition (SC) implies that $u'(w)$ is strictly decreasing in c, hence for any w, $\lambda_s(w) > (<) \lambda_t(w)$ implies $u_s'(w) < (>) u_t'(w)$. Thus a clockwise rotation of $F(w)$, which increases the inequality of collateral, implies a clockwise rotation of $u'(w)$ around w^*. Because $u(w) - u_0$ is the area under the curve $u'(w)$ for $\theta = s,t$ (cf. (U$'$)), any borrower with collateral value $w < w^*$ would gain, whereas a borrower of type $w > w^*$ may gain or lose. Therefore a change in the distribution of collateral shifts bargaining power from one side of the market to the other.

Implications for the equilibrium loan rate and monitoring: numerical results

As the equilibrium loan rate and monitoring are expressed as functions of collateral, any change in $F(w)$ would also change these equilibrium variables. In our static equilibrium model, such an exercise should be viewed as the effect of cross-sectional variations in the investor–entrepreneur market. To analyse the effect of a change in the distribution of collateral on $R(w)$, the equilibrium loan rate, one must first solve the ordinary differential equation (ODE) in (U). When we substitute for $\phi_2(m,w,u(w))$ and $\phi_3(m,w,u(w))$, the ODE reduces to

$$u'(w) = \psi(\lambda(w), w, u(w)) = 2 - \frac{pQ - B}{p[R(\lambda(w), w, u(w)) - w]}.$$

Under imperfectly transferable surplus, the Pareto frontier is a non-linear function of $u(w)$, hence the above ODE does not have an analytical solution. Moreover, it is difficult to determine the direction of shifts in the equilibrium loan rate following a change in $F(w)$, for the following reason. Let $R_\theta(w) \equiv R(\lambda_\theta(w), w, u_\theta(w))$ be the equilibrium loan rate associated with a common collateral value w in economy ξ^θ for $\theta = s,t$. Then

$$\text{(3)} \qquad \Delta R(w) \equiv R_t(w) - R_s(w) \approx \underbrace{\frac{\partial R}{\partial c}[\lambda_t(w) - \lambda_s(w)]}_{\text{matching effect}} + \underbrace{\frac{\partial R}{\partial u}[u_t(w) - u_s(w)]}_{\text{utility effect}}.$$

In this expression, a change in the equilibrium loan rate is decomposed into two effects—namely, a *matching effect* and an *utility effect*, which in general point in opposite directions. To see this, consider the simplest case in Proposition 3, that $F_s \succ F_t$ implies $\lambda_s(w) > (<) \lambda_t(w)$ for $w < (>) w^*$, and $u_s(w) < u_t(w)$ for all $w \in (w_{\min}, w_{\max}]$. In this case, both terms of (3) are negative for $w > w^*$, hence $R_t(w) < R_s(w)$ for all $w > w^*$. However, for $w < w^*$, the first term of (3) is positive because $\partial R/\partial c < 0$ and $\lambda_t(w) < \lambda_s(w)$, but the second term is always negative because $\partial R/\partial u < 0$ and $u_t(w) > u_s(w)$. Therefore the effect of an increase in the inequality of collateral on the equilibrium loan rate is ambiguous for values of $w < w^*$.

We therefore resort to a numerical simulation of the model to examine the shifts in the equilibrium loan rate functions from $R_s(w)$ to $R_t(w)$, and monitoring intensity function from $m_s(w)$ to $m_t(w)$, where $m_\theta(w) \equiv m(\lambda_\theta(w), w, u_\theta(w))$, for $\theta = s,t$, when $F_s \succ F_t$, that is, the distribution of collateral represents greater inequality in economy ξ^t. For this purpose, we take $p = 0.4$, $Q = 5$, $B = 1.5$ and $u_0 = 0.3$. Further, monitoring ability c is assumed to have a truncated beta distribution with parameters $\alpha_c = \beta_c = 2$ on the support [0.4,0.58].[9] Borrower collateral w, on the other hand, is assumed to follow a truncated beta distribution with parameters α and β on the support $[w_{\min}, w_{\max}] \subset (0, 1)$. Recall that if a random variable X follows a truncated beta distribution on $[a,b]$, with $0 \le a < b \le 1$ and parameters $\alpha, \beta > 0$, then its cumulative distribution function is given by

$$\frac{\int_a^x (z-a)^{\alpha-1}(b-z)^{\beta-1} \mathrm{d}z}{(b-a)^{\alpha+\beta-1} \int_0^1 x^{\alpha-1}(1-x)^{\beta-1} \mathrm{d}x}.$$

There is one principal reason for choosing a beta distribution. As we have seen from the theoretical analysis, the heterogeneity in c and w is crucial to determine the relative importance of the matching and utility effects on the equilibrium loan rate; thus a beta distribution is sufficiently flexible to consider alternative specifications for the relative heterogeneity between investors and entrepreneurs. As we have discussed earlier, because two comparable markets differ only in the corresponding distributions of collateral, there is no loss of generality in assuming a given distribution of monitoring ability.

To give an empirical content to the analysis, we use the data constructed by Angelini and Generale (2008) for a sample of Italian firms to match the beta distribution parameters α and β. The dataset includes four surveys run in 1992, 1995, 1998 and 2001 by an Italian credit institution. Each survey includes information about several characteristics of the firm, including the value of its assets. The surveys are representative of Italian manufacturing firms with more than ten employees. One major advantage of this dataset is that firms with fewer than 50 employees are well represented in the surveys. Because we have assumed that the entrepreneur can pledge his entire initial wealth as collateral, we take the natural logarithm of total asset value (i.e. current plus net fixed assets) as our proxy for borrower collateral. For our purpose, we select only the 1992 and 1995 surveys. We take into account only the firms that have reported information on total assets. Thus we are left with 2262 out of 4811, and 3539 out of 4431 observations, respectively. The corresponding cumulative distribution functions of total asset value (in natural logarithms) are shown in Figure 3. It is clear from Figure 3 that the asset distribution of 1992 is more unequal than that of 1995; that is, $F_{1992}(w)$ is a clockwise rotation of $F_{1995}(w)$.[10]

The data in Figure 3 can be conveniently represented by a beta distribution. First, the total asset values are normalized on the support [0,1]. Next, we estimate the mean and variance of each of the two samples corresponding to years 1992 and 1995. Finally, we use these two moments to calibrate the two (unknown) parameters α and β of each distribution, so that the first two moments of the 'artificial' distribution replicate the moments observed in the data. The estimated parameters of the beta distribution on [0,1] corresponding to 1992 and 1995 are given by $\alpha_{1992} = 9.48$, $\beta_{1992} = 7.42$ and $\alpha_{1995} = 11.67$, $\beta_{1995} = 9.19$. It turns out that very low and very high values of collateral imply negative equilibrium utilities for the lenders, hence it is necessary to have $w_{\min} > 0$ and $w_{\max} < 1$. Therefore we truncate the calibrated beta distribution corresponding to each year by

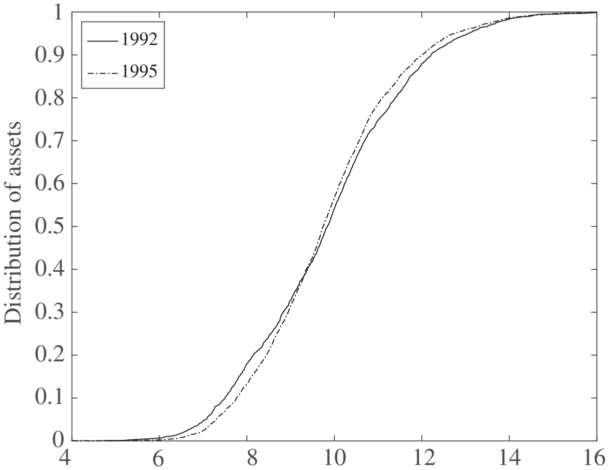

FIGURE 3. The empirical cumulative distribution of borrower collateral of 1992 exhibits a greater inequality than that of 1995.

restricting collateral values in the interval [0.4,0.7]. The rotation point where the two calibrated collateral distributions cross each other is $w^*\approx 0.55$, which corresponds to the collateral value 9.9 in the data.

The dataset does not have information on loan contracts. So we construct the equilibrium functions $R_\theta(w)$ and $m_\theta(w)$ corresponding to year θ = 1992,1995 using the aforementioned parameter values of the truncated beta distributions. The simulated profile of the equilibrium loan rate is presented in Figure 4. Note that the equilibrium loan rate function is clearly non-monotonic in borrower collateral for each sample. As far as the equilibrium monitoring intensity functions are concerned, it is difficult to visually distinguish the one simulated for 1992 from the one corresponding to 1995, as the level does not change much from one year to the other. Thus we plot the difference between the monitoring intensities corresponding to 1995 and 1992—namely, $m_{1995}(w)-m_{1992}(w)$—against the collateral values in Figure 5. It is clearly seen that the equilibrium loan rate in the 1992 sample is higher (lower) for values of collateral below (above) $w\approx 0.55$. On the other hand, the equilibrium monitoring intensity in the 1992 sample is lower (higher) than that for 1995 for collateral values below (above) $w\approx 0.55$. Thus from the numerical exercise we conclude the following.

Result 1 (Effect of a change in collateral inequality) Consider two lender–borrower economies, $\xi^s = (F_s,G)$ and $\xi^t = (F_t,G)$, where $F_s \succ F_t$, that is, the distribution of collateral in ξ^t is more unequal than that in ξ^s. Then there is a unique $\hat{w}_R \in (w_{\min}, w_{\max})$ such that an entrepreneur with collateral value $w<(>)\hat{w}_R$ pays a higher (lower) loan rate in the equilibrium of ξ^t. Moreover, there is a unique $\hat{w}_m \in (w_{\min}, w_{\max})$ such that an entrepreneur with collateral value $w<(>)\hat{w}_m$ suffers less (more) intense monitoring in the equilibrium of ξ^t.

The intuition follows from the cumulative distribution functions in Figure 3. Borrowers with collateral values of 9.9 ($w\approx 0.55$ in the calibrated distribution) or less are more abundant in the 1992 sample than in the 1995 sample. As a result, their bargaining power is lower and thus they pay higher loan rates compared to those in the 1995 sample. By contrast, entrepreneurs with collateral values of 9.9 or more are relatively less

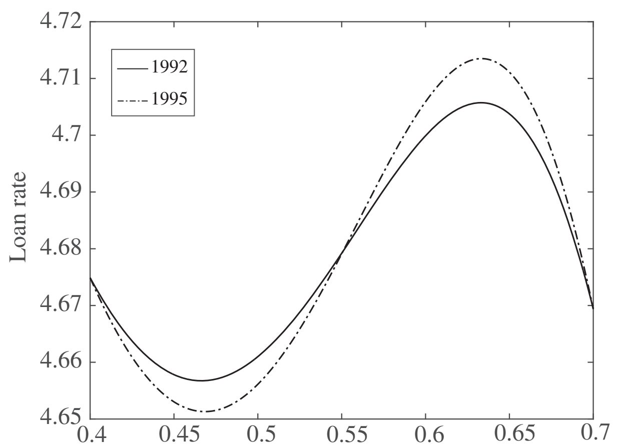

FIGURE 4. The simulated equilibrium loan rate functions corresponding to years 1992 and 1995.

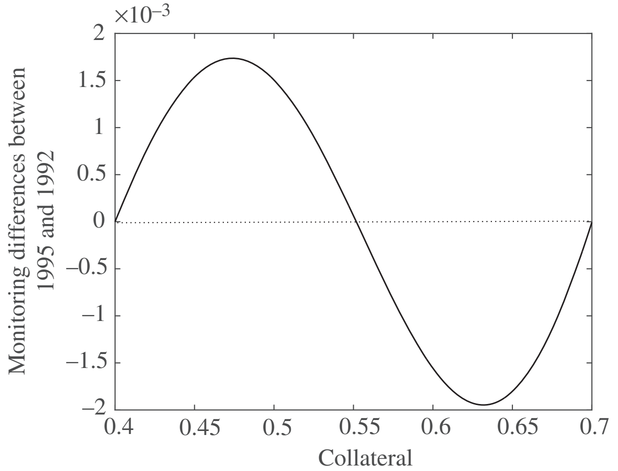

FIGURE 5. Difference between the simulated equilibrium monitoring intensities corresponding to years 1995 and 1992.

abundant in the 1992 sample. For such collateral levels, the bargaining power of entrepreneurs is greater in the 1992 sample than the 1995 sample, which translates into a lower loan rate in the 1992 sample. Thus the overall effect of such a change in the distribution is determined by the resulting change in the bargaining power of entrepreneurs. In the economy with more asset inequality, ξ^t, entrepreneurs with high collateral have greater bargaining power because the competition for better entrepreneurs is more intense compared with that in ξ^s. Conversely, low-collateral borrowers have lower bargaining power in economy ξ^t. Therefore high-collateral entrepreneurs pay lower loan rates in economy ξ^t, whereas low-collateral borrowers face an increased cost of external financing in ξ^t. On the other hand, high-collateral entrepreneurs suffer from higher monitoring in economy ξ^t.

The two empirical distributions depicted in Figure 3 clearly assert that $F_{1992}(w)$ is a clockwise rotation of $F_{1995}(w)$. In fact, the collateral distribution of 1992 is a spread of that of 1995 because the two densities $f_{1992}(w)$ and $f_{1995}(w)$ cross exactly twice. One may wonder how the equilibrium loan rates $R_s(w)$ and $R_t(w)$ compare if F_t first-order stochastically dominates F_s, that is, $F_t(w) \leq F_s(w)$ for all $w \in W$. In particular, would one equilibrium loan rate function lie above or below the other for all collateral values? It turns out that even a first-order shift in the distribution function of collateral may yield a result similar to Result 1. To see this, first note that if $F_t(w) \leq F_s(w)$ for all w, then (a) $\lambda_t(w) \geq \lambda_s(w)$ and (b) $u_t'(w) \leq u_s'(w)$, hence $u_t(w) \leq u_s(w)$ for all w. Thus it follows from (3) that the matching effect and the utility effect always point in opposite directions. Consequently, the two equilibrium loan rate functions $R_s(w)$ and $R_t(w)$ can cross each other as in Figure 4. Therefore a change in the collateral distribution in the sense of a first-order shift may imply an asymmetric treatment of the borrowers in terms of the loan rate to be paid, as does a spread of the collateral distribution.

V. CONCLUDING REMARKS

Compared with contracts for an isolated investor–entrepreneur pair, incentive contracts may be quite different in a market with many heterogeneous investors and entrepreneurs. In the equilibrium of a market, individual contracts are influenced by the two-sided heterogeneity via endogenous investor–entrepreneur matching. In this paper, we have developed a simple two-sided matching model of incentive contracting between lenders and borrowers. Entrepreneurs who differ in collateral values and investors who differ in monitoring ability are matched into pairs in order to accomplish projects of fixed size. In the equilibrium of the market, both the matching and payoffs that accrue to each individual are determined endogenously. We show that the equilibrium lender–borrower matching is negatively assortative, and there is a general non-monotonic association between the loan rate and collateral. Further, we show how a change in the distribution of collateral may affect asymmetrically the cost of external funding and monitoring.

Although our stylized model is built on a number of simplifying assumptions, our conclusions are somewhat general, and can be extended to credit relationships other than those analysed in the present paper. Under double-sided moral hazard, in each investor–entrepreneur partnership, what is crucial is the identification of lenders and borrowers who are 'easier to incentivize'. When investors perform the role of monitors, in equilibrium, investors with stronger incentives to monitor form partnerships with borrowers with greater need of outside equity following a negative assortative matching pattern. Thus any empirical analysis that finds direct or indirect evidence of such negative sorting (e.g. Schwert 2018) by using a set of appropriate measures is consistent with our theoretical finding.

Our model is an extension of Shapley and Shubik (1971) to an environment where matches are pervaded by the underlying incentive problems, which give rise to a non-linear Pareto frontier. However, the agency problem is not the only fundamental that induces imperfect transferability. If at least one of the two contracting parties is risk-averse, then the associated frontier is concave even without the incentive problem. In such markets, as argued by Legros and Newman (2007), partnerships are formed because of pure risk-sharing motives, whereas in our model, each partnership (which is subject to limited liability) implies an optimal trade-off between the provision of incentives and rent extraction. Therefore an extension of the current paper to an environment with risk-averse individuals may shed light on the implications of risk-averse investors in the corporate loan markets.

A more ambitious model would consider many-to-many matching among investors and entrepreneurs. When a lender is allowed to invest in more than one firm, additional complications arise because the monitoring cost function is in general not additively separable. Thus non-zero interaction terms induce externalities across matches. On the other hand, allowing an entrepreneur to borrow from multiple sources may imply the inability of lenders to write binding exclusive contracts. Non-exclusivity may also lead to an externality across matches. Also, an important assumption in the paper is that the relationship between an investor and an entrepreneur lasts for only one period. Such relationships could also involve dynamic considerations, which in turn imply some degree of relaxation of the limited liability constraints, and the conclusions of the current paper could thus change. In a dynamic model, when there are possibilities of wealth accumulation, the income distributions of an economy are generally endogenous. The literature on two-sided matching (e.g. Shapley and Shubik 1971) has largely been silent on the context of dynamic bilateral relationships. In this context, the paper by Mookherjee and Ray (2002) is worth mentioning. Mookherjee and Ray (2002) analyse a dynamic model of equilibrium short-period credit contracts when lenders and borrowers are randomly matched, and the bargaining power is exogenously given. When lenders have all the bargaining power, less wealthy borrowers have no incentive to save, and poverty traps emerge. Conversely, if borrowers have all the bargaining power, then income inequality is reduced as a result of the strong incentives for savings. One significant difference between our model and that of Mookherjee and Ray (2002) is that bargaining power in the current model is distributed endogenously among the principals and agents because the outside option of each individual is endogenous.

APPENDIX: PROOFS

Proof of Lemma 1 We analyse the optimal loan contract for an arbitrary partnership (c,w), which solves the program $(\mathcal{M})$. We ignore the limited liability constraint (LL) for the time being. Substituting $m = cp(R-w)$ into the objective function and the participation constraint, the maximization problem $(\mathcal{M})$ reduces to

$$(\mathcal{M}') \qquad \max_m V(m) \equiv \frac{m^2}{2c} + w - 1,$$

subject to

$$(\text{PCB}') \qquad U(m) \equiv m(pQ - B) - \frac{m^2}{c} + B \geq u.$$

The Lagrangian is given by

$$\mathcal{L} = \frac{m^2}{2c} + w - 1 + \mu\left(m(pQ - B) - \frac{m^2}{c} + B - u\right),$$

where μ is the associated multiplier. The first-order condition with respect to m yields

$$\mu = \frac{m}{2m - c(pQ - B)}.$$

Clearly, $2m \geq c(pQ-B)$ for μ to be non-negative, hence $m > 0$. Note that the first-best monitoring is given by

$$m^{FB} = \underset{m}{\text{argmax}}\left\{mpQ + (1 - m)B - \frac{m^2}{2c} + w - 1\right\} = c(pQ - B).$$

Now we show that (PCB') binds at the optimum. Note that $c(pQ-B) > 0$ implies that $\mu > 1/2$, hence (PCB') binds at the optimum. Given that (PCB') holds with equality, Assumption 1 (i.e. $B > u$) implies that

$$m(pQ - B) - \frac{m^2}{c} < 0 \quad \Longleftrightarrow \quad m > c(pQ - B) = m^{FB};$$

that is, there is always overmonitoring at the optimum, which also implies that $\mu < 1$. The optimal monitoring, which is determined from the binding (PCB'), is given by

$$\text{(A1)} \qquad m(c,w,u) = \frac{1}{2}\left(c(pQ-B) + \sqrt{c^2(pQ-B)^2 + 4c(B-u)}\right).$$

We ignore the smaller root of m because m must be greater than $c(pQ-B)/2$. Note that

$$m(c,w,u) \le 1 \quad \Longleftrightarrow \quad c(pQ-u) \le 1.$$

Thus the second inequality in Assumption 2 guarantees that $m(c,w,u) \le 1$. The optimal loan rate is determined by substituting $m = cp(R-w)$ into (A1), which gives

$$\text{(A2)} \qquad R(c,w,u) = w + \frac{m(c,w,u)}{cp} = w + \frac{1}{2cp}\left(c(pQ-B) + \sqrt{c^2(pQ-B)^2 + 4c(B-u)}\right).$$

We must verify the limited liability constraint (LL). Note that

$$R(c,w,u) \le Q + w \quad \Longleftrightarrow \quad B(1-cpQ) \le u.$$

Thus the first inequality in Assumption 2 guarantees that (LL) holds. For the comparative statics results, differentiating (A1) with respect to c, w and u, respectively, we obtain

$$\frac{\partial m}{\partial c} = \frac{m\mu}{c} > 0, \quad \frac{\partial m}{\partial w} = 0, \quad \frac{\partial m}{\partial u} = -\frac{c\mu}{m} < 0.$$

The above inequalities prove part (a). Finally, differentiating (A2) gives

$$\frac{\partial R}{\partial c} = -\frac{m(1-\mu)}{c^2 p} < 0, \quad \frac{\partial R}{\partial w} = 1, \quad \frac{\partial R}{\partial u} = -\frac{\mu}{pm} < 0.$$

Proof of Lemma 2 The Pareto frontier, which is the value function of the programme ($\mathcal{M}'$) with (PCB$'$) binding, is given by

$$\text{(PF)} \qquad \phi(c,w,u) = \frac{1}{2c} \cdot \left(m(c,w,u)\right)^2 + w - 1.$$

It follows from the Envelope Theorem that

$$\begin{aligned}
\phi_1(c,w,u) &= \frac{\partial \mathcal{L}}{\partial c} = \frac{m^2}{c^2}(\mu - 1/2) > 0,\\
\phi_2(c,w,u) &= \frac{\partial \mathcal{L}}{\partial w} = 1 > 0,\\
\phi_3(c,w,u) &= \frac{\partial \mathcal{L}}{\partial u} = -\mu < 0.
\end{aligned}$$

To prove the single-crossing condition (SC),

$$\frac{\partial}{\partial c}\left(-\frac{\phi_2}{\phi_3}\right) = \frac{\partial}{\partial c}\left(\frac{1}{\mu}\right) = -(pQ-B) \cdot \frac{\partial}{\partial c}\left(\frac{c}{m}\right) = -\frac{(pQ-B)(1-\mu)}{m} < 0.$$

Proof of Proposition 1 The proof of NAM in any equilibrium allocation directly follows from Legros and Newman (2007). Take $c'' > c'$ and $w'' > w'$, and write $u' = u(w')$ and $u'' = u(w'')$, the corresponding equilibrium utilities. Suppose that condition (GDD) holds for all matches, but for the aforementioned two lender–borrower pairs, in an equilibrium allocation, NAM does not hold, that is, $c' = \lambda(w')$ and $c'' = \lambda(w'')$. Assume without loss of generality that $\phi(c'',w',u') = \phi(c'',w'',u'')$. Because (U) holds, that is, $u'(w) > 0$ for all w, we have $u'' > u'$. Thus it follows from (GDD) that $\phi(c',w'',u'') \geq \phi(c',w',u')$, which contradicts the fact that

$$w' = \underset{w}{\operatorname{argmax}}\, \phi(c', w, u(w)).$$

Proof of Proposition 3 The proof can be adapted easily from Määttänen and Terviö (2014, Prop. 4), who analyse a similar result for a one-sided matching market. Note that

$$F_s(w) < (>) F_t(w) \text{ for } w < (>) w^* \iff \lambda_s(w) > (<) \lambda_t(w) \text{ for } w < (>) w^*,$$

because $G(\lambda_\theta(w)) = 1 - F_\theta(w)$ for $\theta = s,t$, and $G'(c) = g(c) > 0$ for all $c \in C$. Moreover, in the equilibria of both markets, $u_s(w_{\min}) = u_t(w_{\min}) = u_0$. Therefore

$$\Delta u(w) \equiv u_t(w) - u_s(w) = \int_{w_{\min}}^{w} [u_t'(x) - u_s'(x)]\mathrm{d}x.$$

The single-crossing condition (SC) implies that $u'(w)$ is strictly decreasing in c, hence the integrand of the above expression, which is the slope of $\Delta u(w)$, is strictly positive (negative) for $w < (>) w^*$. At $w = w^*$, the above definite integral is strictly negative because $\Delta u(w_{\min}) = 0$, and it is strictly increasing on $[w_{\min}, w^*)$. Because $\Delta u(w)$ is strictly decreasing on $(w^*, w_{\max}]$, there are two possibilities: (i) $\Delta u(w)$ does not intersect the horizontal axis, hence it is strictly positive for all $w \in (w_{\min}, w_{\max}]$; or (ii) $\Delta u(w)$ intersects the horizontal axis at some point $\hat{w} \in (w^*, w_{\max}]$, which is unique because $\Delta u(w)$ is strictly decreasing on $(w^*, w_{\max}]$.

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NOTES

1. See Berger and Udell (1990) and Brick and Palia (2007) for a positive relationship, and Agarwal and Hauswald (2010) and Degryse and Van Cayseele (2000) for a negative relationship between loan rate and borrower collateral. Boot *et al.* (1991) analyse a model of secured lending by combining the above two attributes—hidden action and hidden information—and show that default risk, and hence the loan rate, may be either increasing or decreasing with respect to borrower quality.
2. The second class of papers analyses the effect of assigning managerial talent to firm characteristics on the optimal managerial compensation. There is assortative matching; that is, more talented managers run larger firms (e.g. Edmans *et al.* 2009; Terviö 2008), more profitable firms (e.g. Alonso-Paulí and Pérez-Castrillo 2012), safer firms (e.g. Li and Ueda 2012), or firms with greater market power (e.g. Dam 2015). Many of these papers show that assortative matching significantly explains the observed variations in the level and incentive structure of CEO pay.

3. Dam and Pérez-Castrillo (2006) and von Lilienfeld-Toal and Mookherjee (2016) are the two other papers that consider the effect of endogenous matching on principal–agent contracts under one-sided heterogeneity.
4. All our results hold under a general form of monitoring cost function $D(m;c)$ which is strictly increasing and convex in m with $D(0;c) = 0$, $D_{12}(m;c) < 0$ and $D_{112}/D_{12} \leq 1 \leq mD_{11}/D_1$.
5. Monitoring ability may sometimes be difficult to quantify, but can be proxied by the types of institutional investors. For example, Almazan *et al.* (2005) claim that, compared with bank trust departments and insurance companies, investment advisors and investment companies in general entail lower costs of monitoring. Ability may also be proxied by investor attributes such as expertise (e.g. Almazan 2002), experience (e.g. Sorensen 2007), independence of organizational structure (e.g. Bottazzi *et al.* 2008), or size of human capital (e.g. Dimov and Shepherd 2005), which may limit the intensity of investor monitoring. The monitoring cost function that we propose may thus be viewed as a reduced-form function in which ability either represents one of the aforementioned investor attributes or has a strong positive association with one of them.
6. At a constant R, by differentiating (ICL) we get $\partial m/\partial w = -cp < 0$. On the other hand, by differentiating (PCB) we obtain $\partial m/\partial w = pm^2/(B-u) > 0$.
7. Compute the loan rates from the expression $R(c,w,u)$ given in (2).
8. The maximization problem ($\mathcal{P}$) is similar to incentive compatibility in the 'optimal screening problems' (e.g. Maskin and Riley 1984), where the first-order condition (U) is a kind of *local downward incentive constraint*. Moreover, equation (U′ is similar to the *informational rent* of an agent in an optimal screening problem, which is monotonically increasing in w.
9. Given the parameter values, Assumption 2 restricts the values of $c_{\min} \geq 0.4$ and $c_{\max} \leq 0.588$.
10. To confirm that the two distribution functions are distinct from each other, we performed the Kolmogorov–Smirnov test. Because the p-value is 0.004, we can reject the null hypothesis that the two collateral distributions are the same.

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