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On the existence of share contracts under limited liability

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ABSTRACT

In a principal-agent relationship under limited liability, the agent in general receives the full share of revenue. We show that when the agent exerts effort in multiple tasks, the effort substitution effect helps explain the existence of share contracts even under limited liability.

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1. Introduction

In a seminal contribution to the theory of incentive contracts under limited liability and moral hazard, Innes (1990) shows that the optimal financial contract between a risk-neutral investor and a firm is a debt contract when the investor's payoff function is constrained to be non-decreasing in the profit of the firm. But non-debt contracts such as profit or revenue sharing are prevalent in a plethora of real-life contracting situations. Therefore, many attempts have been made to theoretically justify the existence of such institutional arrangements. In the context of tenancy relationships, sharecropping has been shown to be an optimal contract in the presence of moral hazard and limited liability (e.g. Eswaran and Kotwal, 1985; Sengupta, 1997; Ghatak and Pandey, 2000). In the case of franchise contracts, linear non-debt contracts are optimal in the sense that the franchise pays a fixed royalty to the franchisor and the revenue is shared between them according to a fixed sharing rule (e.g. Mathewson and Winter, 1985; Lal, 1990; Bhattacharyya and Lafontaine, 1995). The intuition behind the optimality of debt contracts in setups similar to Innes (1990) is fairly simple. Under limited liability, the principal has claims on the entire revenue when output realizations are sufficiently adverse. Therefore, it is optimal for the principal to offer full incentives (the entire share of revenue) to the agent in order to induce maximum effort so that higher rent [in the form of output] can be extracted when the agent cannot meet his repayment obligations.

The main objective of our paper is to show, under limited liability, that revenue sharing emerges as an optimal contractual arrangement when a risk-neutral agent exerts efforts in multiple tasks. Hölmstrom and Milgrom (1991) analyze optimal contracts between a principal and an agent under moral hazard with multitasking. Although the current paper considers a similar setup, our focus is fundamentally different. In Hömstrom and Milgrom's (1991) model the agent is risk-averse, and hence a share contract emerges as a result of pure risk sharing motives as it would have been the case with a risk-averse agent exerting effort in a single task. We, on the other hand, consider a situation where the agent is risk-neutral and the contracts are subject to limited liability. Such contracting situation with a single task for the agent fails to explain the existence of a revenue sharing contract since the optimal contract is a debt contract (e.g. Innes, 1990). The main point we make in the present paper is that the problem of effort allocation across multiple tasks gives rise to the optimality of revenue sharing (non-debt) contracts.

The economy consists of one principal and one agent. The principal owns two productive assets (plots of land, equipments, or franchise outlets) which differ in quality, and the agent leases both assets to produce revenue. The effort cost function of the agent

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depends on the efforts exerted in both tasks. We first show that when the efforts exerted on different assets are independent of each other, limited liability alone fails to explain the emergence of share contracts. If the agent is given the entire share of revenue from each asset, then it induces him to exert the highest incentive compatible efforts. Thus even for the realizations of adverse productivity shocks on both assets, the principal can be paid a higher rent. Next, we show that when the agent's cost function is not separable in efforts, i.e., there is the so-called *effort substitution* effect, revenue sharing emerges as an optimal contractual agreement. High incentives in one task dampen the incentives to exert high effort in the other since an additional unit of effort in one task increases the marginal effort cost in the other. Thus, a revenue sharing contract serves as an instrument to optimally allocate efforts across different tasks.

One popular approach to explain the existence of non-debt (revenue/profit sharing) contracts is to introduce double-sided moral hazard (e.g. Eswaran and Kotwal, 1985; Bhattacharyya and Lafontaine, 1995). Under double-sided moral hazard, full incentives to the agent exacerbate the moral hazard problem of the principal, and hence a share contract is a way to optimally incentivize both of them. Another set of papers (e.g. Sengupta, 1997; Ghatak and Pandey, 2000) consider situations where the agent has joint moral hazard in the choice of effort and risk, both of which affect the output of a single task to be accomplished by the agent. In such contexts, a debt or fixed rent contract induces the agent to undertake riskier projects, while a wage contract undermines his incentives to exert high effort. Ghatak and Pandey (2000) show that a revenue sharing contract is optimal under joint moral hazard since it encourages the agent to exert higher effort and to undertake lesser risk. The main difference between the current work and that of Ghatak and Pandey (2000) is that in our model a revenue sharing contract emerges so as to optimally allocate efforts across multiple tasks. Finally, Ray (2005) considers a model where the agent undertakes two actions: labor input to grow crop and investment to maintain the land quality in the future periods. He shows that a higher crop share for the agent induces him to exert higher effort that immediately increases the output, but reduces his incentives to invest in land, and hence it is optimal to reduce the agent's revenue share. Ray's (2005) model can be interpreted as a dynamic extension of Ghatak and Pandey (2000), in which share contract serves as an instrument for intertemporal allocation of efforts. On the other hand, Ray (2005) has some flavor of multitask agency models because output from a single asset in two different periods can be viewed as two different tasks.

2. The model

The economy consists of two risk-neutral individuals: one principal (P) and one agent (A). The principal owns two productive assets which differ in quality. The revenue function associated with asset $i = 1, 2$ is given by $Q_i = \theta_i e_i$, where $\theta_i \in [\underline{\theta}_i, \bar{\theta}_i]$ with $0 \leq \underline{\theta}_i < \bar{\theta}_i$ represents the productivity of asset i, and e_i is the labor input or effort exerted by the agent on asset or task i. We assume that Q_i is publicly verifiable. The productivity parameters θ_1 and θ_2 are random variables with the joint probability distribution $F(\theta_1, \theta_2)$, and the respective marginal distributions $F_1(\theta_1)$ and $F_2(\theta_2)$. We assume that the true realizations of productivity are independent across assets, i.e, $F(\theta_1, \theta_2) = F_1(\theta_1)F_2(\theta_2)$. The agent incurs cost of efforts which is given by $\psi(e_1, e_2)$ with $\psi_i, \psi_{ii} > 0$ for $i = 1, 2$, and $\psi_{ij} = \psi_{ji} \geq 0$ for $i, j = 1, 2$ and $i \neq j$. If $\psi_{ij} > 0$, then the marginal cost of effort in one task is increasing in the effort in the other, which is called the 'effort substitution effect'. On the other hand, if $\psi_{ij} = 0$, then the efforts in different tasks are independent.[1] Agent's efforts are not verifiable, and hence there are potential moral hazard problems in effort choice. We normalize the agent's outside option to 0.

A contract is denoted by $\gamma = (\alpha_1, R_1, \alpha_2, R_2)$, where α_i is the agent's share of revenue from asset i, and R_i is his debt or rental obligation on asset i. When $\alpha_i = 1$ and $R_i > 0$ for $i = 1, 2$, then the contract associated with asset i is a *pure rent* or *debt* contract. On the other hand, a *revenue sharing* contract on asset i emerges when $\alpha_i < 1$.[2] Such asset specific contracts are feasible since the returns of each asset is assumed to be publicly verifiable. Under limited liability, the payoffs of the principal and agent from asset $i = 1, 2$ are respectively given by:

$$Y_P^i = \min\{(1 - \alpha_i)\theta_i e_i + R_i,\ \theta_i e_i\},$$
$$Y_A^i = \max\{\alpha_i \theta_i e_i - R_i,\ 0\}.$$

Let $\hat{\theta}_i$ be the value of θ_i such that the limited liability associated with asset i binds, which is given by:

$$\hat{\theta}_i := \frac{R_i}{\alpha_i e_i}.$$

Let $Y_k = Y_k^1 + Y_k^2$ for $k = P, A$ be the aggregate payoff of an individual k (principal or agent).[3] Given a contract γ, the expected payoffs of the principal and the agent are respectively given by:

$$E[Y_P] := \sum_{i=1}^{2} \Big\{ [1 - F_i(\hat{\theta}_i)] \Big[(1 - \alpha_i) e_i E[\theta_i \mid \theta_i \geq \hat{\theta}_i] + R_i \Big] + F_i(\hat{\theta}_i) e_i E[\theta_i \mid \theta_i < \hat{\theta}_i] \Big\},$$

$$E[Y_A] := \sum_{i=1}^{2} \Big\{ [1 - F_i(\hat{\theta}_i)] \times \Big[\alpha_i e_i E[\theta_i \mid \theta_i \geq \hat{\theta}_i] - R_i \Big] \Big\} - \psi(e_1, e_2).$$

Given the independence of the returns of two assets and taking into account the agent's limited liability constraints, the above expressions are intuitive and easy to derive. When asset i has an adverse realization, i.e., $\theta_i < \hat{\theta}_i$, the agent is unable to meet his rental obligation on this asset, and the total asset income is ceased by the principal.

3. Optimality of revenue sharing contracts

Since efforts are not contractible, the principal-agent relationship faces an incentive compatibility constraint which implies that the efforts exerted on both assets must maximize the agent's expected payoff $E[Y_A]$, given a contract γ. Strict concavity of $E[Y_A]$ with respect to efforts implies the following incentive constraints:

$$[1 - F_i(\hat{\theta}_i)]\alpha_i E[\theta_i \mid \theta_i \geq \hat{\theta}_i] = \psi_i(e_i, e_j) \quad \text{for } i, j = 1, 2, \text{ and } i \neq j. \tag{IC$_i$}$$

The second constraint is the participation constraint of the agent, which is given by:

$$E[Y_A] \geq 0, \tag{PC}$$

[1] Effort substitution is absent, for example, when the cost function is additively separable.

[2] When $\alpha_i \in (0, 1)$ and $R_i = 0$, the contract can be interpreted as an *equity* contract.

[3] Limited liability may be joint on both assets where the agent's income is given by:

$$Y_A = \max\{\alpha_1\theta_1 e_1 - R_1 + \alpha_1\theta_2 e_2 - R_2,\ 0\}.$$

This is a situation where cross pledging of loan assets is possible. We abstract from such consideration.

i.e., the agent's expected payoff must be no less than his outside option. It is easy to show that the constraint (PC) must be satisfied with equality, otherwise the principal may increase her expected payoff by slightly increasing the rental payment, and the contract would still be accepted. Finally, feasibility requires that the agent's share of output from each asset cannot be higher than 1, i.e.,

$$\alpha_i \leq 1 \quad \text{for } i = 1, 2. \tag{F_i}$$

After substituting for α_i from (IC_i) for $i = 1, 2$ and for R_i and R_j from the binding participation constraint (PC) into the expressions of the expected payoffs, and into the constraints (F_i), and using the fact that

$$E[\theta_i] = [1 - F_i(\hat{\theta}_i)]E[\theta_i \mid \theta_i \geq \hat{\theta}_i] + F_i(\hat{\theta}_i)E[\theta_i \mid \theta_i < \hat{\theta}_i] \quad \text{for } i = 1, 2, \tag{1}$$

the principal's maximization problem reduces to:

$$\max_{\{e_1, e_2\}} \; e_1E[\theta_1] + e_2E[\theta_2] - \psi(e_1, e_2)$$

$$\text{subject to } [1 - F_i(\hat{\theta}_i)]E[\theta_i \mid \theta_i \geq \hat{\theta}_i] - \psi_i(e_i, e_j) \geq 0 \quad \text{for } i, j = 1, 2, \text{ and } i \neq j. \tag{F'_i}$$

Let λ_i for $i = 1, 2$ be the Lagrange multiplier associated with (F'_i). The first-order condition with respect to e_i is given by:

$$E[\theta_i] - \psi_i(e_i, e_j) - \lambda_i\psi_{ii}(e_i, e_j) - \lambda_j\psi_{ij}(e_i, e_j) = 0 \quad \text{for } i, j = 1, 2, \text{ and } i \neq j. \tag{FOC_i}$$

We first show, in the absence of effort substitution, i.e., $\psi_{ij}(e_i, e_j) = 0$, that debt contract on each asset, i.e., $\alpha_i = 1$ emerges as an optimal contract. Suppose for some i that $\alpha_i < 1$, i.e., the feasibility constraint associated with α_i does not bind. Then the Karush–Kuhn–Tucker conditions imply that $\lambda_i = 0$. Then from the above equation it follows that $\psi_i(e_i, e_j) = E[\theta_i]$. From the incentive constraint (IC_i), it follows that

$$\alpha_i = \frac{E[\theta_i]}{[1 - F_i(\hat{\theta}_i)]E[\theta_i \mid \theta_i \geq \hat{\theta}_i]} = 1 + \frac{F_i(\hat{\theta}_i)E[\theta_i \mid \theta_i < \hat{\theta}_i]}{[1 - F_i(\hat{\theta}_i)]E[\theta_i \mid \theta_i \geq \hat{\theta}_i]} > 1,$$

which is a contradiction. Therefore, the optimal values of the agent's shares are given by $\alpha_1^* = \alpha_2^* = 1$. In other words,

Lemma 1. *In the absence of effort substitution, the optimal contract associated with each asset is a debt contract of the form*

$$D^i = \min\{R_i^*, \theta_i e_i^*\},$$

and the agent gets full share of revenue from each asset.

The optimal effort e_i^* in asset i is determined from the binding feasibility constraint (F'_i) by substituting $\alpha_i^* = 1$, and the level of optimal debt R_i^* associated with asset i is determined from the binding participation constraint. When the marginal cost of effort in one task does not depend on the effort exerted in the other, it is optimal for the principal to offer full incentives, i.e., $\alpha_1 = \alpha_2 = 1$. As a consequence, the agent exerts the highest incentive compatible efforts in both tasks. Hence, even for low realizations of productivity the principal can extract a high rent. This situation is equivalent to hiring two identical agents, one for each task.[4]

Next we show that a revenue sharing contract emerges in the presence of effort substitution, i.e., $\psi_{ij}(e_i, e_j) > 0$. Notice first that share contracts in both tasks, i.e., $\alpha_i < 1$ for $i = 1, 2$ cannot be optimal. If it were the case, then one must have $\lambda_1 = \lambda_2 = 0$. Hence, ($IC_i$) and ($FOC_i$) together would imply

$$\alpha_i = \frac{E[\theta_i]}{[1 - F_i(\hat{\theta}_i)]E[\theta_i \mid \theta_i \geq \hat{\theta}_i]} > 1,$$

a contradiction. Therefore, optimality implies that the contract associated with at least one asset must be a debt contract. We now analyze under what conditions we obtain revenue sharing contract in one of the two tasks. Suppose that $\alpha_i < 1$, while $\alpha_j = 1$. In this case $\lambda_i = 0$, and the first-order conditions reduce to:

$$E[\theta_i] - \psi_i(e_i, e_j) = \lambda_j\psi_{ij}(e_i, e_j), \tag{FOC'_i}$$

$$E[\theta_j] - \psi_j(e_i, e_j) = \lambda_j\psi_{jj}(e_i, e_j). \tag{FOC'_j}$$

And the feasibility condition associated with α_j, which is binding at the optimum, implies

$$[1 - F_j(\hat{\theta}_j)]E[\theta_j \mid \theta_j \geq \hat{\theta}_j] = \psi_j(e_i, e_j). \tag{F''_j}$$

From the above three equations we solve for e_i^*, e_j^* and λ_j^*. Substituting these optimal values into the incentive constraint (IC_i), we get α_i^*, as given in Box I. Clearly, $\alpha_i^* < 1$ if and only if the numerator of the fraction in the above expression is negative since the denominator is always positive as $\psi_{jj} > 0$. Therefore,

Proposition 1. *In the presence of effort substitution, a necessary and sufficient condition for $\alpha_i^* < 1$ and $\alpha_j^* = 1$ for $i, j = 1, 2$ and $i \neq j$ is*

$$\frac{F_i(\hat{\theta}_i)E[\theta_i \mid \theta_i < \hat{\theta}_i]}{F_j(\hat{\theta}_j)E[\theta_j \mid \theta_j < \hat{\theta}_j]} < \frac{\psi_{ij}(e_i^*, e_j^*)}{\psi_{jj}(e_i^*, e_j^*)}. \tag{M_i}$$

In the presence of effort substitution, higher effort in one task makes it more costly for the agent to exert an additional unit of effort in the other. Condition (M_i) is more likely to hold if either $F_j(\hat{\theta}_j)E[\theta_j \mid \theta_j < \hat{\theta}_j]$ is very high relative to $F_i(\hat{\theta}_i)E[\theta_i \mid \theta_i < \hat{\theta}_i]$, or ψ_{ij} is very high relative to ψ_{jj}. In the first case, low realization of revenue is on average less likely in task i than in task j. In the latter, a unit increase in e_i results in a relatively higher increase in the marginal cost of effort in task j than could be caused by a marginal increase in e_j. In both cases, in order to balance the moral hazard problems in two different tasks, it is optimal to incentivize the agent to exert higher effort in task j relative to task i. Notice that there is another equilibrium where $\alpha_1^* = \alpha_2^* = 1$, i.e., debt contract is optimal for each asset if and only if condition (M_i) holds with equality for each $i = 1, 2$.

4. Concluding remarks

In a principal-agent relationship with moral hazard and limited liability of the agent, revenue or profit sharing contracts in general do not appear to be optimal. The present paper makes an attempt to explain the existence of share contracts in the presence of limited liability when the agent exerts efforts in multiple tasks. We have shown that when the effort cost function is not separable in efforts, i.e., there is effort substitution effect, a revenue sharing contract as opposed to debt contract emerges in equilibrium. In the absence of effort substitution effect, a debt contract is optimal in each task.

In many contexts such as franchise contracts multitasking may be more frequently observed than situations where an asset is a taxi cab or a plot of land. But multitasking in contexts like tenancy relationships should not literally be interpreted as one tenant simultaneously cultivating two plots of land. Consider a situation of family farming where the landlord hires two tenants from the same family. High working hours by one family member may then imply

[4] One may wonder whether the full revenue share result holds even if the agent is risk-averse. It is well-known, under moral hazard and risk-aversion, that the optimal contract is an instrument to balance between risk sharing and incentive provision. Although a debt contract provides full incentives for the high realizations of revenue, it undermines the risk sharing motives. It can be shown in a very simple model that the agent will obtain less than a full share of revenue even if the efforts in different tasks are independent. Therefore, the agent's risk aversion is sufficient to explain the existence of revenue sharing non-debt contracts.

$$\alpha_i^* = \frac{\psi_{jj}(e_i^*,\ e_j^*)E[\theta_i] - \psi_{ij}(e_i^*,\ e_j^*)F_j(\hat{\theta}_j)E[\theta_j \mid \theta_j < \hat{\theta}_j]}{\psi_{jj}(e_i^*,\ e_j^*)[1 - F_i(\hat{\theta})]E[\theta_i \mid \theta_i \geq \hat{\theta}]}$$
$$= 1 + \frac{\psi_{jj}(e_i^*,\ e_j^*)F_i(\hat{\theta}_i)E[\theta_i \mid \theta_i < \hat{\theta}_i] - \psi_{ij}(e_i^*,\ e_j^*)F_j(\hat{\theta}_j)E[\theta_j \mid \theta_j < \hat{\theta}_j]}{\psi_{jj}(e_i^*,\ e_j^*)[1 - F_i(\hat{\theta})]E[\theta_i \mid \theta_i \geq \hat{\theta}]}.$$

Box I.

that the other tenant needs to spend more time in the household activities. Irrespective of the different interpretations that can be given to the present model, we show that a multitask agency model helps explain share contracts in the presence of limited liability.

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