

# Optimal Regulation of Access into Bottleneck Industries<sup>☆</sup>

Kaniška Dam\*

*Centro de Investigación y Docencia Económicas  
Carretera México-Toluca 3655, Mexico City, Mexico*

Axel Gautier

*Université de Liège  
7, Bd Rectorat (B31), Liège, Belgium*

Manipushpak Mitra

*Indian Statistical Institute  
203, B. T. Road, Kolkata, India*

---

## Abstract

We investigate how regulatory mechanisms influence the nature of competition in a network industry. In the downstream market, the seller of a differentiated retail product competes with an incumbent firm. The incumbent firm is also the owner of the essential input. The regulator does not observe the cost of the entrant. To maximize social welfare, the regulator designs the retail and access prices which is followed by the firms' decision to produce. We derive the optimal type-dependent retail and access prices that adhere to the traditional Ramsey rule. A duopoly or a monopoly emerges as an optimal market structure in equilibrium. Further, as compared with the symmetric information case, there exist suboptimal entry and a smaller likelihood of duopoly as an optimal market structure under asymmetric information. When the regulator is constraint to offer a non-discriminatory access charge, we show that both the retail and access prices are type-independent.

*JEL classification:* L51.

*Keywords:* One-way access; Ramsey pricing; endogenous market structure.

---

---

<sup>☆</sup>An earlier version of the paper has been circulated under the title “Efficient Access Pricing and Endogenous Market Structure” (see [Dam, Gautier, and Mitra, 2007](#)). We thank Paul Belleflamme, Marcelo Delajara, Perdo Pereira, Nicolas Petit, Jean Tirole, and the participants at the 12th Meeting of the Society for Social Choice and Welfare at Boston College for helpful comments on the earlier and current versions of the paper.

\*Corresponding author.

*Email addresses:* [kaniska.dam@cide.edu](mailto:kaniska.dam@cide.edu) (Kaniška Dam), [agautier@ulg.ac.be](mailto:agautier@ulg.ac.be) (Axel Gautier), [mmitra@isical.ac.in](mailto:mmitra@isical.ac.in) (Manipushpak Mitra)

## 1. Introduction

The optimal design of access prices aims at fostering competition in industries where competitors do not own the essential inputs of production, *bottleneck inputs* in the jargon of regulation. Examples of such input include local loop (in long distance telephony), transmission grid (in electricity generation), pipelines (in natural gas), tracks and stations (in railroad transportation) and local delivery network (in postal services). In many countries a bottleneck input is supplied by a monopolist. Presence of scale economies (due to high fixed costs of the network good) and absence of competing technologies (due to high costs of bypass) are the main reasons for such natural monopolies. In this case the end users of the bottleneck inputs (providers of retail services) pay access charge to cover the costs of the network. In the aforementioned industries, the monopoly owner of the essential input has incentives to charge a prohibitively high or even a discriminatory access price to foreclose the retail market in the case it also provides retail services. Thus, the regulation of access conditions in such industries is of utmost importance.

A typical problem the regulator often faces is that he has incomplete information about the costs of production of the essential inputs, and hence there is a potential adverse selection problem regarding the efficiency of the incumbent firm (the monopoly owner of an essential input). [Laffont and Tirole \(1994\)](#) analyze the problem of designing the optimal access price in a regulated industry where the incumbent firm competes with rival firm(s) in the retail sector. The regulator with limited budget offers a direct mechanism, which consists of the retail and access prices, to the incumbent with the objective of maximizing social welfare. The requirement of budget balance implies that the optimal retail prices adhere to the Ramsey pricing rule. It is often argued that the regulator possesses more information about the firm that is already established in the market than a firm who has not yet entered the market to compete. Hence, an alternative approach (e.g. [Lewis and Sappington, 1999](#); [Gautier and Mitra, 2008](#)) assumes that the regulator has perfect knowledge regarding the production technology of the incumbent, but is uncertain about the costs of production of the potential entrants into the retail market. In this paper we revisit the one-way access problem when the regulator does not possess perfect knowledge about the production cost of a potential entrant, an approach similar to the above mentioned works, which are fundamentally different from the traditional approach pioneered by [Laffont and Tirole \(1994\)](#).

In the present model, there are two competing firms in the downstream market which produce and sell differentiated goods. The incumbent firm is the monopoly owner of an essential input which the entrant firm uses to produce its good for which it pays a per unit access charge to the incumbent. The utilitarian regulator designs optimal retail prices and access charge in order to maximize social welfare. At the price design stage, the regulator does not know the cost conditions of a prospective competitor in the downstream market. The optimal retail prices are incentive compatible prices that aim at eliciting private information regarding the production technology of the entrant. After the regulator decides on the retail and access prices, firms decide to produce the retail goods. Clearly, this decision depends on the regulatory environment. Welfare maximization implies that the retail and access prices adhere to the Ramsey rule. Since the decision of the incumbent and the entrant to produce follows the announcement of the retail and access prices, the structure of the retail market becomes endogenous. The optimal mechanism implies that the downstream market can either be a duopoly or a monopoly. In particular, if the marginal cost of the entrant is very high, then the incumbent firm becomes a monopolist in the retail market. For very low marginal cost of the entrant, on the other hand, the incumbent stays out of the retail business. Only for intermediate values of the marginal cost of the entrant, duopoly emerges as an efficient market structure.

In designing the optimal mechanism, the regulator faces a clear trade-off between efficiency of the production technology of the entrant firm and the degree of substitutability of the downstream products. We show that the likelihood of duopoly as an optimal market structure is smaller as the retail goods become

more homogeneous. When the products are very differentiated, a welfare maximizing regulator allows the rival firm to enter the downstream firm to compete even if this firm is not sufficiently efficient. On the other hand, when the products are close substitutes, the regulator prefers a monopoly where the more efficient firm (either the incumbent or the entrant) serves the retail market. Although this is a common feature of the optimal regulatory mechanism irrespective of whether the regulator may observe the marginal cost of the entrant firm, under asymmetric information the requirements on efficiency for entry into the retail market are more stringent. In other words, under asymmetric information, one observes suboptimal entry in the sense that if the marginal cost of the entrant firm were observable then there should have been more types of this firm competing in the retail market compared to the number of types that actually enter under asymmetric information. Also, the likelihood of duopoly is lower under asymmetric information.<sup>1</sup>

The statutory rules of the competition and regulation authorities in almost all countries assert that the access prices must be ‘transparent’ (that is, the access price must be purely cost-based) and ‘non-discriminatory’ (that is, the owner of the network can neither deny access to any competitor, nor practice price discrimination with respect to the competitors’ demand and cost conditions).<sup>2</sup> We further analyze the optimal regulatory mechanism in which the regulator is constrained to offer a uniform access charge to the entrant firm in the downstream market. We show that, under the non-discriminatory access charge constraint, a separating contract does not exist if the mechanism has to be incentive compatible. In other words, a pooling contract for the entrant, i.e., uniform retail prices, is the only incentive compatible contract. Also, in this situation there exists a cut-off level of the marginal cost of the entrant firm beyond which no types of this firm enter the downstream market. To the best of our knowledge, this a new result in the literature on the optimal design of access charges.

The main contributions of the present paper thus are two-fold. First, we derive the formulae for Ramsey prices and access charge under different regulatory regimes when the cost conditions of the entrant firm is unknown to the regulator. This complements the earlier analyses in the extant literature (e.g. [Laffont and Tirole, 1994](#)) of optimal access prices when there is no uncertainty about the technology of the competitor. Second, our efficient retail and access prices also endogenize the downstream market structure as under different parameter configurations either duopoly or monopoly is efficient.

There are two main reasons to use regulated Ramsey prices as opposed to average cost based prices (e.g. [Fjell, Foros, and Pal, 2010](#)) in the present context. First, under unknown costs estimating demands are more accurate than estimating costs. Ramsey prices are often criticized on the ground that regulatory mechanisms based on demand elasticities are hard to implement especially in a very dynamic industry such as telecommunication where market conditions change rapidly. One way out is to allow the operators to choose prices freely under the constraint of price caps. [Laffont and Tirole \(1996\)](#) argue that under *global price caps*, i.e., when the computation of price cap includes the access charge, prices chosen by the profit-maximizing firms are equivalent to regulated Ramsey prices. [Billette De Villemeur, Cremer, Roy, and Toledano \(2003\)](#) also show such equivalence in the context of access regulation in the postal sector. A second reason thus to use Ramsey prices is that imposing a global price cap with appropriately chosen price-weights is a convenient way to decentralize the implementation of elasticity-based welfare maximizing prices. The fully regulated industry we consider – except for the entry decision of the competitor – is thus equivalent to an industry subject to global price cap regulation.

---

<sup>1</sup>In a context where the government designs auction to reward production rights to a public and a private firms, [Dana and Spier \(1994\)](#) find similar result.

<sup>2</sup>See, for example, the European competition directives on telecommunication (90/388/EEC), electricity (96/92/EC), gas (2003/55/EC), rail (2002/14/EC) and postal services (96/67/EC) that include a ‘non-discrimination’ clause for access charges.

A paper related to our approach is [De Fraja \(1999\)](#), which considers competition in homogeneous retail goods between an incumbent and entrant. The cost conditions of the incumbent firm is unknown, but the regulator has perfect knowledge of the marginal cost of the entrant, a situation exactly opposite to that of ours. As the goods are perfect substitutes, the optimal market structure (both under symmetric and asymmetric information) is always monopoly of one of the firms. Under a regulatory mechanism with discriminatory access price, [De Fraja \(1999\)](#) shows that the likelihood of monopoly of the rival firm increases under asymmetric information, and hence the access regulation is ‘pro-competitive’. We obtain an opposite result since the regulator does not observe the marginal cost of the rival, but possesses perfect knowledge about the production technology of the incumbent. [Lewis and Sappington \(1999\)](#), on the other hand, considers a similar approach to access pricing where the regulator is unaware of the production cost of the entrant firm. In their model, the access charge is type-dependent, but the downstream market structure is exogenously given where both the incumbent and the entrant produce positive quantity. [Lewis and Sappington \(1999\)](#) show that the regulator tends to subsidize more the cost of access of the more efficient type of the entrant in order to induce lower retail prices. The main differences of their model with the current one are the following. First, a duopoly as an optimal market structure is determined endogenously in our model. Second, the regulator subsidizes the access to all types of firm 2 equally since the optimal access charge is non-discriminatory.

Our approach is also related to the following literature on regulation of market structure. There is a class of models (e.g. [Auriol and Laffont, 1993](#); [Dana and Spier, 1994](#); [Jehiel and Moldovanu, 2004](#)) which assume that the regulator designs the market structure and selects the firms which are awarded the right to operate in the retail market as a function of their reported costs. Another class of models consider situations where the market structure is not designed by the regulator though the regulatory environment has a clear influence on the competitors’ behavior, especially its entry decision. [Caillaud \(1990\)](#) considers competition between a regulated network-based firm and a competitive fringe that uses an alternative technology to bypass the existing network. The fringe is active in this market depending on the cost of the alternative technology and the regulated price of the network-based firm. [Gautier and Mitra \(2008\)](#) consider a homogeneous product environment where the option to bypass is not available to the competitors. Depending on the incumbent’s regulated supply and the access conditions, an entrant may compete in or stay out of the retail market. In their model, entry may not be efficient. In all these models, entry decisions are taken once the regulatory mechanism is known but entry itself is not regulated. In that sense, the market structure is endogenous.

## 2. The model

We consider an economy with two firms. Firm 1, the incumbent, is a vertically integrated firm which owns a network good that cannot be cheaply duplicated, and produces a retail good/service. Firm 2 is a potential competitor in the retail market that produces and sells an imperfect substitute of the retail good produced by firm 1. Production of one unit of a retail good uses a unit of the network good. If the retail market is served by at least one firm, the incumbent has to produce positive amount of the network for which it incurs a fixed cost  $k_0$  and per unit cost  $c_0 > 0$ . The production of the retail good  $i$  involves a constant marginal cost  $c_i > 0$  for  $i = 1, 2$ . If firm  $i$  produces an amount  $x_i \geq 0$  of its retail good, then the total cost for firm 1 to provide network is  $k_0 + c_0(x_1 + x_2)$ . If firm 2 operates in the retail market, it pays a per unit access charge  $\alpha$  to firm 1.

The cost parameters  $k_0$ ,  $c_0$  and  $c_1$  of the incumbent are publicly observable, but the entrant’s marginal cost is private information. Let  $c_1, c_2 \in [\underline{c}_2, \bar{c}_2]$  where  $\underline{c}_2 \geq 0$ . The parameter  $c_2$  has a distribution function

$G(c_2)$  with density  $g(c_2) > 0$ , and monotone hazard rate, i.e.,  $m(c_2) := G(c_2)/g(c_2)$  is increasing in  $c_2$ . The distribution of  $c_2$  is common knowledge.

Consumers have quasi-linear preferences. The utility of the consumers from the downstream products is given by  $U(x_1, x_2)$ . Demand functions for the retail goods  $x_i(p_1, p_2)$  for  $i = 1, 2$  are obtained by maximizing  $U(x_1, x_2)$  net of the total expenditure,  $p_1x_1 + p_2x_2$ . We assume that  $\partial x_i/\partial p_i < 0$ , and  $\partial x_i/\partial p_j > 0$  for  $i, j = 1, 2$  and  $i \neq j$ .<sup>3</sup>

We consider an economy where a utilitarian regulator sets the retail prices  $p_1$  and  $p_2$ , and the access charge  $\alpha$  in order to maximize social welfare. The regulator also partly reimburses the incumbent the cost of producing the network good via a lump-sum transfer  $t$ . Firm 1 directly receives the total access receipts  $\alpha x_2$  paid by the entrant. The amount to be reimbursed is raised through distortionary taxes on consumption, and hence the welfare maximization problem induces Ramsey retail prices. In this environment, the only decision the firms make is whether or not to sell a positive quantity of its retail good. Since this decision follows the design of the regulatory mechanism, the structure of the retail market becomes endogenous. If both firms produces strictly positive quantities of the retail good, then the equilibrium market structure is a duopoly.

### 3. Optimal regulation under symmetric information

In this section, as a benchmark, we analyze the optimal regulatory mechanism and market structure under symmetric information. When the marginal cost of firm 2 is publicly known, the objective of the regulator is to maximize social welfare by offering a mechanism  $(p_1, p_2, \alpha)$  that specifies the optimal retail and access prices, and choosing which firm(s) must operate in the downstream market. Welfare is defined as the sum of consumer surplus and profits of firms 1 and 2. We adopt the accounting convention that the regulator collects the total sales revenue  $p_1x_1$  of firm 1, and reimburses the incumbent for incurring the total cost of producing the network via a monetary transfer  $t$ . This is of course without loss of generality. In order to reimburse firm 1 for producing the essential input, the regulator must raise a total amount of  $t + k_0 + c_0(x_1 + x_2) - (p_1 - c_1)x_1$  through distortionary taxes. We assume that the regulator faces a shadow cost of public funds  $\lambda > 0$ . This implies that if the regulator wants to raise \$1 to reimburse the network owner by taxing the consumers, then there is a loss of  $\$(1 + \lambda)$  in the consumer surplus.<sup>4</sup> The net consumer surplus thus is given by:

$$V(p_1, p_2) \equiv U(x_1, x_2) - p_1x_1 - p_2x_2 - (1 + \lambda)\{t + k_0 + c_0(x_1 + x_2) - (p_1 - c_1)x_1\}. \quad (1)$$

Given the regulatory mechanism, both the firms must break even. The regulator makes a transfer of amount  $t$  to the incumbent firm and this firm is paid a total access receipt  $\alpha x_2$  by the entrant. The sum of these two terms, which is its profit, must be non-negative.

$$\Pi_1(p_1, p_2, \alpha) := t + \alpha x_2(p_1, p_2) \geq 0. \quad (2)$$

The net profit of the entrant must also be non-negative, i.e.,

$$\Pi_2(p_1, p_2, \alpha) := (p_2 - c_2 - \alpha)x_2(p_1, p_2) \geq 0. \quad (3)$$

<sup>3</sup>When the utility function is  $U(x_i, x_j)$ , then  $U_i > 0$ ,  $U_{ii}, U_{ij} < 0$  and  $|U_{ii}| > |U_{ij}|$  are sufficient conditions for  $\partial x_i/\partial p_i < 0$ , and  $\partial x_i/\partial p_j > 0$  for  $i, j = 1, 2$  and  $i \neq j$ .

<sup>4</sup>We follow the same approach as [Laffont and Tirole \(1994\)](#) where the regulatory mechanism gives rise to Ramsey prices. In an alternative but similar approach (e.g. [Baron and Myerson, 1982](#)) one may assume that the regulator attaches weights  $1 - \gamma$  to the consumer surplus and  $\gamma$  to the producers surplus, but there are no distortionary taxes. The regulatory mechanism in such models leads to marginal cost pricing, as opposed to Ramsey prices, under symmetric information.

The above restrictions are the participation constraints of firms 1 and 2, respectively. Since along with the mechanism  $(p_1, p_2, \alpha)$ , the regulator also aims at determining the optimal market structure, i.e., which firm(s) must operate in the downstream market, the quantities produced must be non-negative, i.e.,

$$x_i(p_1, p_2) \geq 0 \quad (4)$$

For example, given the mechanism, if  $x_i(p_1, p_2) = 0$  and  $x_j(p_1, p_2) > 0$ , then firm  $j$  is a monopolist in the retail market. The optimal regulatory mechanism thus results from, subject to (2), (3) and (4), the maximization of  $V + \Pi_1 + \Pi_2$ . Since public funds are costly ( $\lambda > 0$ ), the participation constraint of firm 1 binds at the optimum. Also the access price  $\alpha$  is set to ensure that firm 2 breaks even, and hence the participation constraint of firm 2 is satisfied with equality. If one incorporates the above two (binding) constraints into the regulator's objective function, then it reduces to:

$$W(p_1, p_2) = U(x_1(p_1, p_2), x_2(p_1, p_2)) - p_1 x_1(p_1, p_2) - p_2 x_2(p_1, p_2) + (1 + \lambda) \{ \hat{p}_1 x_1(p_1, p_2) + \hat{p}_2 x_2(p_1, p_2) - k_0 \}, \quad (WSI)$$

where  $\hat{p}_i := p_i - c_0 - c_i$  for  $i = 1, 2$ . The regulator maximizes the above expression under the constraints that  $x_i(p_1, p_2) \geq 0$  for  $i = 1, 2$ . If both  $x_1$  and  $x_2$  are strictly positive, then both firms produce strictly positive quantities to serve the downstream demand, and the market structure is a duopoly. If firm 2 is active, i.e.,  $x_2 > 0$ , in the retail market (either as a duopolist or a monopolist) it pays an access charge to firm 1, which is given by  $\alpha = p_2 - c_2$ . The optimal retail prices  $p_i^*$  for  $i = 1, 2$  are a function of the marginal cost of firm 2, and hence are the equilibrium demands  $x_i^*$ , i.e., and  $x_i^* := x_i^*(c_2)$  for  $i = 1, 2$ . Let  $c_2^i$  be the cutoff value of  $c_2$  at which the demand faced by firm  $i$  is zero, i.e.,  $x_i^*(c_2^i) = 0$  for  $i = 1, 2$ . First, we prove a useful result which is stated in the following lemma.

**Lemma 1** *Let  $x_1^* = x_1^*(c_2)$  and  $x_2^* = x_2^*(c_2)$  be the equilibrium demands of firm 1 and firm 2, respectively. Then  $x_1^*$  is non-decreasing and  $x_2^*$  is non-increasing in  $c_2$ .*

*Proof* See Appendix. ■

In the optimal mechanism under symmetric information the retail price of firm 1 is non-increasing and that of firm 2 is non-decreasing in  $c_2$ . The result follows from the facts that  $\partial x_i / \partial p_i < 0$  and  $\partial x_i / \partial p_j > 0$ . Further, define by:

$$\hat{\eta}_i \equiv \frac{\eta_i(\eta_i \eta_j - \eta_{ij} \eta_{ji})}{\eta_i \eta_j + \eta_i \eta_{ij}} \quad \text{for } i, j = 1, 2, \text{ and } i \neq j$$

the *superelasticity* of good  $i$ , where  $\eta_i$  and  $\eta_{ij}$  are respectively the own and cross price elasticity of good  $i$ . The following proposition characterizes the optimal mechanism and market structures under symmetric information.

**Proposition 1** *Let  $(p_1^*, p_2^*, \alpha^*)$  be the optimal regulatory mechanism when the marginal cost of firm 2 is publicly observable.*

(1) *The optimal mechanism under symmetric information is given by:*

$$L_i^* := \frac{p_i^* - c_0 - c_i}{p_i^*} = \frac{\lambda}{1 + \lambda} \frac{1}{\hat{\eta}_i} \quad \text{for } i = 1, 2,$$

$$\alpha^* = c_0 + \frac{\lambda}{1 + \lambda} \frac{p_2^*}{\hat{\eta}_2}.$$

(2) *The optimal market structures under symmetric information are given by the following three cases. There exist  $c_2^1, c_2^2 \in [\underline{c}_2, \bar{c}_2]$  with  $c_2^1 < c_2^2$  such that*

- (2a) when  $c_2 \leq c_2^1$ , then the optimal market structure is the monopoly of firm 2;  
(2b) when  $c_2 \in (c_2^1, c_2^2)$ , the optimal market structure is duopoly;  
(2c) when  $c_2 \geq c_2^2$ , the optimal market structure is the monopoly of firm 1.

*Proof* See Appendix. ■

When firm  $i$  is active in the downstream market, its Lerner index  $L_i$  is inversely proportional to its superelasticity  $\hat{\eta}_i$ , which captures the fact that the firms produce and sell imperfect substitutes in the retail goods market.<sup>5</sup> The optimal access charge is given by the sum of the marginal cost  $c_0$  of producing the network good and the Ramsey term that depends on the retail price and the superelasticity of good 2. The optimal transfer to the incumbent firm is given by the following condition:

$$t^* + \alpha^* x_2(p_1^*, p_2^*) = 0.$$

Since  $\alpha^*$  and  $x_2^*$  are positive, the optimal transfer  $t^*$  is clearly negative. The fact that  $c_2^1 < c_2^2$  follows directly from Lemma 1. Assume  $c_2 = c_1$ . Since the retail goods are imperfect substitutes and there are no fixed costs associated with entry, the duopoly is the optimal market structure, i.e.,  $x_i^* > 0$  for  $i = 1, 2$ . Now increase  $c_2$  a bit from  $c_1$ . Due to Lemma 1,  $x_1^*$  will increase and  $x_2^*$  will decrease from their initial levels. Increasing further the value of  $c_2$  we will thus arrive at  $x_2^* = x_2^*(c_2^2) = 0$  which implies  $c_1 < c_2^2$ . In a similar fashion by decreasing  $c_2$  from  $c_1$ , we can show that  $c_2^1 < c_1$ . Therefore,  $c_2^1 < c_1 < c_2^2$ .

Now suppose that the demand functions for the retail goods 1 and 2 are linear, and are given by:<sup>6</sup>

$$x_i(p_i, p_j) = \frac{1}{1-\beta^2} [a - p_i - \beta(a - p_j)] \quad \text{for } i, j = 1, 2 \text{ and } i \neq j. \quad (5)$$

The parameter  $\beta$  represents the degree of substitutability and we assume that the goods are imperfect substitutes, that is  $\beta \in (0, 1)$ . Thus, higher values of  $\beta$  imply that the degree of substitutability is higher across retail goods. It is easy to show, under linear demands, that

$$\begin{aligned} c_2^1 &= \frac{1}{\beta} [c_1 - (1-\beta)(a-c_0)] < c_1, \\ c_2^2 &= \beta c_1 + (1-\beta)(a-c_0) > c_1. \end{aligned}$$

Notice that  $c_2^1$  is increasing and  $c_2^2$  is decreasing in  $\beta$ , i.e., as the retail goods become close substitutes, the likelihood of duopoly in equilibrium diminishes. In the polar case when  $\beta = 1$ ,  $c_2^1 = c_2^2 = c_1$ , i.e., when the products are perfect substitutes, duopoly is never an optimal market structure, and the retail market is served by the more efficient firm. These findings are summarized in the following corollary.

**Corollary 1** *Suppose the demand functions are linear, and given by (5). Then as the degree of product substitutability increases, a duopoly as an optimal market structure becomes less likely. If, at the limit, the products are perfect substitutes and  $c_1 < (>)c_2$ , then the incumbent (entrant) serves the entire retail market.*

<sup>5</sup>The representation of the optimal prices as in the above proposition is quite standard (e.g. Laffont and Tirole, 1994).

<sup>6</sup>The linear demand functions are obtained by maximizing the following quadratic utility function:

$$U(x_1, x_2) = a(x_1 + x_2) - \frac{1}{2}(x_1^2 + x_2^2) - \beta x_1 x_2,$$

with  $a \geq c_0 + c_1$  and  $\beta \in (0, 1)$ . For the derivation of the demand functions, see Singh and Vives (1984).

*Proof* See Appendix. ■

Obviously, when the products become very heterogeneous, i.e.,  $\beta$  decreases, a duopoly is more likely to emerge as an optimal market structure. In the limiting case when  $\beta \rightarrow 0$ , duopoly becomes the only optimal market structure.

#### 4. Optimal regulation under asymmetric information

In this section we assume that the regulator knows only the distribution function  $G(c_2)$  of the marginal cost of firm 2, not the true value of  $c_2$ . Since the marginal cost of firm 2 is not publicly observable, there is a potential adverse selection problem. A direct regulatory mechanism, denoted by  $\Gamma(c_2) = (p_1(c_2), p_2(c_2), \alpha(c_2))$ , will be offered on the basis announced marginal cost of firm 2.<sup>7</sup> The regulator chooses the retail prices and the access charge in order to maximize the expected social welfare. After being offered the regulatory mechanism, the firms decide whether to produce or not. Therefore, as in the case of symmetric information, the regulator also indirectly decides on the cut-offs for the optimal market structures.

##### 4.1. The optimal regulatory mechanism: menu contracts

Because of the potential adverse selection problem due to the non-observability of  $c_2$ , an optimal regulatory mechanism must be incentive compatible, i.e., in a direct mechanism firm 2 must truthfully reveal its marginal cost. We first analyze an optimal incentive compatible mechanism. Define by

$$\Pi_2(c'_2, c_2) := [p_2(c'_2) - c_2 - \alpha(c'_2)]x_2(p_1(c'_2), p_2(c'_2)),$$

which is the profit of firm 2 if it announces to have a marginal cost  $c'_2$  when its true marginal cost is  $c_2$ . Further, let  $\Pi_2(c_2) := \Pi_2(c_2, c_2)$ . An optimal regulatory mechanism [under asymmetric information] solves the following maximization problem.

$$\begin{aligned} \max_{\{p_1(c_2), p_2(c_2), \alpha(c_2)\}} & \int_{\underline{c}_2}^{\bar{c}_2} V(p_1(c_2), p_2(c_2))dG(c_2) \\ & + \int_{\underline{c}_2}^{\bar{c}_2} \Pi_1(p_1(c_2), p_2(c_2), \alpha(c_2))dG(c_2) \\ & + \int_{\underline{c}_2}^{\bar{c}_2} \Pi_2(c_2)dG(c_2) \end{aligned} \quad (W_{AI})$$

subject to

$$\Pi_1(p_1(c_2), p_2(c_2), \alpha(c_2)) \geq 0 \text{ for all } c_2 \in [\underline{c}_2, \bar{c}_2], \quad (PC_1)$$

$$\Pi_2(c_2) \geq 0 \text{ for all } c_2 \in [\underline{c}_2, \bar{c}_2], \quad (PC_2)$$

$$\Pi_2(c_2) \geq \Pi_2(c'_2, c_2) \text{ for all } c_2, c'_2 \in [\underline{c}_2, \bar{c}_2], \quad (IC_2)$$

$$x_1(p_1(c_2), p_2(c_2)) \geq 0 \text{ for all } c_2 \in [\underline{c}_2, \bar{c}_2], \quad (NN_1)$$

$$x_2(p_1(c_2), p_2(c_2)) \geq 0 \text{ for all } c_2 \in [\underline{c}_2, \bar{c}_2], \quad (NN_2)$$

The first two constraints are the participation constraints of the firms which say that each firm must earn non-negative profits. The third constraint is the incentive compatibility of firm 2 which asserts that this firm

---

<sup>7</sup>In general, one should also include  $t(c_2)$ , the transfer to the incumbent firm in the mechanism. Since our objective is to analyze the optimal retail and access prices, to save on notations we omit  $t(c_2)$  from the arguments of  $\Gamma(c_2)$ .

will truthfully reveal its type. The constraints  $(\text{NN}_1)$  and  $(\text{NN}_2)$  are the non-negativity restrictions on retail outputs. A regulatory mechanism  $\Gamma(c_2)$  that satisfies  $(\text{PC}_1)$ - $(\text{NN}_2)$  is called a *feasible* mechanism. Prior to analyzing the optimal regulatory mechanism, in the following lemma we characterize a feasible mechanism.

**Lemma 2** *A regulatory mechanism  $\Gamma(c_2)$  is feasible if and only if it satisfies the following conditions for all  $c_2 \in [\underline{c}_2, \bar{c}_2]$ :*

- (a)  $x_i(c_2) := x_i(p_1(c_2), p_2(c_2)) \geq 0$  for  $i = 1, 2$ ;
- (b)  $\Pi_1(p_1(c_2), p_2(c_2), \alpha(c_2)) \geq 0$ ;
- (c)  $\Pi_2(\bar{c}_2) \geq 0$ ;
- (d)  $x_2(c_2)$  is non-increasing in  $c_2$ ;
- (e)  $\Pi_2'(c_2) + x_2(c_2) = 0$ .

*Proof* See Appendix. ■

The above are a standard result in the mechanism design literature (e.g. [Baron and Myerson, 1982](#); [Guesnerie and Laffont, 1984](#)), which are useful for the subsequent analysis. Conditions (a) and (b) are simply the constraints  $(\text{NN}_1)$ ,  $(\text{NN}_2)$  and  $(\text{PC}_1)$  rewritten. Condition (c) asserts that, given a feasible mechanism, even the most inefficient type of firm 2 makes non-negative profits. Condition (d) is a *monotonicity* condition. Intuitively, if firm 2 reports a high marginal cost of production, then it must sell a lower quantity in the retail market since producing a higher quantity would be inefficient from view point of social welfare. Since  $x_2(c_2)$  is a non-increasing function of  $c_2$ , it is differentiable almost everywhere in  $[\underline{c}_2, \bar{c}_2]$ . The last condition is called the *local incentive compatibility constraint*. Notice that the incentive compatibility constraint  $(\text{IC}_2)$  is equivalent to:

$$\Pi_2(c_2) = \max_{c_2'} \Pi_2(c_2', c_2). \quad (\text{IC}'_2)$$

The local incentive compatibility constraint is precisely the first-order condition of the above maximization problem of the entrant firm, which must be satisfied at  $c_2' = c_2$ . The second-order condition of the maximization problem, evaluated at  $c_2' = c_2$ , is given by:

$$\Pi_2''(c_2) + 2x_2'(c_2) \leq 0. \quad (\text{SOC})$$

Local incentive compatibility implies that  $\Pi_2''(c_2) = -x_2'(c_2)$ , and hence  $x_2'(c_2) \leq 0$  is a necessary and sufficient condition for  $(\text{SOC})$  to hold. It also follows from condition (e) of the above lemma that

$$\Pi_2(c_2) - \Pi_2(\bar{c}_2) = \int_{c_2}^{\bar{c}_2} x_2(\theta) d\theta \equiv R(c_2). \quad (\text{R})$$

The term  $R(c_2)$  is the *informational rent* of each type  $c_2 < \bar{c}_2$  of firm 2. Clearly, this rent is zero for the least efficient type of firm 2, i.e.,  $c_2 = \bar{c}_2$ . Now consider any  $c_2 \in [\underline{c}_2, \bar{c}_2]$ . Since  $c_1$  is known, the regulator will set  $\Pi_1 = 0$  which implies that

$$t(c_2) = -\alpha(c_2)x_2(c_2) = \Pi_2(c_2) - [p_2(c_2) - c_2]x_2(c_2).$$

Using the above along with  $\Pi_1 = 0$ , we get the expression for welfare as follows:

$$\begin{aligned} W(p_1(c_2), p_2(c_2), \alpha(c_2)) &= V(p_1(c_2), p_2(c_2)) + \Pi_2(p_1(c_2), p_2(c_2), \alpha(c_2)) \\ &= U(x_1(p_1(c_2), p_2(c_2)), x_2(p_1(c_2), p_2(c_2))) \\ &\quad - \{p_1(c_2)x_1(p_1(c_2), p_2(c_2)) + p_2(c_2)x_2(p_1(c_2), p_2(c_2))\} \\ &\quad + (1 + \lambda)\{\hat{p}_1(c_2)x_1(p_1(c_2), p_2(c_2)) + \hat{p}_2(c_2)x_1(p_1(c_2), p_2(c_2))\} \\ &\quad - (1 + \lambda)k_0 - \lambda\Pi_2(c_2), \end{aligned} \quad (6)$$

where  $\hat{p}_i(c_2) := p_i(c_2) - c_0 - c_i$  for  $i = 1, 2$ . Since  $\Pi_2$  enters the objective function with a negative sign, we must have  $\Pi_2(\bar{c}_2) = 0$ . Condition (R) implies that the participation constraint (PC<sub>2</sub>) is always satisfied, and hence can be ignored. Integrating by parts condition (R) over  $c_2$ , we obtain

$$\int_{\underline{c}_2}^{\bar{c}_2} \Pi_2(c_2) dG(c_2) = \int_{\underline{c}_2}^{\bar{c}_2} x_2(c_2) m(c_2) dG(c_2). \quad (\star)$$

Using the above facts, the regulator's objective function reduces to:

$$\int_{\underline{c}_2}^{\bar{c}_2} \widehat{W}(p_1(c_2), p_2(c_2)) dG(c_2), \quad (W'_{AI})$$

where  $\widehat{W}(p_1(c_2), p_2(c_2))$  is equal to the following expression:

$$\begin{aligned} & U(x_1(p_1(c_2), p_2(c_2)), x_2(p_1(c_2), p_2(c_2))) \\ & - p_1(c_2)x_1(p_1(c_2), p_2(c_2)) - p_2(c_2)x_2(p_1(c_2), p_2(c_2)) \\ & + (1 + \lambda)\{\hat{p}_1(c_2)x_1(p_1(c_2), p_2(c_2)) + \hat{p}_2(c_2)x_2(p_1(c_2), p_2(c_2)) - k_0\} \\ & - \lambda x_2(p_1(c_2), p_2(c_2))m(c_2). \end{aligned}$$

Notice that the objective function is now independent of  $\alpha(c_2)$ . Hence, the constrained welfare maximization problem reduces to the maximization of  $(W'_{AI})$  with respect to  $p_1(c_2)$  and  $p_2(c_2)$  subject to the non-negativity constraints (NN<sub>1</sub>) and (NN<sub>2</sub>). Solving the above maximization problem, we first find the optimal values of  $p_1(c_2)$  and  $p_2(c_2)$ , and the cut-offs  $\hat{c}_2^1$  and  $\hat{c}_2^2$  for the optimal market structures where  $x_i(\hat{c}_2^i) = 0$  for  $i = 1, 2$ . In order to find the optimal regulatory mechanism, it only remains to determine the optimal access charge  $\alpha(c_2)$ , which solves equation (R) and is given by:

$$\alpha(c_2)x_2(c_2) = [p_2(c_2) - c_2]x_2(c_2) - R(c_2). \quad (A)$$

Finally, define by

$$z(c_2) := c_2 + \frac{\lambda}{1 + \lambda} m(c_2),$$

the *virtual* type (or marginal cost) of firm 2. Notice that  $z(c_2) \geq c_2$  for all  $c_2$  with equality at  $c_2 = \underline{c}_2$ . The following proposition characterizes the optimal mechanism and market structures under asymmetric information.

**Proposition 2** *Let  $(p_1(c_2), p_2(c_2), \alpha(c_2))$  be the optimal regulatory mechanism when the regulator cannot observe the marginal cost of firm 2.*

(1) *The optimal mechanism is given by:*

$$\begin{aligned} L_1(c_2) &:= \frac{p_1(c_2) - c_0 - c_1}{p_1(c_2)} = \frac{\lambda}{1 + \lambda} \frac{1}{\hat{\eta}_1(c_2)}, \\ \widehat{L}_2(c_2) &:= \frac{p_2(c_2) - c_0 - z(c_2)}{p_2(c_2)} = \frac{\lambda}{1 + \lambda} \frac{1}{\hat{\eta}_2(c_2)}, \\ \alpha(c_2) &= c_0 + \frac{\lambda}{1 + \lambda} \frac{p_2(c_2)}{\hat{\eta}_2(c_2)} + [z(c_2) - c_2] - \frac{R(c_2)}{x_2(c_2)}. \end{aligned}$$

(2) *The optimal market structures under asymmetric information are given by the following three cases. There exist  $\hat{c}_2^1, \hat{c}_2^2 \in [\underline{c}_2, \bar{c}_2]$  with  $\hat{c}_2^1 < \hat{c}_2^2$  such that*

(2a) *when  $c_2 \leq \hat{c}_2^1$ , then the optimal market structure is the monopoly of firm 2;*

- (2b) when  $c_2 \in (\hat{c}_2^1, \hat{c}_2^2)$ , the optimal market structure is duopoly;  
(2c) when  $c_2 \geq \hat{c}_2^2$ , the optimal market structure is the monopoly of firm 1.

*Proof* See Appendix. ■

The optimal values of the Lerner indices under asymmetric information depend on the marginal cost of firm 2 since the retail prices and quantities are now type-dependent. The Lerner index of firm 1 is similar to that under symmetric information. But it is different for the entrant firm whenever it is active in the downstream market. Notice that  $\hat{L}_2(c_2)$  in the above proposition is the virtual value of  $L_2(c_2)$  since the former depends on  $z(c_2)$ . The true Lerner index of firm 2, given its true type, can be written as

$$L_2(c_2) := \frac{p_2(c_2) - c_0 - c_2}{p_2(c_2)} = \underbrace{\frac{\lambda}{1 + \lambda} \frac{1}{\hat{\eta}_2(c_2)}}_{\text{Ramsey term}} + \underbrace{\frac{\lambda}{1 + \lambda} \frac{m(c_2)}{p_2(c_2)}}_{\text{Incentive-correction term}},$$

which is composed of a standard Ramsey term and an incentive-correction term. Notice first that for the most efficient type of firm 2, i.e.,  $c_2 = \underline{c}_2$ , the true and virtual values coincide implying the ‘no distortion at the top’ property. This is because there is no incentive correction necessary for the most efficient entrant. For the other types of firm 2 such correction is necessary because of the informational costs due to private information imposed by these types. When the entrant announces that it is of type  $c_2$ , the regulator offers a mechanism which is meant for its virtual value  $z(c_2)$ , which is greater than  $c_2$ , because he is interested in reducing such informational costs. Note also that the optimal type-dependent access charge also consists of an incentive correction term which is the sum of  $z(c_2) - c_2$  and  $R(c_2)/x_2(c_2)$ , apart from the standard Ramsey markup term. Finally, as in Proposition 1, using Lemma 1, we can show that  $\hat{c}_2^1 < \hat{c}_2^2$ , i.e., there the duopoly is an optimal market structure with a positive probability.

## 4.2. The optimal market structures

Proposition 2 suggests that when firm 2 is very efficient, i.e.,  $c_2 \leq \hat{c}_2^1$ , it will serve the retail market as a monopolist. On the other hand, for very high levels of inefficiency, firm 2 stays out of business. For intermediate values of efficiency, the optimal market structure is a duopoly. We will now compare these cut-off values with those obtained under symmetric information. Observe that informational asymmetry is equivalent to an increase in the entrant’s marginal cost equal to  $z(c_2) - c_2$ , and hence  $x_i > 0$  for  $i = 1, 2$  if  $z(c_2) \in (c_2^1, c_2^2)$ . Then by definition,  $z(\hat{c}_2^i) = c_2^i$ , which implies

$$z(\hat{c}_2^i) - \hat{c}_2^i = c_2^i - \hat{c}_2^i = \frac{\lambda}{1 + \lambda} m(\hat{c}_2^i) > 0 \implies \hat{c}_2^i < c_2^i. \quad (7)$$

Also, since  $m(c_2)$  is increasing in  $c_2$  and  $\hat{c}_2^1 < \hat{c}_2^2$ , from the above equation we have

$$\hat{c}_2^2 - \hat{c}_2^1 < c_2^2 - c_2^1. \quad (8)$$

The above findings are stated in the following proposition.

**Proposition 3** *As compared with the equilibrium under symmetric information, the optimal regulatory mechanism under asymmetric information implies that*

- (a) *there is always suboptimal entry and firm 2 is less likely to be a monopolist in the retail market;*
- (b) *the likelihood of duopoly as an optimal market structure is smaller.*

The probability under which firm 2 is allowed to operate in the retail market is given by  $G(\hat{c}_2^2)$  which is lower than  $G(c_2^2)$ , the probability under which firm 2 would have been allowed to enter the downstream market if its marginal cost were publicly observable. This occurs because of the private information possessed by this firm. Under symmetric information, the regulator designs the optimal mechanism taking the true value of the marginal cost of firm 2 into account; whereas, he takes the virtual marginal cost into account under the non-observability of  $c_2$ . Hence, entry is always suboptimal in the sense that a lower fraction of firm 2 is allowed to enter the retail market relative to the symmetric information equilibrium. In designing the optimal mechanism, the regulator clearly faces a trade-off between the efficiency of production of the retail goods and competitiveness of the retail market. When efficiency is a concern for the regulator, and he does not have perfect knowledge about the cost conditions of the rival firm, he is reluctant to let the more inefficient types of firm 2 operate in the downstream market. On the other hand, since  $\hat{c}_2^1 < c_2^1$ , the likelihood of the monopoly of firm 2 is also smaller. In this sense the optimal regulatory mechanism with non-discriminatory access charge is efficient but ‘anti-competitive’.

Notice also that the likelihood of a duopoly as an optimal market structure is also smaller under asymmetric information since

$$G(\hat{c}_2^2) - G(\hat{c}_2^1) < G(c_2^2) - G(c_2^1).$$

The term in the left-hand-side of the above inequality is the probability of duopoly under asymmetric information, while that in the right-hand-side is the probability of duopoly in the symmetric information equilibrium. Under the optimal regulatory mechanism with non-observability of marginal cost, firm 2 needs to be relatively more efficient to enter the retail market as well as to serve the market as a monopolist. In a context where the government designs auction to reward production rights to firms, [Dana and Spier \(1994\)](#) find similar result. Notice also that inequality (8) is equivalent to

$$c_2^1 - \hat{c}_2^1 < c_2^2 - \hat{c}_2^2,$$

i.e., the regulator requires firm 2 to be relatively more efficient [compared with the case of symmetric information] in order to let the firm enter the market than to let it operate as a monopolist. This is because the efficiency costs due to informational asymmetry is higher when  $c_2$  is relatively high. The following figure summarizes the above findings.

[Insert Figure 1 about here]

Conclusions similar to Corollary 1 can be drawn if we assume that the demand functions are linear as specified in (5). We further assume that  $G(\cdot)$  is a Uniform cumulative distribution function. With linear demands it is easy to compute the cut-off values for the optimal market structures, which are given by:

$$\hat{c}_2^i \equiv \left( \frac{1+\lambda}{1+2\lambda} \right) c_2^i + \left( \frac{\lambda}{1+2\lambda} \right) \underline{c}_2 \quad \text{for } i = 1, 2. \quad (9)$$

Notice first that

$$c_2^2 - \hat{c}_2^2 = \frac{\lambda}{1+2\lambda} \{c_2^2 - \underline{c}_2\}. \quad (10)$$

Since under linear demands,  $c_2^2$  is decreasing in  $\beta$ , the term in the left-hand-side of the above equation, which is the measure of inefficiency due to suboptimal entry, also decreases in  $\beta$ . Further, the equality in (8) implies that whenever  $c_2^2 - c_2^1 \rightarrow 0$  the difference  $\hat{c}_2^2 - \hat{c}_2^1$  also goes to 0. We know that the first difference of cut-offs decreases with  $\beta$ , and hence a higher  $\beta$  also implies a lower probability of duopoly under asymmetric information. Therefore,

**Proposition 4** *When the demand functions of the retail goods are linear and the marginal cost of the entrant firm is uniformly distributed, an increase in the degree of product substitutability implies (a) a decrease in the inefficiency of entry, and (b) a smaller likelihood of duopoly under asymmetric information.*

It is worth noting that, although the inefficiency of entry decreases as the retail products become more homogeneous, such inefficiency cannot be completely eliminated. Notice that  $c_2^2 = c_1$  for  $\beta = 1$ , and hence the difference in (10) remains strictly positive unless  $c_1 = c_2$ . In other words, if firm 1 is more efficient than the most efficient type of firm 2, the regulator will always deter entry of some types of the rival firm. Duopoly as an optimal market structure under asymmetric information disappears in the polar case when the products are perfect substitutes as has been the case with symmetric information. Notice that, at  $\beta = 1$ ,

$$\hat{c}_2^1 = \hat{c}_2^2 = \left( \frac{1 + \lambda}{1 + 2\lambda} \right) c_1 + \left( \frac{\lambda}{1 + 2\lambda} \right) c_2 < c_1,$$

which implies that even if monopoly is optimum, if firm 2 wants to serve the retail market it has to be ‘much more’ efficient compared to the incumbent firm. Note that, under symmetric information,  $c_2 < c_1$  was sufficient for firm 2 to be the monopolist. But under asymmetric information, the cost differential has to be much larger, i.e., ‘just being more efficient’ is not sufficient for firm 2 to drive the incumbent out of business in the retail market.

## 5. Regulation under non-discriminatory access charge: pooling contracts

In the previous section we have analyzed the optimal regulatory mechanism when the marginal cost of the entrant firm is unknown to the regulator. The optimal retail prices at the downstream market and the access charge have been shown to be dependent on the types or marginal cost of firm 2. In most of the countries type-dependent access charges are often deemed discriminatory as different types of users of the same services are obliged to pay different access charges. In other words, the access price charged to the entrant firm cannot be made contingent on the revealed cost of this firm.<sup>8</sup>

Lemma 2 and condition (R) together imply that a regulatory mechanism under symmetric information with type-dependent retail prices and a uniform access charge is clearly not incentive compatible. In other words, access charge must be discriminatory along with type-dependent retail prices in order to guarantee information disclosure by the entrant firm. The following lemma shows, in an incentive compatible mechanism, that the regulated retail prices must also be type-independent if the regulator is constrained to offer a non-discriminatory access charge to firm 2.

**Lemma 3** *In an incentive compatible regulatory mechanism the retail prices  $p_1$  and  $p_2$  must be constant with respect to  $c_2$  if the regulator is constrained to offer a uniform access charge. Moreover, there exists a*

---

<sup>8</sup>Discrimination in access is a sensible issue, especially when the access provider offers also competing downstream services. Whether second degree price discrimination (menu pricing) should be considered as discriminatory is an issue of debate. In 1998, Deutsche Bahn, the German train operator, introduced TPS 98 access tariff, which included two possible payment options for using the rail tracks: a two-part tariff and a flat charge. The Bundeskartellamt, the German Cartel Office, considered this scheme as discriminatory on the grounds that marginal and average prices differed across companies. Following that, Deutsche Bahn replaced this payment scheme by a single tariff (see Pittman, 2004). According to Article 102(c) of the European Treaty, the application by a dominant firm of dissimilar trading conditions to equivalent transactions is prohibited if it places some firms at a competitively disadvantageous position. This, per se, does not rule out second degree price discrimination by the access provider. However, the European Commission and the European Court of Justice have always taken a tough stance on discriminatory prices adopted by dominant firms which are not justified by cost saving (Motta, 2004, p. 499).

unique cut-off level  $\tilde{c}_2$  of the marginal cost of firm 2 such that no types of this firm with  $c_2 \geq \tilde{c}_2$  enter the retail market.

*Proof* Notice first that, under a non-discriminatory access charge, i.e.,  $\alpha(c_2) = \alpha$  for all  $c_2$ , a given type  $c_2$  of firm 2 will enter the retail market if this type makes a strictly positive profit, i.e.,

$$(p(c_2) - c_2 - \alpha)x_2(c_2) > 0.$$

The above inequality defines the cut-off  $\tilde{c}_2$  beyond which no types of firm 2 enter the downstream market. The cut-off is determined by:

$$p_2 - \tilde{c}_2 - \alpha = 0. \quad (11)$$

Then condition (R) reduces to:

$$(p_2(c_2) - c_2 - \alpha)x_2(c_2) = \int_{c_2}^{\tilde{c}_2} x_2(\theta) d\theta \quad \text{for all } c_2 \in [\underline{c}_2, \tilde{c}_2].$$

The above implies that  $\alpha = p_2(\tilde{c}_2) - \tilde{c}_2$ . Substituting for  $\alpha$  in the incentive compatibility constraint of any  $c_2 \in [\underline{c}_2, \tilde{c}_2]$  we get:

$$\begin{aligned} (\tilde{c}_2 - c_2)x_2(c_2) - (p_2(\tilde{c}_2) - p_2(c_2))x_2(c_2) &= \int_{c_2}^{\tilde{c}_2} x_2(\theta) d\theta \\ \implies \underbrace{\int_{c_2}^{\tilde{c}_2} \left[ \left(1 - \frac{dp_2(\theta)}{d\theta}\right) x_2(c_2) - x_2(\theta) \right] d\theta}_{H(c_2)} &= 0 \quad \text{for all } c_2 \in [\underline{c}_2, \tilde{c}_2]. \end{aligned} \quad (12)$$

From the above it follows that

$$\begin{aligned} \frac{dH(c_2)}{dc_2} &= - \left[ \left(1 - \frac{dp_2(c_2)}{dc_2}\right) x_2(c_2) - x_2(c_2) \right] = 0 \quad \text{for all } c_2 \in (\underline{c}_2, \tilde{c}_2), \\ \implies \left[ \frac{dp_2(c_2)}{dc_2} \right] x_2(c_2) &= 0, \\ \implies \frac{dp_2(c_2)}{dc_2} &= 0 \quad \text{for all } c_2 \in (\underline{c}_2, \tilde{c}_2) \end{aligned}$$

since  $x_2(c_2) > 0$ . The above implies that  $p_2(c_2) = p_2(\tilde{c}_2)$  for all  $c_2 \in [\underline{c}_2, \tilde{c}_2]$  which proves that  $p_2$  is constant with respect to  $c_2$ . Then from condition (12) we get:

$$\begin{aligned} (\tilde{c}_2 - c_2)x_2(c_2) &= \int_{c_2}^{\tilde{c}_2} x_2(\theta) d\theta, \\ \implies \int_{c_2}^{\tilde{c}_2} x_2(c_2) d\theta &= \int_{c_2}^{\tilde{c}_2} x_2(\theta) d\theta, \\ \iff \int_{c_2}^{\tilde{c}_2} [x_2(c_2) - x_2(\theta)] d\theta &= 0. \end{aligned}$$

Since  $x_2(c_2) \geq x_2(\theta)$  for all  $\theta \in [c_2, \tilde{c}_2]$  and  $\int_{c_2}^{\tilde{c}_2} [x_2(c_2) - x_2(\theta)] d\theta = 0$ , it follows that  $x_2(c_2) = x_2(\theta)$  for all  $\theta \in [c_2, \tilde{c}_2]$ . Moreover, since the last argument is true for  $c_2 = \underline{c}_2$ , it follows that  $x_2(c_2) \equiv x_2(p_1(c_2), p_2(c_2)) = x_2(p_1, p_2)$  for all  $c_2 \in [\underline{c}_2, \tilde{c}_2]$ . Finally, since  $x_2(c_2) = x_2$  and  $p_2(c_2) = p_2$  for all  $c_2 \in [\underline{c}_2, \tilde{c}_2]$ , it must be the case that  $p_1(c_2) = p_1$  for all  $c_2 \in [\underline{c}_2, \tilde{c}_2]$ . ■

Clearly, any type of firm 2 with marginal cost higher than  $\tilde{c}_2$  will make non-positive profits, and will not enter the retail market. All types that enter the market receive a contract where the retail prices and the access charge are not type-dependent. In other words, the optimal regulatory mechanism offers a pooling contract, which is trivially incentive compatible, as opposed to a separating contract described in the previous section. The intuition is as follows. As in any adverse selection setup when the type space is one dimensional, the regulator requires two instruments, namely  $x_2(c_2)$  and  $\alpha(c_2)$ , in order to guarantee information disclosure and participation by firm 2. A uniform access charge implies that the regulator lacks sufficient instruments, and hence the only way to make the mechanism incentive compatible is to offer a pooling contract to the entrant. Notice also that, for the regulator, choosing a uniform access price is equivalent to choosing the cut-off  $\tilde{c}_2$ . Therefore, the retail market is a duopoly with probability  $G(\tilde{c}_2)$ .<sup>9</sup>

Using equation (11), the expression for the expected welfare reduces to:

$$\begin{aligned}\tilde{W}(p_1, p_2, \tilde{c}_2) &= U(x_1(p_1, p_2), x_2(p_1, p_2)) - p_1 x_1(p_1, p_2) - p_2 x_2(p_1, p_2) \\ &\quad + (1 + \lambda)[\hat{p}_1 x_1(p_1, p_2) + \{(p_2 - c_0)G(\tilde{c}_2) - \mu(\tilde{c}_2)\}x_2(p_1, p_2) - k_0]\end{aligned}$$

which the regulator maximizes with respect to  $p_1$ ,  $p_2$  and  $\tilde{c}_2$ . The term  $\mu(\tilde{c}_2)$  is given by:

$$\mu(\tilde{c}_2) := \int_{c_2}^{\tilde{c}_2} z(c_2) dG(c_2) = G(\tilde{c}_2) E[z(c_2) | c_2 < \tilde{c}_2],$$

where  $E[z(c_2) | c_2 < \tilde{c}_2]$  is the conditional expectation of  $z(c_2)$  for  $c_2 < \tilde{c}_2$ . The following proposition describes the optimal retail and access prices in the optimal regulatory mechanism in which the regulator is constrained to offer a uniform access charge.

**Proposition 5** *Let  $(p_1, p_2, \alpha)$  be the optimal regulatory mechanism when the regulator cannot observe the marginal cost of firm 2, and is obliged to offer a uniform access charge to the entrant. The optimal mechanism is given by:*

$$\begin{aligned}L_1 &:= \frac{p_1 - c_0 - c_1}{p_1} = \frac{\lambda}{1 + \lambda} \frac{1}{\hat{\eta}_1}, \\ \tilde{L}_2 &:= \frac{G(\tilde{c}_2)\{p_2 - c_0 - E[z(c_2) | c_2 < \tilde{c}_2]\}}{p_2} = \frac{\lambda}{1 + \lambda} \frac{1}{\hat{\eta}_2}, \\ \alpha &= c_0 + [z(\tilde{c}_2) - \tilde{c}_2],\end{aligned}$$

where  $\tilde{c}_2$  solves  $p_2 - c_0 - z(\tilde{c}_2) = 0$ .

*Proof* See Appendix. ■

The Lerner index of the incumbent firm is the same as the one under symmetric information since there is no uncertainty over the technology of the incumbent firm and the regulatory mechanism does not depend on  $c_2$ . The Lerner index of firm 2 is expressed in terms of its expected virtual marginal cost conditional on entry. This is because the types of firm 2 that enter the downstream market, i.e.,  $c_2 < \tilde{c}_2$  are pooled in the optimal regulatory mechanism. Finally, the optimal access charge comprises of a mark-up over and above the marginal cost of producing the bottleneck input, which is equal to  $z(\tilde{c}_2) - \tilde{c}_2$ .

At this juncture, a few caveats are warranted.

---

<sup>9</sup>It is difficult to compare the likelihood of duopoly under uniform access charge with that under menu contracts since they are determined by two different sets of conditions. It can be shown that under linear demand system and uniform distribution of  $c_2$ , duopoly is less likely in the pooling mechanism than under the separating mechanism for some particular parameter configurations, but any general conclusions in this regard are hard to draw.

- When the regulator must offer a uniform access charge, an incentive compatible mechanism is necessarily a pooling mechanism, i.e., the retail prices and access charge are independent of  $c_2$ . An interesting question is to ask if, under a constraint of a non-discriminatory access charge, is there a way for the regulator to offer a separating mechanism which is incentive compatible. It turns out that for a separating mechanism to be optimal, the regulator needs another instrument to regulate firm 2, for instance the possibility to offer a type-contingent transfer to the entrant firm. Our objective is not to analyze the optimality of a separating mechanism, rather we are interested in analyzing the effects of uniform access charge on the efficient Ramsey prices, and their implications for the structure of the retail market as a non-discriminatory access constraint is often imposed to have a “fair” competition at the downstream level.
- Which of the two incentive compatible mechanisms (discriminatory or uniform) yields higher social welfare would also be an interesting research question. Often such comparison is called for in the context of cost based pricing. [Fjell, Foros, and Pal \(2010\)](#) show that when optimal access prices are based on average costs, under certain conditions, welfare under endogenous access charge is higher than that under exogenous (non-discriminatory) access price. In the present context such welfare comparison is not feasible since the optimal mechanisms under different regulatory regimes emerge under different sets of constraints.

## 6. Conclusions

In this paper we have analyzed how a utilitarian regulator determines the optimal retail and access prices when he is uncertain about the cost of producing the retail good by a potential competitor. Since the incumbent and entrant firms decide on how much to produce after observing the regulatory mechanism, the structure of the downstream market is endogenous. Whether a duopoly or a monopoly would emerge as an optimal market structure depends on the difference in efficiency of the firms that serve the downstream market. In a simple model where the downstream segment of a network industry consists of two firms, we have shown that if the entrant firm is neither too efficient nor too inefficient, it is optimal to allow both the firms to serve the retail market. In the polar case when the products are perfect substitutes (as shown in the case when the demands are linear), the retail market is served by only one firm depending on which firm is more efficient. When the marginal cost of the entrant is not publicly observable, the requirements on cost conditions for entry are more stringent since one of the objectives of the regulatory mechanism is to reduce inefficiency due to private information of the entrant firm. Therefore, the welfare maximizing retail prices are Ramsey prices which the regulator designs to fine-tune the nature of downstream competition.

The main focus of the current paper has been on the design of optimal regulatory mechanisms when the regulator cannot observe the entrant’s cost of production. In a regulated industry, the regulator designs the retail prices and access charge which depend on the marginal cost of the entrant firm. Since the design of optimal retail prices aims at elicit private information regarding the production technology the entrant firm, these prices are Ramsey prices that take into account the costs imposed by informational asymmetry. We consider also uniform access pricing scheme simply because a type-dependent access price is often viewed as a discriminatory practice by the regulation and competition authorities. We have shown that under a non-discriminatory access charge, the only incentive compatible mechanism is a pooling mechanism which implies that the retail prices are also independent of the types of the entrant firm.

As efficiency and endogenous market competition are the main concerns of our approach to the pricing of access to bottleneck inputs, the optimal pricing rule has some flavor of the *efficient component pricing rule* (ECPR). We improve upon such pricing rules in the following ways: (a) our rule is efficient because

it maximizes the expected social welfare by taking into account the social opportunity cost of network provision (the shadow cost of public funds influences the pricing rules), and (b) our access pricing formula can be implemented in a second-best world with differentiated products, a situation in which the regulator would not be able to implement the simplest version of the ECPR.

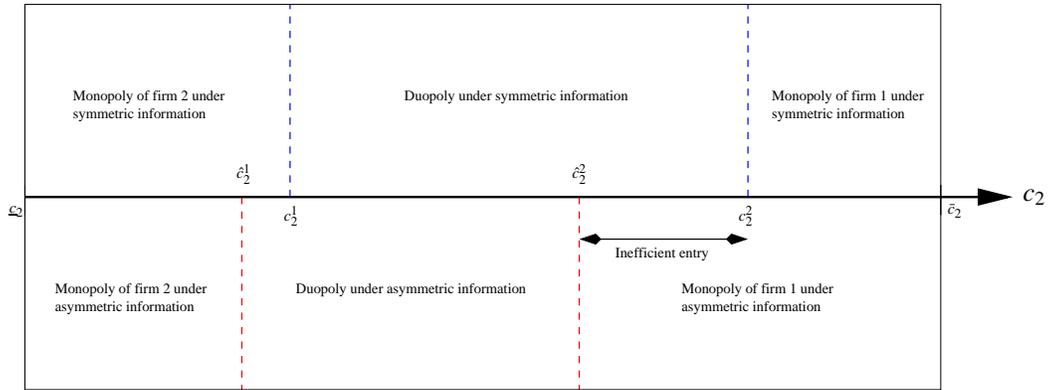


Figure 1: Equilibrium market structures

## Appendix

### Proof of Lemma 1

Consider the constrained welfare maximization problem which is given by:

$$\begin{aligned} & \max_{\{p_1, p_2\}} W(p_1, p_2; c_2), \\ & \text{subject to } x_1(p_1, p_2) \geq 0, \\ & \quad \quad \quad x_2(p_1, p_2) \geq 0. \end{aligned}$$

The Lagrange function is given by:

$$\mathcal{L}(p_1, p_2; c_2) = W(p_1, p_2; c_2) + \gamma_1 x_1(p_1, p_2) + \gamma_2 x_2(p_1, p_2),$$

where  $\gamma_1$  and  $\gamma_2$  are the corresponding Lagrange multipliers. By Envelope theorem, we have

$$\frac{\partial \mathcal{L}}{\partial c_2} = -(1 + \lambda)x_2(p_1, p_2).$$

Then,

$$\begin{aligned} \frac{\partial^2 \mathcal{L}}{\partial p_1 \partial c_2} &= -(1 + \lambda) \frac{\partial x_2}{\partial p_1} < 0, \\ \frac{\partial^2 \mathcal{L}}{\partial p_2 \partial c_2} &= -(1 + \lambda) \frac{\partial x_2}{\partial p_2} > 0, \end{aligned}$$

i.e.,  $\mathcal{L}(p_1, p_2; c_2)$  has *decreasing differences* in  $(p_1, c_2)$  and *increasing differences* in  $(p_2, c_2)$ . From [Topkis \(1998, chapter 2\)](#) it follows that  $p_1^*$  is non-increasing and  $p_2^*$  is non-decreasing in  $c_2$ .<sup>10</sup> Given the above, notice that

$$\begin{aligned} \frac{\partial x_1^*}{\partial c_2} &= \frac{\partial x_1^*}{\partial p_1} \frac{\partial p_1^*}{\partial c_2} + \frac{\partial x_1^*}{\partial p_2} \frac{\partial p_2^*}{\partial c_2} \geq 0, \\ \frac{\partial x_2^*}{\partial c_2} &= \frac{\partial x_2^*}{\partial p_1} \frac{\partial p_1^*}{\partial c_2} + \frac{\partial x_2^*}{\partial p_2} \frac{\partial p_2^*}{\partial c_2} \leq 0. \end{aligned}$$

The above hold since  $\partial x_i^*/\partial p_i < 0$  and  $\partial x_i^*/\partial p_j > 0$  for  $i \neq j$ .

### Proof of Proposition 1

Consider the demand functions  $x_1(p_1, p_2)$  and  $x_2(p_1, p_2)$  of firms 1 and 2. We assume that  $\partial x_i/\partial p_i < 0$  for  $i = 1, 2$ , and  $\partial x_i/\partial p_j > 0$  for  $i \neq j$ . We also assume the required properties of the demand functions such that the welfare function  $W(p_1, p_2)$  is concave. The welfare maximization problem of the regulator is given by:

$$\begin{aligned} & \max_{\{p_1, p_2\}} W(p_1, p_2) = U(x_1(p_1, p_2), x_2(p_1, p_2)) - p_1 x_1(p_1, p_2) - p_2 x_2(p_1, p_2) \\ & \quad \quad \quad + (1 + \lambda) \{ \hat{p}_1 x_1(p_1, p_2) + \hat{p}_2 x_2(p_1, p_2) - k_0 \} && \text{(W}_{SI}) \\ & \text{subject to } x_1(p_1, p_2) \geq 0, && \text{(NN}_1) \\ & \quad \quad \quad x_2(p_1, p_2) \geq 0. && \text{(NN}_2) \end{aligned}$$

<sup>10</sup>A function  $f(x, c)$  has increasing (decreasing) differences in  $(x, c)$  if  $f_{xc}(x, c) > (<) 0$ .

Let  $\gamma_1$  and  $\gamma_2$  be the Lagrange multipliers associated with (NN<sub>1</sub>) and (NN<sub>2</sub>). The Kuhn-Tucker conditions are given by:

$$[(1 + \lambda)\hat{p}_1 + \gamma_1] \frac{\partial x_1(p_1, p_2)}{\partial p_1} + [(1 + \lambda)\hat{p}_2 + \gamma_2] \frac{\partial x_2(p_1, p_2)}{\partial p_1} = -\lambda x_1(p_1, p_2), \quad (\text{FOC}_1)$$

$$[(1 + \lambda)\hat{p}_1 + \gamma_1] \frac{\partial x_1(p_1, p_2)}{\partial p_2} + [(1 + \lambda)\hat{p}_2 + \gamma_2] \frac{\partial x_2(p_1, p_2)}{\partial p_2} = -\lambda x_2(p_1, p_2), \quad (\text{FOC}_2)$$

$$x_1(p_1, p_2) \geq 0, \quad \gamma_1 \geq 0, \quad \gamma_1 x_1(p_1, p_2) = 0, \quad (\text{FOC}_3)$$

$$x_2(p_1, p_2) \geq 0, \quad \gamma_2 \geq 0, \quad \gamma_2 x_2(p_1, p_2) = 0. \quad (\text{FOC}_4)$$

Consider now the following cases:

**Case 1:** Monopoly of the entrant firm, i.e.,  $x_1(p_1, p_2) = 0$  and  $x_2(p_1, p_2) > 0$ . Then  $\gamma_1 \geq 0$  and  $\gamma_2 = 0$ . Then conditions (FOC<sub>1</sub>) and (FOC<sub>2</sub>) reduce to:

$$(1 + \lambda) \left[ \hat{p}_1 \frac{\partial x_1(p_1, p_2)}{\partial p_1} + \hat{p}_2 \frac{\partial x_2(p_1, p_2)}{\partial p_1} \right] = -\gamma_1 \frac{\partial x_1(p_1, p_2)}{\partial p_1} \quad (\text{FOC}'_1)$$

$$(1 + \lambda) \left[ \hat{p}_1 \frac{\partial x_1(p_1, p_2)}{\partial p_2} + \hat{p}_2 \frac{\partial x_2(p_1, p_2)}{\partial p_2} \right] = -\gamma_1 \frac{\partial x_1(p_1, p_2)}{\partial p_2} - \lambda x_2(p_1, p_2) \quad (\text{FOC}'_2)$$

Remember that  $L_i p_i = \hat{p}_i := p_i - c_0 - c_i$  for  $i = 1, 2$ . From the above, we have

$$\begin{aligned} & \frac{\lambda x_2}{\partial x_1 / \partial p_2} + (1 + \lambda) \left[ \hat{p}_2 \frac{\partial x_2 / \partial p_2}{\partial x_1 / \partial p_2} + \hat{p}_1 \right] = (1 + \lambda) \left[ \hat{p}_2 \frac{\partial x_2 / \partial p_1}{\partial x_1 / \partial p_1} + \hat{p}_1 \right] \\ \implies & \frac{\lambda x_2}{\partial x_1 / \partial p_2} + (1 + \lambda) \hat{p}_2 \left[ \frac{\partial x_2 / \partial p_2}{\partial x_1 / \partial p_2} - \frac{\partial x_2 / \partial p_1}{\partial x_1 / \partial p_1} \right] = 0 \\ \iff & L_2 p_2 \left[ \frac{\partial x_2 / \partial p_1}{\partial x_1 / \partial p_1} \frac{\partial x_1}{\partial p_2} - \frac{\partial x_2}{\partial p_2} \right] = \frac{\lambda x_2}{1 + \lambda} \\ \iff & L_2 \left[ \frac{p_2}{x_2} \frac{\partial x_2 / \partial p_1}{\partial x_1 / \partial p_1} \frac{\partial x_1}{\partial p_2} - \frac{p_2}{x_2} \frac{\partial x_2}{\partial p_2} \right] = \frac{\lambda}{1 + \lambda} \end{aligned} \quad (13)$$

Now analyze the term inside the square brackets in (13). Notice in equilibrium that  $x_1(p_1, p_2) = 0$  which defines implicitly  $p_1$  as a function  $p_1(p_2)$  of  $p_2$ . Then  $p'_1(p_2)$  is given by:

$$-p'_1(p_2) = \frac{\partial x_1 / \partial p_2}{\partial x_1 / \partial p_1}. \quad (14)$$

Hence the term inside the square brackets becomes

$$\frac{p_2}{x_2} \left[ -p'_1(p_2) \frac{\partial x_2}{\partial p_1} - \frac{\partial x_2}{\partial p_2} \right]. \quad (15)$$

Now, in equilibrium we have  $\tilde{x}_2(p_2) := x_2(p_1(p_2), p_2)$ . Then

$$\frac{d\tilde{x}_2(p_2)}{dp_2} = \frac{\partial x_2}{\partial p_1} p'_1(p_2) + \frac{\partial x_2}{\partial p_2} \iff -p'_1(p_2) \frac{\partial x_2}{\partial p_1} = \frac{\partial x_2}{\partial p_2} - \frac{d\tilde{x}_2(p_2)}{dp_2}.$$

Substitute the above into the expression in (15) which reduces to:

$$\tilde{\eta}_2 := -\frac{p_2}{\tilde{x}_2(p_2)} \frac{d\tilde{x}_2(p_2)}{dp_2}.$$

Then equation (13) gives

$$L_2^* := \frac{p_2^* - c_0 - c_2}{p_2^*} = \frac{\lambda}{1 + \lambda} \frac{1}{\tilde{\eta}_2}.$$

Given that  $\alpha^* = p_2^* - c_2$ , we get

$$\alpha^* = c_0 + \frac{\lambda}{1 + \lambda} \frac{p_2^*}{\tilde{\eta}_2}.$$

**Case 2:** Duopoly, i.e.,  $x_i(p_1, p_2) > 0$  for  $i = 1, 2$ . Then  $\gamma_1 = \gamma_2 = 0$ . Then (FOC<sub>1</sub>) and (FOC<sub>2</sub>) reduce to:

$$\hat{p}_1 \frac{\partial x_1(p_1, p_2)}{\partial p_1} + \hat{p}_2 \frac{\partial x_2(p_1, p_2)}{\partial p_1} = -\frac{\lambda x_1(p_1, p_2)}{1 + \lambda}, \quad (\text{FOC}'_1)$$

$$\hat{p}_1 \frac{\partial x_1(p_1, p_2)}{\partial p_2} + \hat{p}_2 \frac{\partial x_2(p_1, p_2)}{\partial p_2} = -\frac{\lambda x_2(p_1, p_2)}{1 + \lambda}, \quad (\text{FOC}'_2)$$

Since  $L_i p_i = \hat{p}_i$  for  $i = 1, 2$ , the above system reduces to:

$$\begin{bmatrix} \eta_1 & -\eta_{21} \left( \frac{p_2 x_2}{p_1 x_1} \right) \\ -\eta_{12} \left( \frac{p_1 x_1}{p_2 x_2} \right) & \eta_2 \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} = \begin{bmatrix} \frac{\lambda}{1 + \lambda} \\ \frac{\lambda}{1 + \lambda} \end{bmatrix}.$$

Solving the above system of equations we obtain

$$L_i^* := \frac{p_i^* - c_0 - c_i}{p_i^*} = \frac{\lambda}{1 + \lambda} \frac{1}{\hat{\eta}_i} \quad \text{for } i = 1, 2, \quad (16)$$

where  $\hat{\eta}_i$  is the superelasticity of firm  $i$ . It is easy to show that  $\hat{\eta}_i = \tilde{\eta}_i$  when  $x_j(p_i, p_j) = 0$  for  $i, j = 1, 2$  and  $i \neq j$ . Given that  $\alpha^* = p_2^* - c_2$ , we get

$$\alpha^* = c_0 + \frac{\lambda}{1 + \lambda} \frac{p_2^*}{\hat{\eta}_2}.$$

**Case 3:** Monopoly of the incumbent firm, i.e.,  $x_1(p_1, p_2) > 0$  and  $x_2(p_1, p_2) = 0$ . This case is similar to the case where the entrant firm is a monopolist. The Lerner index of firm 1 is given by:

$$L_1^* := \frac{p_2^* - c_0 - c_1}{p_1^*} = \frac{\lambda}{1 + \lambda} \frac{1}{\tilde{\eta}_1}. \quad (17)$$

In this case, firm 2 does not enter the market, and hence the optimal access charge  $\alpha^*$  remains unchanged. This completes the proof of the proposition.

#### *Proof of Corollary 1*

Define by  $a_i := a - c_0 - c_i$  and remember that  $\hat{p}_i := p_i - c_0 - c_i$ . Then the linear demand system is given by:

$$x_1(p_1, p_2) = \frac{1}{1 - \beta^2} [a - p_1 - \beta(a - p_2)] = \frac{1}{1 - \beta^2} [a_1 - \hat{p}_1 - \beta(a_2 - \hat{p}_2)], \quad (18)$$

$$x_2(p_1, p_2) = \frac{1}{1 - \beta^2} [a - p_2 - \beta(a - p_1)] = \frac{1}{1 - \beta^2} [a_2 - \hat{p}_2 - \beta(a_1 - \hat{p}_1)]. \quad (19)$$

From the above we get for  $i, j = 1, 2$  and  $i \neq j$ ,

$$\frac{\partial x_i(p_i, p_j)}{\partial p_i} = -\frac{1}{1-\beta^2},$$

$$\frac{\partial x_i(p_i, p_j)}{\partial p_j} = \frac{\beta}{1-\beta^2}.$$

First consider the case when firm 2 is a monopolist in the downstream market. Notice first that  $x_1(\hat{p}_1, \hat{p}_2) = 0$  implies  $\hat{p}_1 = \beta\hat{p}_2 + (a_1 - \beta a_2)$ . Substituting this and the values of the partial derivatives into the first order condition (FOC'<sub>1</sub>), we get

$$\gamma_1 = (1 + \lambda)(\beta a_2 - a_1).$$

Since  $\gamma_1 \geq 0$ , we have

$$\beta a_2 - a_1 = \beta(a - c_0 - c_2) - (a - c_0 - c_1) \geq 0 \implies c_2 \leq \frac{1}{\beta}[c_1 - (1 - \beta)(a - c_0)] \equiv c_2^1.$$

Next, consider the case when the optimal market structure is duopoly. Substituting the values of the partial derivatives,  $x_1(\hat{p}_1, \hat{p}_2)$  and  $x_2(\hat{p}_1, \hat{p}_2)$  into the first order conditions (FOC''<sub>1</sub>) and (FOC''<sub>2</sub>), and solving for  $\hat{p}_1$  and  $\hat{p}_2$  we get  $\hat{p}_i = [\lambda/(1 + \lambda)]a_i$ . Then the demands for goods 1 and 2 are respectively given by:

$$x_1^* = \frac{1 + \lambda}{1 + 2\lambda}(a_1 - \beta a_2),$$

$$x_2^* = \frac{1 + \lambda}{1 + 2\lambda}(a_2 - \beta a_1).$$

The facts that  $x_1^* > 0$  and  $x_2^* > 0$  imply that

$$c_2 > \frac{1}{\beta}[c_1 - (1 - \beta)(a - c_0)] \equiv c_2^1,$$

$$c_2 < \beta c_1 + (1 - \beta)(a - c_0) \equiv c_2^2.$$

Finally, consider the case when firm 1 is the monopolist, i.e.,  $x_2 = 0$  and  $\gamma_2 \geq 0$ . This case is symmetric to the case when there is monopoly of firm 2. It is easy to show that this case occurs if  $c_2 \geq c_2^2(c_1, c_2)$ .

Simple algebra shows that  $c_2^2 - c_2^1 = [(1 - \beta^2)a_1]/\beta > 0$ , and hence in the limiting case  $\beta = 1$  implies  $c_2^1 = c_2^2 = c_1$ . Finally, we have

$$\frac{\partial [c_2^2 - c_2^1]}{\partial \beta} = -\left(1 + \frac{1}{\beta^2}\right)a_1 < 0,$$

i.e., as  $\beta$  increases, the interval of values of  $c_2$  over which a duopoly market structure is optimum shrinks. This completes the proof of the corollary.

### *Proof of Lemma 2*

Notice first that conditions (a) and (b) are simply constraints (NN<sub>1</sub>), (NN<sub>2</sub>) and (PC<sub>1</sub>) rewritten. Hence, will prove the necessity and sufficiency of conditions (c)-(e). We first prove the necessity part. Since (PC<sub>2</sub>) holds for all  $c_2 \in [\underline{c}_2, \bar{c}_2]$ , in particular it holds for  $c_2 = \bar{c}_2$  which implies condition (c). Notice that

$$\Pi_2(c'_2, c_2) = [p_2(c'_2) - c'_2 + c'_2 - c_2 - \alpha(c'_2)]x_2(c'_2) = \Pi_2(c'_2) + (c'_2 - c_2)x_2(c'_2).$$

Hence, incentive compatibility implies that

$$\Pi_2(c'_2) - \Pi_2(c_2) \leq -(c'_2 - c_2)x_2(c'_2). \quad (20)$$

Similarly one can show that

$$\Pi_2(c'_2) - \Pi_2(c_2) \geq -(c'_2 - c_2)x_2(c_2). \quad (21)$$

Now suppose that  $c'_2 > c_2$ . Then from (20) and (21) we have

$$-(c'_2 - c_2)x_2(c_2) \leq \Pi_2(c'_2) - \Pi_2(c_2) \leq -(c'_2 - c_2)x_2(c'_2). \quad (22)$$

From (22) we get

$$(c'_2 - c_2)[x_2(c_2) - x_2(c'_2)] \geq 0. \quad (23)$$

Given  $c'_2 > c_2$ , we have  $x_2(c'_2) \leq x_2(c_2)$  as a necessary condition for feasibility. Since  $x_2(c_2)$  is non-decreasing, it is differentiable almost everywhere in its domain. Notice also that the incentive compatibility constraint can be written as

$$\Pi_2(c_2) = \max_{c'_2} \{ \Pi_2(c'_2) + (c'_2 - c_2)x_2(c'_2) \}. \quad (\text{IC}'_2)$$

The first- and second-order conditions of the above maximization problem are respectively given by:

$$\Pi'_2(c'_2) + (c'_2 - c_2)x'_2(c'_2) + x_2(c'_2) = 0, \quad (\text{FOC})$$

$$\Pi''_2(c'_2) + (c'_2 - c_2)x''_2(c'_2) + 2x'_2(c'_2) \leq 0. \quad (\text{SOC})$$

The above two conditions must be satisfied at  $c'_2 = c_2$  since (IC<sub>2</sub>) holds for all types of firm 2, and hence we have

$$\Pi'_2(c_2) + x_2(c_2) = 0, \quad (\text{FOC}')$$

$$\Pi''_2(c_2) + 2x'_2(c_2) \leq 0. \quad (\text{SOC}')$$

Condition (FOC') is the local incentive compatibility constraint. This completes the proof of necessity of conditions (c)-(e).

Now we prove the sufficiency part of the lemma. Since  $x_2(c_2) \geq 0$  for all  $c_2$ , and  $\Pi_2(\bar{c}_2) \geq 0$ , it follows from (R) that (PC<sub>2</sub>) is satisfied for all  $c_2 \in [c_2, \bar{c}_2]$ . To prove the sufficiency of conditions (d) and (e), suppose that these two conditions hold for all  $c_2 \in [c_2, \bar{c}_2]$ , and suppose on the contrary that for at least one type  $c_2$  the incentive compatibility is violated, i.e.,

$$\begin{aligned} \Pi_2(c'_2, c_2) &= \Pi_2(c'_2) + (c'_2 - c_2)x_2(c'_2) > \Pi_2(c_2) \\ \iff [\Pi_2(c'_2) - \Pi_2(c_2)] + (c'_2 - c_2)x_2(c'_2) &> 0 \text{ for at least one } c'_2 \neq c_2. \end{aligned}$$

Integrating the above we get

$$\int_{c_2}^{c'_2} [\Pi'_2(\theta) + x_2(c'_2)] d\theta > 0. \quad (24)$$

Now assume without loss of generality that  $c'_2 > c_2$ . Since (FOC') holds for all  $c_2$ , we have

$$x_2(c'_2) \leq x_2(\theta) \leq x_2(c_2) \text{ for all } \theta \in [c_2, c'_2].$$

The first of the above two inequalities and (FOC') together imply that

$$\Pi'_2(\theta) + x_2(c'_2) \leq 0 = \Pi'_2(\theta) + x_2(\theta).$$

Integrating the above we get

$$\int_{c_2}^{c'_2} [\Pi'_2(\theta) + x_2(c'_2)] d\theta \leq 0. \quad (25)$$

Inequalities (24) and (25) contradict each other. Now if  $c'_2 < c_2$ , then the same logic leads us to a similar contradiction. This establishes the sufficiency of conditions (d) and (e).

*Proof of Proposition 2*

After setting  $\Pi_2(\bar{c}_2) = 0$  and  $\alpha(c_2) = \alpha$  for all  $c_2$ , and integrating (R) by parts with respect to  $c_2$ , we obtain

$$\int_{c_2}^{\bar{c}_2} \Pi_2(c_2) dG(c_2) = \int_{c_2}^{\bar{c}_2} x_2(c_2) m(c_2) dG(c_2), \quad (\star)$$

Substituting the above, the regulator's maximization problem boils down to:

$$\begin{aligned} \max_{\{p_1(c_2), p_2(c_2)\}} \int_{c_2}^{\bar{c}_2} \widehat{W}(p_1(c_2), p_2(c_2)) dG(c_2), & \quad (W'_{AI}) \\ \text{subject to } x_1(p_1(c_2), p_2(c_2)) \geq 0, & \quad (\text{NN}_1) \\ x_2(p_1(c_2), p_2(c_2)) \geq 0, & \quad (\text{NN}_2) \end{aligned}$$

Now the above problem can be solved point-wise for each  $c_2$ . Define by  $\bar{p}_2(c_2) := \hat{p}_2(c_2) - z(c_2)$ . Let  $\gamma_1$  and  $\gamma_2$  be the Lagrange multipliers associated with (NN<sub>1</sub>) and (NN<sub>2</sub>). With slight abuse of notations, let  $p_i = p_i(c_2)$  and  $\hat{p}_i = \hat{p}_i(c_2)$  for  $i = 1, 2$ , and  $\bar{p}_2 = \bar{p}_2(c_2)$ . The Kuhn-Tucker conditions are given by:

$$[(1 + \lambda)\hat{p}_1 + \gamma_1] \frac{\partial x_1(p_1, p_2)}{\partial p_1} + [(1 + \lambda)\bar{p}_2 + \gamma_2] \frac{\partial x_2(p_1, p_2)}{\partial p_1} = -\lambda x_1(p_1, p_2), \quad (\text{foc}_1)$$

$$[(1 + \lambda)\hat{p}_1 + \gamma_1] \frac{\partial x_1(p_1, p_2)}{\partial p_2} + [(1 + \lambda)\bar{p}_2 + \gamma_2] \frac{\partial x_2(p_1, p_2)}{\partial p_2} = -\lambda x_2(p_1, p_2), \quad (\text{foc}_2)$$

$$x_1(p_1, p_2) \geq 0, \quad \gamma_1 \geq 0, \quad \gamma_1 x_1(p_1, p_2) = 0, \quad (\text{foc}_3)$$

$$x_2(p_1, p_2) \geq 0, \quad \gamma_2 \geq 0, \quad \gamma_2 x_2(p_1, p_2) = 0. \quad (\text{foc}_4)$$

The above first order conditions are very similar to those of the case of symmetric information. In the optimal solutions for  $(p_1(c_2), p_2(c_2))$  under asymmetric information,  $\hat{p}_1$  is replaced by  $\hat{p}_1(c_2)$ ,  $\hat{p}_2$  by  $\bar{p}_2(c_2)$ , and  $c_2$  by  $z(c_2)$ . Therefore, we omit the details of the equilibrium analysis. Using the first order conditions, and proceeding as in the case of symmetric information, we get the expressions for  $L_1$  and  $L_2$ , and the optimal access price under three different equilibrium market structures. Notice that the price elasticities of demand, and hence the superelasticities are now dependent on  $c_2$ .

*Proof of Proposition 5*

As in Section 4.1, the expected social welfare is given by the following expression:

$$\int_{c_2}^{\bar{c}_2} [U(x_1, x_2) - p_1 x_1 - p_2 x_2 + (1 + \lambda)(\hat{p}_1 x_1 + \hat{p}_2 x_2 - k_0) - \lambda x_2 m(c_2)] dG(c_2),$$

where  $\hat{p}_i := p_i - c_0 - c_i$  for  $i = 1, 2$ . Since we have a pooling equilibrium we have an entry cut-off  $\bar{c}_2$ . Let

$$\mu(\bar{c}_2) := \int_{c_2}^{\bar{c}_2} z(c_2) dG(c_2) = G(\bar{c}_2) E[z(c_2) | c_2 < \bar{c}_2].$$

Using  $\alpha = p_2 - \tilde{c}_2$  and the definition of  $\mu(\tilde{c}_2)$ , the regulator's objective function boils down to:

$$\begin{aligned} \tilde{W}(p_1, p_2, \tilde{c}_2) := & U(x_1(p_1, p_2), x_2(p_1, p_2)) - p_1 x_1(p_1, p_2) - p_2 x_2(p_1, p_2) \\ & + (1 + \lambda) [\hat{p}_1 x_1(p_1, p_2) + \{(p_2 - c_0)G(\tilde{c}_2) - \mu(\tilde{c}_2)\} x_2(p_1, p_2) - k_0]. \end{aligned}$$

To obtain the pooling equilibrium, the regulator needs to select  $p_1$ ,  $p_2$  and the entry cut-off  $\tilde{c}_2$  to maximize the above expression subject to (1)  $x_1(p_1, p_2) \geq 0$ , and (2)  $x_2(p_1, p_2) \geq 0$ . Let  $\tilde{p}_2 := (p_2 - c_0)G(\tilde{c}_2) - \mu(\tilde{c}_2)$ . The relevant first order conditions are given by:

$$[(1 + \lambda)\hat{p}_1 + \gamma_1] \frac{\partial x_1}{\partial p_1} + [(1 + \lambda)\tilde{p}_2 + \gamma_2] \frac{\partial x_2}{\partial p_1} + \lambda x_1 = 0, \quad (26)$$

$$[(1 + \lambda)\hat{p}_1 + \gamma_1] \frac{\partial x_1}{\partial p_2} + [(1 + \lambda)\tilde{p}_2 + \gamma_2] \frac{\partial x_2}{\partial p_2} + \lambda x_2 = 0, \quad (27)$$

$$p_2 = c_0 + z(\tilde{c}_2). \quad (28)$$

Solving the above system and using  $\alpha = p_2 - \tilde{c}_2$  we get the optimal retail prices, access charge and  $\tilde{c}_2$ .

## References

- Auriol, Emmanuelle and Jean-Jacques Laffont (1993), "Regulation by Duopoly." *Journal of Economics and Management Strategy*, 1, 507–533.
- Baron, David and Roger Myerson (1982), "Regulating a Monopolist with Unknown Costs." *Econometrica*, 50, 911–930.
- Billette De Villemeur, Etienne, Helmuth Cremer, Bernard Roy, and Joëlle Toledano (2003), "Optimal Pricing and Price-Cap Regulation in the Postal Sector." *Journal of Regulatory Economics*, 24, 49–62.
- Caillaud, Bernard (1990), "Regulation, Competition and Asymmetric Information." *Journal of Economic Theory*, 52, 87–110.
- Dam, Kaniska, Axel Gautier, and Manipushpak Mitra (2007), "Efficient Access Pricing and Endogenous Market Structure." CORE Discussion Paper 2007/4.
- Dana, James and Kathryn Spier (1994), "Designing a Private Industry. Government Auction with Endogenous Market Structure." *Journal of Public Economics*, 53, 127–147.
- De Fraja, Gianni (1999), "Regulation and Access Pricing with Asymmetric Information." *European Economic Review*, 43, 109–134.
- Fjell, Kenneth, Oystein Foros, and Debashis Pal (2010), "Endogenous Average Cost Based Access Pricing." *Review of Industrial Organization*, 36, 149–162.
- Gautier, Axel and Manipushpak Mitra (2008), "Regulation of an Open Access Essential Facility." *Economica*, 75, 662–682.
- Guesnerie, Roger and Jean-Jacques Laffont (1984), "A Complete Solution to a Class of Principal-Agent Problems with an Application to the Control of a Self-Managed Firm." *Journal of Public Economics*, 25, 329–369.
- Jehiel, Philippe and Benny Moldovanu (2004), "The Design of an Efficient Private Industry." *Journal of the European Economic Association, Papers and Proceedings*, 2-3, 516–525.
- Laffont, Jean-Jacques and Jean Tirole (1994), "Access Pricing and Competition." *European Economic Review*, 38, 1637–1710.
- Laffont, Jean-Jacques and Jean Tirole (1996), "Creating Competition Through Interconnection: Theory and Practice." *Journal of Regulatory Economics*, 10, 227–256.
- Lewis, Tracy and David Sappington (1999), "Access Pricing with Unregulated Downstream Competition." *Information Economics and Policy*, vol. 11, pp. 72–100.
- Motta, Massimo (2004), *Competition Policy: Theory and Practice*. Cambridge University Press.
- Pittman, Russell (2004), "Russian Railroad Reform and the Problem of Non-Discriminatory Access to Infrastructure." *Annals of Public and Cooperative Economics*, 75, 167–192.
- Singh, Nirvikar and Xavier Vives (1984), "Price and Quantity Competition in a Differentiated Duopoly." *RAND Journal of Economics*, 15, 546–554.
- Topkis, Donald (1998), *Supermodularity and Complementarity*. Princeton University Press.