A price theory of vertical and lateral integration under two-sided productivity heterogeneity*

Kaniṣka Dam† Konstantinos Serfes‡

Abstract

We analyze the interplay between product market prices and firm boundary decisions. Heterogeneous supplier units are matched to form enterprises. Each enterprise can choose between two ownership structures – centralized ownership (integration) performs well in coordinating managerial actions but ignores private benefits; dispersed ownership (non-integration), on the other hand, overvalues private benefits but is conducive to poor coordination. A more productive unit on one side of the supplier market matches with a more productive unit on the other side following a positive assortative matching pattern. The equilibrium ownership structure is monotone, i.e., high-revenue suppliers integrate while the low-revenue ones stay separate. Apart from the standard price effect we identify a novel utility effect that emerges from endogenous matching. Price can be positively or negatively correlated with the incidence of integration in the market. In the latter case, the industry supply may be backward-bending. Our model delivers new empirical and policy implications.

*We are grateful to Mark Armstrong (the editor) and two anonymous referees for insightful comments that helped improve on the earlier version of the paper. We would also like to thank Eva Chalioti, Berta Gamboa de Buen, Joseph Harrington, Matthew Nagler, David Pérez-Castrillo, Patrick Rey, Ramya Shankar, Jack Stecher, Chad Syverson, Galina Vereshchagina, Nikos Vettas, the seminar participants at CCNY, CIDE, SWUFE, the conference participants at CRETE 2013, SAET 2013 and EARIE 2014 for helpful suggestions, and Victor Hugo de la Vega for excellent research assistance. Dam acknowledges the research grant, "Fondo de Apoyo a la Investigación" from CIDE.

†Center for Research and Teaching in Economics (CIDE), 3655 Carretera Mexico-Toluca, Lomas de Santa Fe, 01210 Mexico City, Mexico. E-mail: kaniska.dam@cide.edu

‡Corresponding author. School of Economics, LeBow College of Business, Drexel University, 3220 Market Street, Philadelphia PA 19104. E-mail: ks346@drexel.edu

1 Introduction

There is a plethora of evidence on heterogeneity of firm productivity within an industry, which is also correlated with organizational variation at the firm level (e.g. Gibbons, 2010; Syverson, 2011) and endogenous sorting among firms (e.g. Hortaçsu and Syverson, 2007; Atalay, Hortaçsu, and Syverson, 2014). In this paper, we analyze the interplay between product market competition and firm boundary decisions (choice of ownership structure) under ex-ante productivity heterogeneity.[1] We intend to shed light on the following two basic sets of questions pertaining to the Organizational Industrial Organization (OIO) literature which lies at the intersection of organizational and industrial economics:

1. Does competition for heterogeneous suppliers in the input market imply that enterprises are formed following an assortative matching pattern? Do differences in productivity imply differences in organizational design?
2. To what extent changes in the market price explain the cross-sectional variation in modes of organization and production decisions of firms? In this setting, what is the effect of market price on aggregate output?

We analyze the interaction between a competitive input market, in which input suppliers are vertically differentiated and match with each other to form enterprises, and a perfectly competitive product market. Production of a homogeneous consumption good uses the assets owned by two distinct supplier units (A and B) run by a manager apiece. Within each enterprise, the managers choose between two modes of organization – *non-integration* where each manager possesses the decision right, and *integration* where the decision right is conferred on an outsider. When the individual supplier units decide on their actions, the managers tend to overvalue private costs arising from non-verifiable actions against the socially valuable enterprise revenue, and hence, such organization is conducive to poor coordination. In an integrated firm, on the other hand, managerial actions are well coordinated because the outsider maximizes revenue, but private costs are ignored. Hart and Hölmstrom (2010) present this generic trade-off between the two modes of organization which arises from purely coordination motives.[2] Neither organizational structure is efficient; utility between the two suppliers under integration is transferred perfectly, while under non-integration it is transferred imperfectly. Balanced revenue shares between two non-integrated suppliers (i.e., close to $50\% - 50\%$), and hence balanced utilities, provide the strongest incentives for coordination, making non-integration more likely to be the preferred structure. By contrast, more unbalanced revenue share or utility allocation induces integration.

When heterogeneous suppliers match with each other endogenously, in the equilibrium of the market high-productivity suppliers form enterprises following a positively assortative matching pattern irrespective of the organizational mode they choose. An enterprise comprising of high-productivity A-

[1] Two-sided productivity heterogeneity and matching have also become increasingly important in the context of international trade, as there are easily available datasets with detailed information about the matching between exporters and importers (e.g. Bernard, Moxnes, and Ulltveit-Moe, 2014; Dragusanu, 2014; Sugita, Teshima, and Seira, 2015).

[2] Atalay et al. (2014) offer empirical support for such trade-off. They find that very few of the inputs produced by one subsidiary are shipped to other subsidiaries of the same firm, implying that the motive for integration is something different from the standard elimination of double marginalization. Rather, integration facilitates efficient transfers of intangible inputs (e.g., managerial oversight) within firms. Alonso, Dessein, and Matouschek (2008) analyze the optimal choice of ownership structure which is motivated by the trade-off between coordination and adaptation. Integration induces better coordination of managerial actions at the expense of poor adaptation to local uncertainty. By contrast, under non-integration, communication is cost-less and the managers are able to accurately adapt decisions to a changing environment.

and B-suppliers (through endogenous matching) generates high revenue as opposed to the matches of low-productivity supplier units. Therefore, ex-ante exogenous heterogeneity of input suppliers implies ex-post endogenous heterogeneity of firms. Because in the high-revenue firms there is greater incentive for coordination, high-productivity suppliers integrate, whereas the low-productivity ones stay separate.

We assume A-suppliers have all the bargaining power, making take-it-or-leave it offers to B-suppliers. As we alluded to the above, the total revenue under non-integration depends on how balanced the utility allocations are across the two suppliers. A higher product market price induces the bargaining (Pareto) frontier of each enterprise to shift out both under integration and non-integration. If the utility of B-suppliers is fixed at low levels, and hence the utility of A-supplier is high, a higher price results in even more unbalanced utilities, making integration more likely to dominate. The converse is true when the utility of B-supplier is fixed at high levels. This is the *price effect*, which rests on the assumption that the utility of B-supplier remains unaltered following a change in the market price. There is a second novel effect, which we call a *revenue sharing* or *utility effect*. It emerges, very naturally, when input suppliers of heterogeneous productivities match with each other and the shares are determined *endogenously* to ensure a stable market equilibrium. An increase in the product market price increases the equilibrium utility of the B-suppliers. As the price and utility effects may pull in opposite directions, the net effect of an increase in price depends on which of the two countervailing forces is stronger. We show that market price can be either *negatively* or *positively* correlated with integration, depending upon how competitive the market is (i.e., the level of the market price).

Perfect coordination in an integrated firm implies that the output produced in such firms is independent of the market price and greater than that in a non-integrated firm. Therefore, the price effect implies that there is more aggregate output after the market price has increased, holding the organizational structures in the market fixed. The negative correlation between price and integration, on the other hand, introduces a second opposing force on aggregate output. If the second force is stronger, then industry output may decrease with price. In other words, the organizationally augmented supply curve (OAS) can be *backward-bending* due to the organizational restructuring a price change triggers.

In the present model, the bargaining set of an enterprise may be non-convex due to the possibility of choosing between two different modes of organization. Thus, we require stronger complementarity conditions to guarantee positive sorting, which may be of independent interest for analyzing matching under imperfect transferability arising due to contract incompleteness.

Related literature. The literature on Organizational Industrial Organization, which is concerned with how market structure affects firm boundaries and vice versa, is still in its early stages of development.[3] Our paper adds to the recent contributions to this strand of literature.[4] Legros and Newman's (2013) work is the closest to ours; they also build on Hart and Hölmstrom (2010) and analyze how changes in the price level of a perfectly competitive product market affect the enterprises' decision to integrate when the input market comprises of identical supplier units. They show the co-existence of both ownership structures in the same marketplace even if the supplier units are ex-ante identical. An exogenous variation in the enterprise revenue implies high-revenue firms are more likely to integrate.

[3]See Legros and Newman (2014) for an excellent survey. As these authors argue: "Nascent efforts at developing an OIO already suggest that market conditions or industrial structure matter for organization design. At the same time, organizational design will affect the productivity of firms, hence eventually the total industry output, the quality of products and information about this quality for consumers. Organizational design matters for consumers, hence for IO."

[4]There is another strand of literature which studies the effect of market competition on the incentive structure in an organization (e.g. Besley and Ghatak, 2005; Sørensen, 2007; Terviö, 2008) under two-sided heterogeneity and endogenous matching.

A higher market price increases the foregone revenue when firms remain unintegrated and therefore it increases the incentives for integration (when the utility of the supplier who receives the offers is fixed at zero, i.e., a low level). This is the standard price effect which is also present in the current model. We differ from Legros and Newman (2013) in the following respects. First, we allow input suppliers (units) to be ex-ante heterogeneous, and hence, competition for "good-quality" units arises naturally which leads to an endogenous matching and an endogenous distribution of enterprise revenue. In contrast, the issue of endogenous matching does not emerge in a model with ex-ante homogeneous suppliers. Even when Legros and Newman (2013) allow for ex-ante heterogeneity in firm productivities, because the matching is exogenous, all supplier units who receive offers consume their reservation payoff (zero). In light of our framework, the same utility allocation cannot be part of a stable equilibrium since more productive units must receive higher utility. This generates a novel revenue sharing or utility effect which in general pulls in opposite direction relative to the price effect. Therefore, in Legros and Newman (2013), higher price always implies more integration and greater industry output, whereas in our model price may be negatively correlated with integration and the industry supply curve may be backward-bending. Our model is thus flexible enough, in the sense that it can be applied to markets with any degree of firm heterogeneity and with general distributions of firm productivities. We obtain Legros and Newman's (2013) model as the limit of our model, i.e., when firm heterogeneity vanishes.

Grossman and Helpman (2002) analyze a market for differentiated products with free entry where firms can either vertically integrate or become specialized producers by outsourcing.[5] Large governance costs in the integrated firms are balanced against costs arising from a holdup problem (as in Grossman and Hart, 1986) and search for suitable partners under outsourcing. Thus, the trade-off between the two modes of organization does not arise from coordination motives. In the equilibrium of the market with identical participants, either all firms vertically integrate or there is pervasive outsourcing, except in a knife-edge case, as opposed to the robust co-existence of both ownership structures in our model due to two-sided heterogeneity. Extreme revenue shares for the intermediate good producers makes an equilibrium with integration more likely to occur because low share implies excess demand for intermediate inputs and high share, on the other hand, increases the search friction due to excessive entry of the specialized input producers. Thus, pervasive outsourcing is sustainable as an industry equilibrium if the (exogenous) distribution of bargaining power among the intermediate and final good producers is more balanced. It is worth noting that the revenue sharing effect in Grossman and Helpman (2002) is exogenous and does not interact with the degree of product market competition.

Gibbons, Holden, and Powell (2012) obtain generic heterogeneity of ownership by analyzing a rational-expectations equilibrium of price formation and endogenously chosen governance structures. They show that the informativeness of the price mechanism can induce ex-ante homogeneous firms to choose heterogeneous governance structures. Aghion, Griffith, and Howitt (2006) provide evidence of a U-shaped relationship between product market competition and vertical integration. Integration is more likely when competition is either soft or intense. In addition to the role of product market competition, we emphasize the significance of competition for heterogeneous supplier units in the input market in an endogenous matching framework.

Alfaro, Conconi, Fadinger, and Newman (2016) use changes in trade policy, e.g. tariffs, as an exogenous source of price variation. They find empirical evidence that the level of product prices do affect vertical integration decisions. Acemoglu, Griffith, Aghion, and Zilibotti (2010) show that technology intensity affects the likelihood of integration. They find that technology intensity of the downstream

[5] Lafontaine and Slade (2007) review the findings of empirical studies on vertical integration.

producers is positively correlated with integration, while technology intensity of the upstream suppliers is negatively correlated. Our model also disentangles the contributions of the two units to overall firm productivity.

2 The Model

2.1 Technology and matching

Our model builds on Hart and Hölmstrom (2010) and Legros and Newman (2013) by introducing two-sided productivity heterogeneity. On each side of the market there is a continuum of input suppliers of measure 1. Each supplier is a collection of assets and workers overseen by a manager. Suppliers are vertically differentiated.[6] In particular, let $J_A = [0, 1]$ be the set of "A-suppliers" on one side of the market and $J_B = [0, 1]$, the set of "B-suppliers" on the other side. Each supplier $i \in J_A$ is assigned a type or 'productivity' $a = a(i) \in A$ and each $j \in J_B$ has an assigned type $b = b(j) \in B$ where the type spaces $A = [a_{min}, a_{max}]$ and $B = [b_{min}, b_{max}]$ are subintervals of $\mathbb{R}_{++}$. Let $G(a)$ be the fraction of A-suppliers with productivity lower than a, i.e., $G(a)$ is the cumulative distribution function of a with the associated density function $g(a) > 0$ for all $a \in A$. Similarly, let $F(b)$ be the distribution function of b with the associated density function $f(b) > 0$ for all $b \in B$.

Production of a homogeneous consumer good requires one A-supplier and one B-supplier who are matched one to one to form an 'enterprise'. All decisions and payoffs of each enterprise will only depend on the types of the two participating units, and hence, a typical enterprise will be denoted by (a, b). A matching is a one-to-one mapping $\beta : A \to B$ which assigns to each $a \in A$ a type $b = \beta(a) \in B$. Let $\alpha \equiv \beta^{-1}$ denote the inverse matching function. Such enterprises may include lateral as well as vertical relationships. The stochastic output of an enterprise (a, b) is given by:

$$\tilde{y}(a, b) = \begin{cases} z(a, b) & \text{with probability } \pi(e_A, e_B) \equiv 1 - (e_B - e_A), \\ 0 & \text{otherwise.} \end{cases}$$

The success output $z(a, b)$ can be thought of as the productivity of an enterprise (a, b).[7] We assume that $z(a, b)$ is twice continuously differentiable and strictly increasing on $A \times B$. Moreover, $z(a, b)$ is assumed to be *log-supermodular* in (a, b), i.e., $zz_{ab} \geq z_a z_b$ for all $(a, b) \in A \times B$.[8] Each supplier must make a non-contractible production decision: $e_A \in [0, 1]$ by an A-supplier and $e_B \in [0, 1]$ by a B-supplier. These decisions can be made by the manager of the assets or by someone else. If the two managers coordinate their decisions and set $e_A = e_B$, then inefficiencies disappear and the enterprise reaches its full

[6] Bloom and van Reenen (2007), using a survey from medium-sized manufacturing firms from four countries, document that management practices are heterogeneous and affect firm performance. Gibbons (2010) offers a more detailed account of various empirical studies that document persistent performance differences (PPDs). In the computer industry, computer systems manufacturers rely on networks of independent component suppliers. These suppliers are of various 'qualities' and produce components that are used as inputs in the production of the final product (see Fallick, Fleischman, and Rebitzer, 2006).

[7] Legros and Newman (2013) assume a quadratic probability of success of the form $\pi(e_A, e_B) \equiv 1 - (e_A - e_B)^2$. We use a linear one instead for tractability. The linear probability of success is everywhere below the quadratic one implying that non-integration is more profitable under the quadratic probability. That is why in Legros and Newman (2013) integration is never a strictly dominant organizational mode for all (a, b, u) and all P. Nevertheless, the important qualitative features of the model are not affected by the choice of linear probability.

[8] We use subscripts to denote partial derivatives.

potential $z(a, b)$ with probability 1. The manager of each supplier unit is risk neutral and incurs a private cost for the managerial action.[9] The private cost of an A unit is ce_A^2 and that of a B unit is $c(1-e_B)^2$ with $c > 0$. Clearly, there is a disagreement about the direction of the decisions, what is easy for one is hard for the other.[10] Also, managers with zero cash endowments are protected by limited liability, i.e., their state-contingent incomes must always be nonnegative. The importance of this assumption is that the division of surplus between the managers will affect the organizational choice.

2.2 Ownership structures and contracts

The ownership structure can be contractually determined. We assume two different options, each of which implies a different allocation of decision rights. First, the production units can remain separate firms (the *non-integration* regime, denoted by N). In this case, managers have full control over their decisions. Second, the two input suppliers can integrate, a regime denoted by I, into a single firm, giving control over managerial decisions, e_A and e_B, to a third party who always has enough cash to finance the acquisition. The third party is motivated entirely by income and incurs no costs from the managerial decisions. These costs are still borne by the managers. As argued by Hart and Hölmstrom (2010), integration results in an organization where less weight is placed on private benefits than under non-integration. This, however, is offset by the fact that under integration total profit, rather than individual unit profits, is maximized.[11]

Each enterprise's revenue is publicly verifiable, and hence, contractible. We assume that each A-supplier has all the bargaining power in an arbitrary enterprise (a, b) and makes take-it-or-leave-it offers to the B-supplier. A contract $(s, d) \in [0, 1] \times \{N, I\}$ specifies a revenue share s for the B-supplier and an ownership structure d. Consider an arbitrary enterprise (a, b). If the members of this enterprise stay separate, then a revenue-sharing contract is simply a share s of the total revenue that accrues to the B-supplier. As we assume limited liability, the units get nothing in the case of failure.

When the two units integrate, a third party, called the *headquarters*, HQ, buys the assets of the A- and B-suppliers for predetermined prices in exchange for a share contract $\mathbf{s} = (s_A, s_B, s_{HQ}) \in \mathbb{R}_+^3$ with $s_A + s_B + s_{HQ} = 1$. HQs are supplied perfectly elastically with an opportunity cost normalized to zero.

2.3 The product market

The product market is perfectly competitive where consumers and suppliers take the product price P as given. Identical consumers maximize a smooth quasi-linear utility which gives rise to a downward-sloping demand curve $D(P)$. Suppliers correctly anticipate price P when they sign contracts and make

[9] The private cost can represent, for example, job satisfaction or a way to capture different beliefs held by managers and workers about the consequences of strategic choices (see Hart and Hölmstrom, 2010).

[10] For example, as discussed in Hart and Hölmstrom (2010), the two units may want to adopt a common standard, as in the Cisco's acquisition of Stratacom. The benefits for the organization from a common standard can be enormous but the private benefits within the firms may decrease because of the change the new standard introduces. Moreover, there is no agreement between the firms about which 'approach' should be adopted. However, agreeing on a common approach (coordination) boosts firm revenue.

[11] In the airline industry, for instance, major carriers subcontract portion of their network to regional partners. In some cases the majors own the regional partner, while in other cases the two operate as separate units and a contract governs their relationship (see Forbes and Lederman, 2009, 2010).

their production decisions. Define by $R(a,b) \equiv Pz(a,b)$, the revenue of an enterprise (a,b) in the event of success. We assume $0 \leq R(a,b) \leq 2c$ for all $(a,b) \in A \times B$.

2.4 Timing of events

The economy lasts for three dates, $t = 0, 1, 2$. At date 0, one A-supplier and one B-supplier match one to one to form an enterprise (a,b). At $t = 1$, and each A-supplier makes a take-it-or-leave-it contract offer (s,d) to each B-supplier. At date 1, the manager of each unit chooses e_A and e_B. Finally at $t = 2$, the revenue of each enterprise is realized and the agreed upon payments are made. We solve the model by backward induction.

2.5 Equilibrium

An equilibrium of the economy consists of a set of enterprises formed through feasible contracts, i.e., ownership structures and corresponding revenue shares, for each enterprise and a market-clearing price. Recall that there are two possible ownership structures for each enterprise – integration (I) and separation (N). In general, choice of ownership structures depends on the revenue share that accrues to each member of an enterprise, the output of each enterprise and the market price. An allocation for the market (β, u, v) specifies a one-to-one matching rule $\beta : A \to B$, and payoff functions $u : B \to \mathbb{R}_+$ and $v : A \to \mathbb{R}_+$ for the B- and A-suppliers, respectively.

Definition 1 (Equilibrium) *Given the type distributions $F(b)$ and $G(a)$, an allocation (β, u, v) and a product-market price P constitute an equilibrium allocation of the economy if they satisfy the following conditions:*

(a) **Feasibility:** *the revenue shares and the corresponding payoffs to the agents in each equilibrium enterprise are feasible given the output of the enterprise and the equilibrium price P;*

(b) **Optimization:** *Each A-supplier of a given type chooses optimally a B-supplier to form an enterprise, i.e., given $u(b)$ for each $b \in B$,*

$$\beta(a) = argmax_b \; \phi(a,b,u(b);P), \qquad (\mathscr{M}_a)$$

for each $a \in A$. The function $\phi(a,b,u(b);P)$ is the bargaining frontier *or* Pareto frontier *of the enterprise (a,b), which is the maximum payoff that can be achieved by a type a A-supplier given that the B-supplier of type b consumes $u(b)$ at a given market price P.*

(c) **Supplier market clearing:** *The equilibrium matching function satisfies the following 'measure consistency' condition. For any subinterval $[i_0, i_1] \subseteq J_A$, let $i_k = G(a_k)$ for $k = 0, 1$, i.e., a_k is the productivity of the A supplier at the i_k-th quantile. Similarly, for any subinterval $[j_0, j_1] \subseteq J_B$, let $j_h = F(b_h)$ for $h = 0, 1$. If $[a_0, a_1] = \lambda([b_0, b_1])$, then it must be the case that*

$$i_1 - i_0 = G(a_1) - G(a_0) = F(b_1) - F(b_0) = j_1 - j_0. \qquad \text{(MC)}$$

(d) **Product market clearing:** *The aggregate (expected) supply in the industry $Q(P)$ is equal to the demand $D(P)$.*

Definition 1-(b) asserts that each A-supplier chooses her partner optimally. Part (c) of the above definition simply says that one cannot match say two-third of the A-suppliers to one-third of the B-suppliers because the matching is constrained to be one-to-one.

3 Equilibrium sorting and ownership structures

We proceed as follows. In Section 3.1, we derive the bargaining frontiers under N and I and establish the optimal organization for a given enterprise, as a function of the exogenously given utilities. In Section 3.2, we allow the units to match endogenously, we endogenize the utilities and we show that the equilibrium matching is positive assortative. In Section 3.3, we derive the equilibrium ownership structures in the supplier market.

3.1 Optimal ownership structure for an arbitrary enterprise

We analyze the optimal contract for an arbitrary enterprise (a, b). We assume that the A-supplier possesses all the bargaining power in the relationship and makes take-it-or-leave-it offer to the B-supplier. We first analyze each ownership structure separately.

3.1.1 Non-integration

For the time being we suppress the argument (a, b) from the contract terms. Under this organizational structure the shares affect both the size and the distribution of surplus between the two units (imperfectly transferable utility). An optimal contract for a non-integrated enterprise (a, b) solves the following maximization problem:

$$\max_{s} \quad V_A \equiv \pi(e_A, e_B)(1-s)R(a, b) - ce_A^2, \qquad (\mathscr{P}_N)$$

$$\text{subject to} \quad U_B \equiv \pi(e_A, e_B)sR(a, b) - c(1-e_B)^2 = u, \qquad (\text{PC}_B)$$

$$e_A = \text{argmax}_e \left\{\pi(e, e_B)(1-s)R(a, b) - ce^2\right\} = \frac{(1-s)R(a, b)}{2c}, \qquad (\text{IC}_A)$$

$$e_B = \text{argmax}_e \left\{\pi(e_A, e)sR(a, b) - c(1-e)^2\right\} = 1 - \frac{sR(a, b)}{2c}, \qquad (\text{IC}_B)$$

where u is the outside option of the B-supplier. We assume that $u \geq u_0$, where $u_0 \geq 0$ is the reservation utility of all B-suppliers, i.e., the utility any B-supplier obtains if he does not join any firm. Constraint (PC_B) is the *participation constraint* of the B-supplier, whereas constraints (IC_A) and (IC_B) are the *incentive compatibility constraints* of the A-supplier and the B-supplier, respectively. Note that (IC_A) and (IC_B) together imply that $\pi(e_A, e_B) = R(a, b)/2c$, and hence, $\pi(e_A, e_B) \in [0, 1]$ under the assumption that $R(a, b) \in [0, 2c]$. The (maximum) value function of the above maximization problem, denoted by $\phi^N(a, b, u; P)$, is the maximum payoff that accrues to the A-supplier given that the B-supplier consumes u at a given market price P. The following lemma characterizes the optimal revenue share and the value function under non-integration.

Lemma 1 *When the supplier units in an arbitrary enterprise (a,b) stay separate, for a given product market price P, the optimal revenue share (accruing to the B-supplier) and the associated value function are respectively given by:*

$$s(a,b,u;P) = 1 - \frac{\sqrt{R(a,b)^2 - 4cu}}{R(a,b)}, \tag{1}$$

$$\phi^N(a,b,u;P) = \frac{1}{4c}\left[2R(a,b)\sqrt{R(a,b)^2 - 4cu} - \left(R(a,b)^2 - 4cu\right)\right], \tag{2}$$

for $0 \leq u \leq \frac{R(a,b)^2}{4c}$.

The participation constraint of the B-supplier determines the optimal revenue share $s = s(a,b,u;P)$ of each type b B-supplier. Note that u must lie between 0, which corresponds to $s = 0$, and $R(a,b)^2/4c$, the level corresponding to $s = 1$. The value function is symmetric with respect to the 45^0 line (on which $\phi^N(a,b,u;P) = u(b)$ and $s = 0.5$). This implies that total surplus is maximized when the shares across the two non-integrated units are equal (see Figure 1). Equal, or more broadly 'balanced', shares yields strong incentives to the managers to better coordinate their decisions.

3.1.2 Integration

When the two supplier units integrate, the enterprise is acquired by HQ who is conferred with the decision making power. Motivated entirely by incomes, HQ will choose e_A and e_B to maximize the expected revenue $\pi(e_A, e_B)R(a,b)$ as long as $s_{HQ} > 0$. This induces $e_A = e_B$. The HQ breaks even as the market for headquarters is perfectly competitive. The private costs of managerial actions are still borne by the individual supplier units. The aggregate managerial cost, $ce_A^2 + c(1-e_B)^2$ is minimized when $e_A = e_B = \frac{1}{2}$. The maximum payoff accruing to the A-supplier, given that the B-supplier consumes u, in each enterprise (a,b) is given by:

$$\phi^I(a,b,u;P) = R(a,b) - \frac{c}{2} - u \quad \text{for } 0 \leq u \leq R(a,b) - \frac{c}{2}. \tag{3}$$

The above function is linear in u, i.e., surplus is fully transferable between the two managers since neither the action taken by HQ nor the costs borne by the managers depends on the revenue share. The function $\phi^I(a,b,u;P)$ is strictly increasing in a and b, strictly decreasing in u (with slope -1) and symmetric with respect to the 45^0 line.

Although surplus is fully transferable between the A- and B-suppliers, this form of organization is not the efficient one as HQ, having a stake in the firm's revenue, puts too little weight on the managers' private costs while maximizing the expected revenue.[12]

[12] Note that the first-best surplus, $\frac{R^2}{2c}$, is strictly higher than $R - \frac{c}{2}$, the surplus accrued to an integrated firm as well as $\frac{3R^2}{8c}$, the maximum surplus in a non-integrated firm, which corresponds to $s = \frac{1}{2}$. The diminished output under non-integration reflects the distortionary effect of incomplete contracting. It is similar to the double-marginalization that creates incentives for vertical integration in a world with perfect contracts (see Perry, 1989).

3.1.3 Choice of organization and the bargaining frontier

Having analyzed the optimal contract of an arbitrary enterprise under each ownership structure separately, it is now convenient to analyze the optimal organization and the bargaining frontier associated with a given enterprise. At any given price P and utility u accruing to the B-supplier, each enterprise (a,b) would choose N over I if and only if $\phi^N(a,b,u;P) \geq \phi^I(a,b,u;P)$. We assume that an enterprise would choose to stay separate whenever it is indifferent between N and I. Thus, at any given product market price P, the bargaining frontier of each enterprise (a,b) is given by:

$$\phi(a,b,u;P) = \max\left\{\phi^N(a,b,u;P), \phi^I(a,b,u;P)\right\} \quad \text{for } 0 \leq u \leq u_{max}(a,b),$$

where

$$u_{max}(a,b) \equiv \max\left\{\frac{R(a,b)^2}{4c}, R(a,b) - \frac{c}{2}\right\}.$$

The equality between $\phi^N(a,b,u;P)$ and $\phi^I(a,b,u;P)$ gives two cut-off levels $u_L(a,b)$ and $u_H(a,b)$ of the utility of the B-supplier with $0 < u_L(a,b) \leq u_H(a,b) < u_{max}(a,b)$. The corresponding revenue shares (of the B-supplier) are $s_L(a,b)$ and $s_H(a,b)$, respectively, which are given by (1).

In an arbitrary enterprise (a,b), the optimal choice of ownership structure depends on the revenue of the enterprise, $R(a,b)$. One can find two threshold values $R^- \equiv (2-\sqrt{2})c$ and $R^+ \equiv \frac{2c}{3}$ of $R(a,b)$ with $0 < R^- < R^+ < 2c$ such that when $R(a,b) < R^-$ the managers of the supplier units prefer to stay separate because in this case $\phi^N(a,b,u;P) > \phi^I(a,b,u;P)$ for all $(a,b,u;P)$. On the other hand, the suppliers prefer to integrate when $R(a,b) > R^+$. Interestingly, when $R^- \leq R(a,b) \leq R^+$ there is no clear dominance of one mode of organization over the other. This case is depicted in Figure 1 where $\phi^N(a,b,u;P)$ is the strictly concave frontier and $\phi^I(a,b,u;P)$ is the linear frontier. Clearly, they intersect twice at $u_L(a,b)$ and $u_H(a,b)$. Therefore, for $R^- \leq R(a,b) \leq R^+$, non-integration is chosen by each enterprise (a,b) if and only if $u \in [u_L(a,b), u_H(a,b)]$. The resultant bargaining frontier is given by the upper envelope of $\phi^N(a,b,u;P)$ and $\phi^I(a,b,u;P)$, which is non-concave. Note that when $R(a,b) = R^+$, we have $u_L(a,b) = u_H(a,b)$. Thus to summarize,

Proposition 1 (Bargaining frontier of a given enterprise) *In a given enterprise (a,b), there exist two threshold values R^- and R^+ of the enterprise revenue $R(a,b)$ with $0 < R^- < R^+ < 2c$ such that*

(a) *When $R(a,b) < R^-$, the enterprise chooses non-integration over integration. The bargaining frontier is given by $\phi(a,b,u;P) = \phi^N(a,b,u;P)$;*

(b) *When $R(a,b) \in [R^-, R^+]$, non-integration is preferred if and only if $u \in [u_L(a,b), u_H(a,b)]$. The bargaining frontier is given by*

$$\phi(a,b,u;P) = \begin{cases} \phi^N(a,b,u;P) & \text{if } u \in [u_L(a,b), u_H(a,b)], \\ \phi^I(a,b,u;P) & \text{if } u \in [0, u_{max}(a,b)] \setminus \in [u_L(a,b), u_H(a,b)]; \end{cases}$$

(c) *When $R(a,b) > R^+$, the enterprise chooses integration over non-integration. The bargaining frontier is given by $\phi(a,b,u;P) = \phi^I(a,b,u;P)$.*

Low revenue, i.e., $R < R^-$ implies that an organization puts more emphasis on private benefits relative to the benefits accruing from coordination and chooses non-integration over integration for all levels of u.

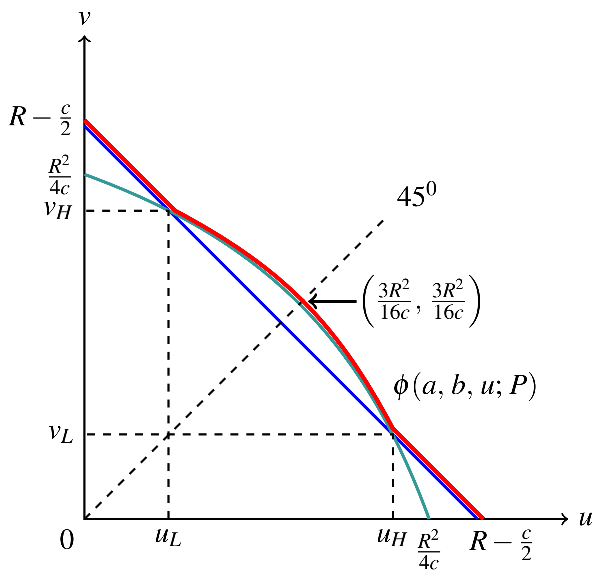

Figure 1: The two bargaining frontiers, ϕ^N and ϕ^I, when $R(a,b) \in [R^-, R^+]$. For intermediate values of the B supplier utility u, N is preferred to I, while for high or low values of u, I is chosen over N. The combined frontier is the upper envelope of ϕ^N and ϕ^I.

On the other hand, for the high-revenue (or high-productivity) enterprises with $R > R^+$, more weight is placed on coordination and revenue maximization, and hence, integration is the optimal choice for all u. For intermediate productivity enterprises, $R \in [R^-, R^+]$, either organizational structure may be optimal, depending on the levels of u, or the revenue share s. For intermediate values of u, an enterprise prefers to stay separate because the corresponding shares s and $1-s$ are more balanced and so coordination among the two units can be achieved without being integrated. But for the extreme values of u, either high or low, integration is preferred because the shares are tilted in favor of one of the two units and the incentives for revenue maximization are weak.

Proposition 1 allows us to reduce the set $[0, 1] \times \{N, I\}$ of feasible contracts (s, d) to the set of feasible shares $[0, 1]$ only, or equivalently to the set of feasible transfers (to the B-supplier) $[0, u_{max}(a, b)]$. For example, if $R^- \leq R(a, b) \leq R^+$, then a revenue share in the interval $[s_L(a, b), s_H(a, b)]$ or a transfer in $[u_L(a, b), u_H(a, b)]$ is equivalent to the fact that an enterprise (a, b) would never choose to integrate because integration is the strictly dominated mode of organization. On the other hand, a revenue share in $[0, s_L(a, b)) \cup (s_H(a, b), 1]$ or a transfer in $[0, u_L(a, b)) \cup (u_H(a, b), u_{max}(a, b)]$ implies that the enterprise strictly prefers to integrate. Thus, in the equilibrium of the supplier market where the supplier units match with each other optimally, the only thing that would matter is the set of feasible revenue shares, $[0, 1]$, or equivalently the set of feasible transfers, $[0, u_{max}(a, b)]$ for each enterprise (a, b).[13]

[13]In the proofs of many subsequent results, we will use revenue share s and utility transfer u interchangeably.

3.2 Equilibrium matching

In this section, we analyze the equilibrium matching function $b = \beta(a)$ and show that the matching is positive assortative (PAM), i.e., $\beta(a)$ is non-decreasing in a. PAM in an equilibrium allocation relies on the properties of the bargaining frontier $\phi(a, b, u(b); P)$. Until Section 4, the product market price will be taken as constant, and hence, we will disregard the dependence of ϕ on P for the time being. In the enterprise formation stage at date 0, each type a A-supplier solves the program ($\mathscr{M}_a$) to choose a B-supplier. In Lemma 6 (in Appendix) we show that $\phi(a, b, u(b))$ is strictly *supermodular* or has strict *increasing differences* in (a, b), i.e., for any $a' > a$ and $b' > b$, we have

$$\phi(a', b', u(b')) - \phi(a', b, u(b)) > \phi(a, b', u(b')) - \phi(a, b, u(b)). \tag{SPM}$$

The following proposition characterizes an equilibrium allocation.

Proposition 2 *Under condition (SPM) for all $(a, b) \in A \times B$, $\beta(a)$ solves the maximization problem ($\mathscr{M}_a$) for each $a \in A$ only if the following two conditions hold:*

(a) *The matching is positive assortative, i.e., $\beta(a)$ is non-decreasing in a;*

(b) *The equilibrium utility $u(b)$ of each type b is a strictly increasing function, and is given by the following ordinary differential equation (ODE):*

$$u'(b) = -\frac{\phi_2^d(a, b, u(b))}{\phi_3^d(a, b, u(b))} \quad \text{for } b = \beta(a), \tag{FOC}$$

whenever ownership structure d is the preferred choice for the enterprise $(a, \beta(a))$.

Proof To show the necessity of (a), suppose the equilibrium matching is not positive assortative, i.e., there are a and a' with $a' > a$, but we have $\beta(a') < \beta(a)$. Then, it follows from condition (SPM) that

$$\phi(a', \beta(a), u(\beta(a))) - \phi(a', \beta(a'), u(\beta(a'))) > \phi(a, \beta(a), u(\beta(a))) - \phi(a, \beta(a'), u(\beta(a'))), \tag{4}$$

since, by assumption, $a' > a$ and $\beta(a) > \beta(a')$. We argue that if condition (4) holds for a and a', then the maximization problem ($\mathscr{M}_a$) is not solved for at least one of a and a'. If both a and a' were maximizing, then it must be the case that

$$\phi(a, \beta(a), u(\beta(a))) \geq \phi(a, \beta(a'), u(\beta(a'))),$$
$$\phi(a', \beta(a'), u(\beta(a'))) \geq \phi(a', \beta(a), u(\beta(a))).$$

The above two inequalities together imply

$$\phi(a', \beta(a), u(\beta(a))) - \phi(a', \beta(a'), u(\beta(a'))) \leq \phi(a, \beta(a), u(\beta(a))) - \phi(a, \beta(a'), u(\beta(a'))),$$

which is a contradiction to condition (4).

Now we prove the second part of the proposition, i.e., the necessity of (FOC). We first prove that $u(b)$ is strictly increasing in b. Consider an equilibrium allocation where $b = \beta(a)$ for some $a \in A$. Suppose on the contrary that $u(b)$ is not strictly increasing, i.e., there is a $b' > b$ such that $u(b') \leq u(b)$. Since the bargaining frontier $\phi(a, b, u(b))$ of a is strictly increasing in b and strictly decreasing in $u(b)$, in the aforementioned allocation we have

$$\phi(a, b', u(b')) > \phi(a, b, u(b)),$$

which is a contradiction to the fact that $\beta(a)$ maximizes $\phi(a, b, u(b))$ for a. As $u(b)$ is strictly increasing in b it is *differentiable almost everywhere*.[14]

Now suppose in an equilibrium allocation that we have $b = \beta(a)$, and the enterprise $(a, \beta(a))$ chooses ownership structure d. Then for all $b' \in B$, $d, d' = N, I$ and $d \neq d$, the following is true.

$$\begin{aligned}\phi^d(a, b, u(b)) &= \phi(a, b, u(b)) \\ &\geq \phi(a, b', u(b')) \\ &= \max\left\{\phi^d(a, b', u(b')), \phi^{d'}(a, b', u(b'))\right\} \\ &\geq \phi^d(a, b', u(b')).\end{aligned}$$

First, let $b' = b + h$ with $h > 0$. Then, $\phi^d(a, b, u(b)) \geq \phi^d(a, b', u(b'))$ implies that

$$\frac{\phi^d(a, b+h, u(b+h)) - \phi^d(a, b, u(b))}{h} \leq 0.$$

As $h \to 0$, the above inequality reduces to

$$\phi_2^d(a, b, u(b)) + u'(b)\phi_3^d(a, b, u(b)) \leq 0. \tag{5}$$

Next, let $b' = b - h$ with $h > 0$. Then, $\phi^d(a, b, u(b)) \geq \phi^d(a, b', u(b'))$ implies that

$$\frac{\phi^d(a, b, u(b)) - \phi^d(a, b-h, u(b-h))}{h} \geq 0.$$

Letting $h \to 0$, the above inequality reduces to

$$\phi_2^d(a, b, u(b)) + u'(b)\phi_3^d(a, b, u(b)) \geq 0. \tag{6}$$

Inequalities (5) and (6) together yield (FOC). ■

The differential equation (FOC) is similar to the *local downward incentive constraint* in an optimal screening problem, which is the equality between the marginal earnings of each type b B-supplier and his marginal contribution to the match surplus, given an ownership structure. As in a classic assignment model with transfers, supermodularity of the bargaining frontier $\phi(a, b, u(b))$ implies PAM. Proving this property of the bargaining frontier, i.e., condition (SPM) in the present context is not a trivial task because the maximum of two supermodular functions is not necessarily supermodular. Recall that the bargaining frontier of each enterprise (a, b) where $b = \beta(a)$ at any equilibrium allocation is given by:

$$\begin{aligned}\phi(a, b, u(b)) &= \max\left\{\phi^N(a, b, u(b)), \phi^I(a, b, u(b))\right\} \\ &= \frac{1}{2}\left[\phi^N(a, b, u(b)) + \phi^I(a, b, u(b))\right] + \frac{1}{2}|\Delta(a, b)|,\end{aligned}$$

[14]The function $u(b)$ is not differentiable at b's at which $\phi^N(\cdot, b, u(b)) = \phi^I(\cdot, b, u(b))$. At these points the the graph of $u'(b)$ will have discontinuities because at each such point the left-hand derivative and the right-hand derivative are not equal. Yet, this does not contradict optimization by each a since we assume that any enterprise chooses N whenever it is indifferent between stay separate and integrate.

where $\Delta(a,b) \equiv \phi^N(a,b,u(b)) - \phi^I(a,b,u(b))$. It is easy to show that each of $\phi^N(a,b,u(b))$ and $\phi^I(a,b,u(b))$ is supermodular in (a,b), and hence, the first part of the above expression is supermodular in (a,b) since it is the sum of two supermodular functions. Therefore, a sufficient condition for the bargaining frontier to be supermodular in (a,b) is that $|\Delta(a,b)|$ is supermodular, i.e.,

$$\Delta_{ab}(a,b) \geq (\leq) 0 \quad \text{if } \Delta(a,b) \geq (\leq) 0. \tag{7}$$

We show that, under mild parameter restrictions, the above property is satisfied, and hence, (SPM) holds for all (a,b) (see Lemma 6). To summarize, in order to guarantee an equilibrium allocation with PAM, not only we require that the bargaining frontier under each ownership structure is supermodular, but also condition (7) which is a stronger complementarity (between the two supplier types) property in the sense that the marginal gain for an A-supplier by switching to a more profitable ownership structure is increasing in her partner's type.

Legros and Newman (2007) propose a necessary and sufficient condition for PAM, called the *generalized increasing difference* (GID), and show that $\phi_{21} > 0$ and $\phi_{31} > 0$ together imply GID. In our model, GID is less likely to hold because the objective function $\phi(a,b,u(b))$ is not quasi-concave with respect to b for the medium-revenue enterprises, and hence, one requires stronger complementarity conditions such as (7) in order to guarantee positive sorting. In a model of managerial incentive schemes under two-sided endogenous matching, Alonso-Paulí and Pérez-Castrillo (2012) analyze the choice between incentive and Codes of Best Practice (CBP) contracts. The presence of two different contracting modes gives rise to a non-concave bargaining frontier for each shareholder-manager pair. They show that for considerable range of parameter values the equilibrium matching may be negative assortative with a high-revenue shareholder offering a CBP contract to a less efficient manager, whereas a low-revenue shareholder offers an incentive contract to lure a more efficient manager.

Recall that $\alpha(\cdot)$ denote the inverse matching function, which exists because in any equilibrium allocation the matching function $\beta(a)$ is monotone. The equilibrium matching function must satisfy the measure consistency condition (MC) which together with PAM implies that $G(\alpha(b)) = F(b)$, from which it follows that $\alpha'(b) = f(b)/g(a)$ for $a = \alpha(b)$. Thus, although the equilibrium matching pattern is invariant to any changes in the type distributions, the shape of the matching function is not. Since densities are local measures of dispersion, the equilibrium matching function is steeper (flatter) at any given b if the A suppliers are more dispersed (concentrated) at $\alpha(b)$ relative to the B-suppliers at b. In a special case when a and b follow the same distribution on $[x_{min}, x_{max}]$, then the equilibrium matching function is linear with slope equal to 1.

3.3 Equilibrium ownership structures

We now intend to determine the equilibrium organizational pattern in the supplier market. In Section 3.1, we have analyzed the optimal contract of each enterprise (a,b) for given levels of enterprise revenue $R(a,b)$ and the utility of each B-supplier. In the market equilibrium, both revenue and utility are endogenized through the equilibrium matching function $a = \alpha(b)$.

Consider first an enterprise $(\alpha(b), b)$ along the equilibrium path whose productivity is given by $\tilde{z}(b) \equiv z(\alpha(b), b)$. Since $z_a, z_b > 0$ and the equilibrium exhibits PAM, we have $\tilde{z}'(\cdot) > 0$ and hence, an inverse function $\tilde{z}^{-1}$ exists. Therefore, for a given level of revenue R of enterprise $(\alpha(b), b)$, we may write

$$b = \tilde{z}^{-1}\left(\frac{R}{P}\right) \equiv Z(R). \tag{8}$$

We further assume that $R_{min} \equiv P\tilde{z}(b_{min}) \in (0, R^-)$. This is equivalent to saying that the minimum productivity is low enough, i.e., $b_{min} < b^- \equiv Z(R^-)$, which ensures that the low-productivity enterprises (with productivity close to b_{min}) choose N in the market equilibrium. Since the stand-alone utility u_0 of all the B-suppliers is the outside option of a B-supplier with type b_{min}, in equilibrium we must have $u(b_{min}) = u_0$.

Using the expression for ϕ_2^d and ϕ_3^d for $d = N, I$ (see Lemma 5), (FOC) can be expressed as follows

$$u'(b) = \begin{cases} R_b(\alpha(b), b) & \text{if } u(b) \in [u_L(\alpha(b), b), u_H(\alpha(b), b)], \\ \frac{R(\alpha(b),b)R_b(\alpha(b),b)}{2c} \cdot \frac{1-s(b;P)+s(b;P)^2}{s(b;P)} & \text{otherwise,} \end{cases} \tag{9}$$

where the equilibrium share function $s(b; P) \equiv s(\alpha(b), b, u(b); P)$ is given by (1).

According to the Picard-Lindelöf Theorem (see Birkhoff and Rota, 1989) a unique solution to the ODE exists (at least in the neighborhood of the initial condition) and is given (implicitly) by

$$u(b) = u_0 + \int_{b_{min}}^{b} u'(x)dx, \tag{10}$$

provided that $u'(b)$ is bounded, Lipschitz continuous in u and continuous in b. Whenever $u(b) < u_L(\alpha(b), b)$ and $u(b) > u_H(\alpha(b), b)$, the ODE assumes a simple form; all the aforementioned properties are satisfied and an analytical solution can be easily obtained. However, for values of u in the interval $[u_L(\alpha(b), b), u_H(\alpha(b), b)]$, the ODE is much more complicated and an analytical solution to it does not exist. In this region, we require to establish the existence and uniqueness of a solution. Our assumptions ensure that it is continuous in b, because b enters u' through $\tilde{z}(b)$ which is continuous as $z(\cdot)$ is assumed to be a continuous function. The term u enters through the share. If a differentiable function has a derivative that is bounded everywhere by a real number, then the function is Lipschitz continuous, which holds in this case. Moreover, u' is bounded above as long as $u_0 > 0$. If $u_0 = 0$, then $s = 0$ and u' becomes unbounded. Hence, we require u_0 to be strictly positive.

We first derive the *indifference locus* on which any equilibrium enterprise $(\alpha(b), b)$ is indifferent between choosing both ownership structures. In Figure 2, a point (b, u) in the 'type-utility' space represents the type of the B-supplier and the utility consumed by this type, respectively of an enterprise $(\alpha(b), b)$ formed in an equilibrium allocation. Let $b^- = Z(R^-)$ and $b^+ = Z(R^+)$.

- If for any equilibrium enterprise $(\alpha(b), b)$ we have $b \in [b_{min}, b^-)$, clearly this enterprise would strictly prefer non-integration over integration irrespective of the utility allocations. For values of $b \in (b^+, b_{max}]$, on the other hand, an enterprise would strictly prefer to integrate. Hence, if b equals either b^- or b^+, then enterprise $(\alpha(b), b)$ is indifferent between N and I.
- Now let $b \in [b^-, b^+]$. We know that, in this case, there is no clear dominance of one ownership structure over the other. Define $\tilde{u}_L(b) \equiv u_L(\alpha(b), b)$ and $\tilde{u}_H(b) \equiv u_H(\alpha(b), b)$ where u_L and u_H are given by equations (A2) and (A3) in Appendix. Note that, by construction, $\tilde{u}_L(b^-) = 0 < (3/2 - \sqrt{2})c = \tilde{u}_H(b^-)$ and $\tilde{u}_L(b^+) = \tilde{u}_H(b^+) = \frac{c}{12}$. Thus, $\tilde{u}_L(b)$ and $\tilde{u}_H(b)$ together define the *parabola-shaped curve* in Figure 2 any point (b, u) on which represents that an equilibrium enterprise $(\alpha(b), b)$ is indifferent between choosing to stay separate and integrate.

Therefore, $\tilde{u}_L(b)$, $\tilde{u}_H(b)$ and the portion of the vertical line at $b = b^-$ above the point $(b^-, (3/2-\sqrt{2})c)$ together define the indifference locus for an enterprise $(\alpha(b), b)$, which partitions the 'type-utility space' into two disjoint regions in which one ownership structure is preferred to the other by all enterprises $(\alpha(b), b)$ formed in equilibrium.

The equilibrium organization pattern is determined by the intersection between the equilibrium utility $u(b)$ of each type b B-supplier and the indifference locus. Let (b^*, u^*) be the intersection point(s), i.e., an enterprise $(\alpha(b^*), b^*)$ is an indifferent enterprise in the equilibrium allocation.

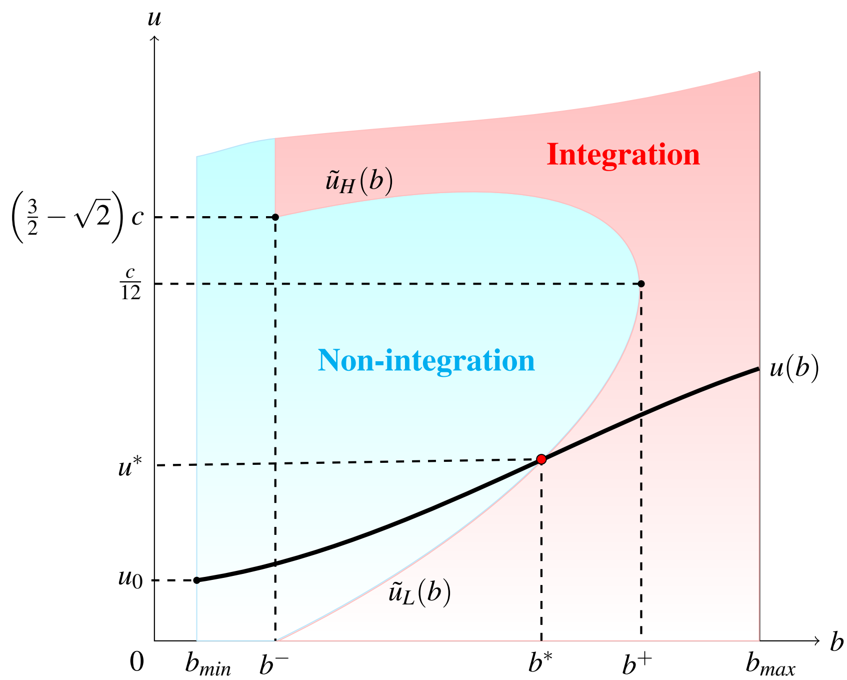

Figure 2: Monotonic ownership structure in the market equilibrium. Enterprises with $b \leq b^*$ stay separate, whereas those with $b > b^*$ integrate.

Our goal is to provide conditions under which the equilibrium ownership structure is monotonic in types, i.e., enterprises comprising of low-productivity B-suppliers, and hence, low-productivity A-suppliers since the equilibrium matching is PAM, would choose to stay separate, while high-productivity enterprises would prefer integration. In other words, b^* is unique. Let

$$r(b) \equiv \frac{z_a(\alpha(b), b)/g(\alpha(b))}{z_b(\alpha(b), b)/f(b)},$$

which is the ratio of the average marginal productivities of the A- and B-suppliers in a typical equilibrium enterprise $(\alpha(b), b)$. The following proposition provides sufficient conditions under which b^* is unique.

Proposition 3 *Suppose that $\sqrt{2}-1 < r(b) < \sqrt{2}+1$ for all $b \in [b^-, b^+]$. Then, there is a unique threshold productivity $b^* \in [b^-, b^+]$ of the B-suppliers such that each equilibrium enterprise $(\alpha(b), b)$ chooses to stay separate (integrate) if and only if $b \leq (>)\, b^*$.*

Since the indifference locus divides the type-utility space into two disjoint region and $u(b)$ is continuous and bounded on $[b_{min}, b_{max}]$, $u(b)$ necessarily intersects the indifference locus, i.e., $b^* \in [b^-, b^+]$ exists. Clearly, the slope of the indifference locus and that of $u(b)$ determine whether b^* is unique. Also, any enterprise with $b < b^-$ will stay separate and that with $b > b^+$ will choose to integrate in equilibrium irrespective of the utility allocations. The first inequality in the above proposition guarantees that $u(b)$ is flatter everywhere than $\tilde{u}_L(b)$ in the interval $[b^-, b^+]$ so that if there is an intersection on $\tilde{u}_L(b)$, then it is unique (as drawn in Figure 2). The second inequality, on the other hand, implies that $u(b)$ is steeper than $\tilde{u}_H(b)$, and hence, the intersection of $u(b)$ with $\tilde{u}_H(b)$ or with the vertical part of the indifference locus (in this case, $b^* = b^-$) is unique.

Intuitively, $r(b)$ can be very large if in an equilibrium enterprise $(\alpha(b), b)$, either the marginal productivity of the A-supplier is very high relative to that of the B-supplier, or the B-suppliers of this particular type are abundant relative to the A-suppliers of the matched type. Thus, if neither the supplier units are too different in terms of marginal productivity, nor one type is too scarce relative to its matched type, then the equilibrium ownership structure is monotone. For example, if $z_a(a, b) = z_b(a, b)$ and $g(a) = f(b)$ for all $(a, b) \in A \times B$, then $r(b) = 1$ for all b, and the above sufficient condition is trivially satisfied. In this case, b^* is unique. Note that even though the above sufficient condition is written in terms of the equilibrium variable $\alpha(b)$, it is easy to pin down the types a and b along the equilibrium path because the equilibrium exhibits PAM and the type distributions are known.

4 Effect of price changes on the equilibrium

4.1 Incidence of integration

We examine how a change in the product market price P affects the fraction of integrated firms in the market equilibrium. We will identify two countervailing effects, namely a *price effect* and a *utility effect*. The price effect, which refers to a shift in the indifference locus keeping the equilibrium utility $u(b)$ of each type b B-supplier unaltered, is similar to the one in Legros and Newman (2013). The utility effect, which represents a shift in $u(b)$ keeping the indifference locus unaltered, is novel and arises because the utility of each supplier is endogenized through the equilibrium matching. Recall that $(\alpha(b^*), b^*)$ is the enterprise indifferent between integrate and stay separate (see Figure 2). The above two effects thus refer to how the threshold productivity b^* responds to a change in the product market price. Let us write the threshold productivity as a function of the product market price, i.e., $b^* = b^*(P)$.

In order to analyze the price effect, note the following changes in the indifference locus which is depicted in Figure 3. Let the corresponding $\tilde{u}_k$'s for $k = L, H$ be denoted by $\tilde{u}_k(b; P)$ and $\tilde{u}_k(b; P')$. When the product market price increases from P to P', both b^- and b^+ decrease, but $\tilde{u}_H(b^-) = (3/2 - \sqrt{2})c$ and $\tilde{u}_L(b^+) = \tilde{u}_H(b^+) = c/12$ remain unaltered. As a result, the new indifference locus intersects the old one only once at (b^0, u^0) with $b^0 \in (b^-(P), b^+(P'))$ and $u^0 > (3/2 - \sqrt{2})c$. For values of b in $[b^-(P), b^0]$ the new indifference locus stays above the old ones and both of them are upward-sloping. Therefore, if both the intersections of $u(b)$ with the old and the new loci occur for $b \in [b^-(P), b^0]$ but for $u \geq (3/2 - \sqrt{2})c$, then b^* increases and hence, a larger fraction of the enterprises prefer to stay separate. Otherwise, a increase in the product market price implies more integration. One such case is depicted in Figure 3 where $u(b)$ intersects both $\tilde{u}_L(b; P)$ and $\tilde{u}_L(b; P')$. The price effect is summarized in the following Lemma.

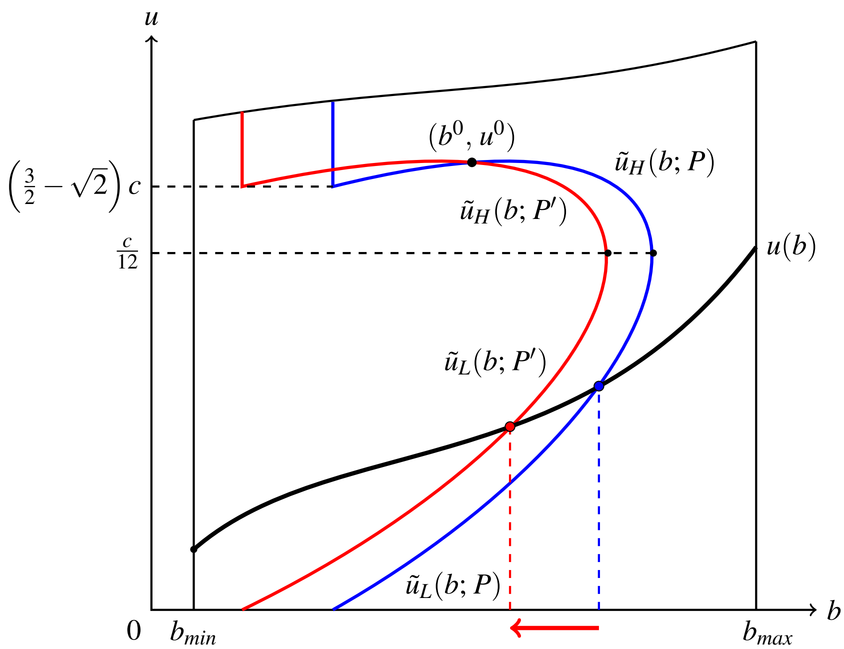

Figure 3: Price effect: More enterprises prefer to integrate, i.e., $b^*(P') < b^*(P)$, when utility of B-suppliers is low.

Lemma 2 (Price effect) *Suppose that, following a change in the product market price, the equilibrium utility $u(b)$ of each type b B-supplier remains unaltered. Let the product market price increases from P to P'. Then,*

(a) *if $u(b)$ intersects both $\tilde{u}_L(b;P)$ and $\tilde{u}_L(b;P')$, or both $\tilde{u}_H(b;P)$ and $\tilde{u}_H(b;P')$ at any $b \geq b^0$, then $b^*(P) > b^*(P')$, i.e., a greater fraction of the enterprises will integrate in the equilibrium;*

(b) *if $u(b)$ intersects both $\tilde{u}_H(b;P)$ and $\tilde{u}_H(b;P')$ at any $b < b^0$, then $b^*(P) < b^*(P')$, i.e., fewer enterprises will integrate in the equilibrium.*

Lemma 2 is qualitatively not very different from the price effect in Legros and Newman (2013) where $\tilde{u}_L(b)$ and $\tilde{u}_H(b)$ never intersect each other and $u(b)$ is flat at $u_0 = 0$ because all units are homogeneous. For example, if $u(b)$ intersects only $\tilde{u}_L(b;P)$ and $\tilde{u}_L(b;P')$, then a higher price increases the fraction of integrated enterprises, but if the intersection of $u(b)$ is with $\tilde{u}_H(b;P)$ and $\tilde{u}_H(b;P')$, then a higher price decreases the fraction of integrated enterprises. The latter case never arises in Legros and Newman (2013) because the utility of B suppliers is fixed at low values (zero).[15]

The intuition behind the price effect has to do with whether a higher price places the shares, when the shares of B-suppliers are fixed, closer or farther to the efficient ones (i.e., balanced shares), as the bargaining frontiers under N and I shift. If the shares move closer to the efficient shares non-integration

[15]Nevertheless, Legros and Newman (2013, footnote 16) do recognize the different impact price may have when utility is fixed at high levels.

becomes more likely. The level of utility of B-suppliers (which depends on the shares they receive) determines which case will arise.

Lemma 2 has been derived keeping the equilibrium utility of each type b B-suppliers unaltered when the price in the product market changes. But the utility, being endogenously determined through the equilibrium matching, will respond to a price change. This is our key difference with a model with homogeneous enterprises. Let $u(b;P)$ and $u(b;P')$ be the equilibrium utility functions corresponding to the price levels P and P', respectively. We first show that

Lemma 3 *If the product market price increases from P to P', then $u(b;P') > u(b;P)$ for all $b > b_{min}$. Moreover, $u(b;P') - u(b;P)$ is an increasing function of b.*

Since in equilibrium $u(b_{min}) = u_0$, both in the old and new equilibria the utility of any B-supplier of type b_{min} is unaffected by a change in the product market price. Higher market price benefits all the B suppliers (except the lowest-productivity ones) and more so the higher types. The utility effect refers to the change in the fraction of integrated enterprises when the equilibrium utility function changes that is attributed to a change in price, but the indifference locus remains unchanged.

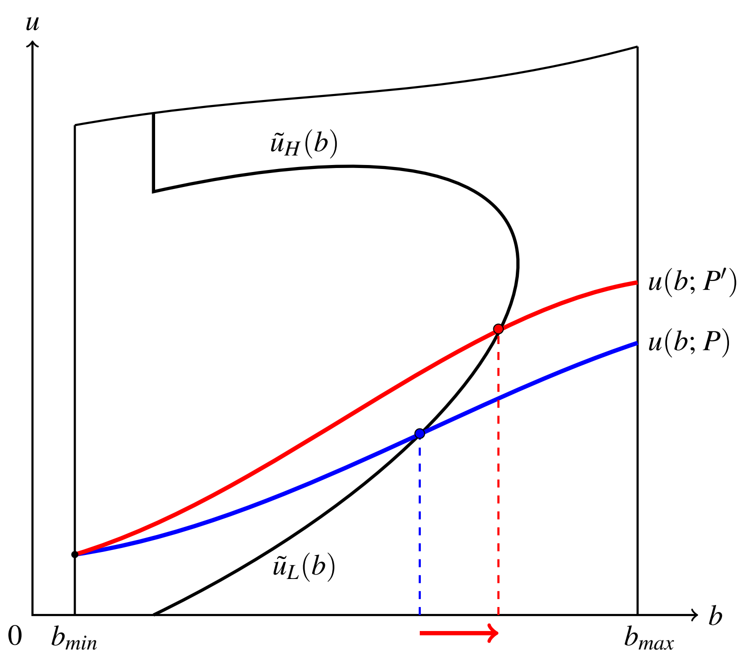

Figure 4: Utility effect: More enterprises prefer to stay separate, i.e., $b^*(P') > b^*(P)$, when utility of B-suppliers is low.

Lemma 4 (Utility effect) *Suppose that, following a change in the product market price, the indifference locus remains unaltered. Let the product market price increases from P to P'. Then,*

(a) *if both $u(b;P)$ and $u(b;P')$ intersect the $\tilde{u}_L(b)$ curve, then $b^*(P) < b^*(P')$, i.e., fewer enterprises will integrate in the equilibrium;*

(b) *If both* $u(b;P)$ *and* $u(b;P')$ *intersect the* $\tilde{u}_H(b)$ *curve, then* $b^*(P) > b^*(P')$*, i.e., a greater fraction of the enterprises will integrate in the equilibrium;*

The above results are fairly intuitive. Consider the indifferent enterprise $(\alpha(b^*(P)), b^*(P))$ in the initial equilibrium, and let $(b^*(P), u(b^*(P)))$ is on $\tilde{u}_L(b)$. When the product market price goes up, Lemma 3 implies that utilities of B suppliers increase. When these utilities are relatively low to begin with, as it is the case when the intersection of $u(b)$ is with $\tilde{u}_L(b)$, then shares between the two suppliers become more balanced, and hence, the indifferent enterprise strictly prefers to stay separate rather than being indifferent between N and I. Therefore, integration decreases following an increase in the product market price. This case is depicted in Figure 4. On the other hand, if $(b^*(P), u(b^*(P)))$ is on $\tilde{u}_H(b)$ (i.e., utility is high), then an increase in the product market price implies that the share of $b^*(P)$ moves closer to 1 (i.e., shares become more unbalanced), and hence enterprise $(\alpha(b^*(P)), b^*(P))$ strictly prefers to integrate. Therefore, integration increases following an increase in the product market price. Note that if $u(b;P)$ intersects $\tilde{u}_L(b)$ and $u(b;P')$ intersects $\tilde{u}_H(b)$, then the effect of a rise in the product market price on the threshold productivity b^* is ambiguous.

It is clear that the price and utility effects may pull in opposite directions, and hence, the net effect of an increase in P on the incidence of integration depends on the relative strength of these two effects. Because a closed form solution for $u(b)$ does not exist it is impossible to come up with clean conditions which characterize the net effect of P on the incentives to integrate. The following numerical example reveals that the utility effect may dominate, and there may be less integration following an increase in the market price.

Example 1 (higher prices lead to less integration) Let $z(a,b) = a^{0.01}b^{0.575}$. We set $c = 0.2$ and $u_0 = 0.012$. Furthermore, we assume that both a and b are uniformly distributed on $[0.05, 0.1]$ so that $\alpha(b) = b$. Using the above data we solve the ODE (9) numerically using Matlab.[16] The range of the product market prices has been taken to be $[0.54, 0.63]$. Figure 5 depicts the threshold productivity $b^*(P)$. Initially, $b^*(P)$ is decreasing in P, i.e., the fraction of enterprises that integrate increases following an increase in the market price. For the intermediate level of prices, for $P \in [0.565, 0.58]$, $b^*(P)$ is increasing in P implying that a greater measure of enterprises now stay separate. Finally, for high product market prices, $b^*(P)$ is decreasing in P.

It is worth mentioning that, in the above example, $u(b;P)$ intersects the indifference loci at $\tilde{u}_H(b;P)$ for all $P \in [0.54, 0.63]$. Recall that $s \geq \bar{s} \approx 0.243$ is a sufficient condition for PAM in an equilibrium allocation. Since $R(a,b)^2 s(2-s) = 4cu$, an equivalent sufficient condition is that $u(b) \geq 0.00183$. As we have taken $u_0 = 0.012$, the above example satisfies the conditions of Lemma 5.

4.2 Organizationally augmented industry supply

We derive now the industry supply curve (OAS) as a function of the product market price. Consider an arbitrary enterprise (a,b). If this enterprise stays separate, then its expected output is given by:

$$q^N(a,b,P) = \pi(e_A(a,b), e_B(a,b))z(a,b) = \frac{R(a,b)}{2c} \cdot z(a,b) = \frac{Pz(a,b)^2}{2c}.$$

[16]The Matlab codes are available upon request.

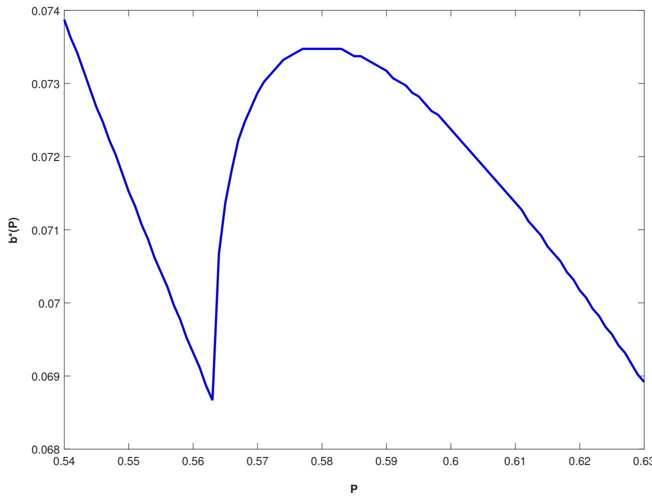

Figure 5: Incidence of integration as a function of the product market price. Following an increase in P, for low and high values of market price integration increases, whereas for intermediate values of P integration decreases.

The output of enterprise (a,b) is strictly increasing in the product market price P. On the other hand, if this enterprise integrates, its expected output is given by:

$$q^I(a,b,P) = \pi(e_A(a,b), e_B(a,b))z(a,b) = z(a,b).$$

The output of an integrated enterprise does not depend on the product market price. Clearly, an integrated enterprise produces greater expected output because $R(a,b) \le 2c$ for all (a,b). The organizationally augmented industry supply is the expected output aggregated across all the enterprises in equilibrium, which is given by:

$$Q(P) = \int_{b_{min}}^{b^*(P)} q^N(\alpha(b), b, P)dF(b) + \int_{b^*(P)}^{b_{max}} q^I(\alpha(b), b, P)dF(b). \quad \text{(OAS)}$$

A change in the product market price P affects $Q(P)$ via two channels – a rise in P (i) augments the output q^N of each non-integrated enterprise, but leaves the integrated output q^I unaltered and (ii) changes the fraction of integrated enterprises by changing the threshold productivity $b^*(P)$ of the indifferent enterprise. Moreover, these two effects may be countervailing. Whether the augmented industry supply curve $Q(P)$ is increasing or decreasing in P depends on the sign of $db^*(P)/dP$. Clearly, if the values of P are close to zero, then non-integration is the preferred ownership structure for all the enterprises because $b^-(P)$ is arbitrarily high. Therefore, $Q(P)$ must be increasing for low product market prices. But for high values of P if the threshold productivity $b^*(P)$ increases, the reduction in the number of integrated enterprises, and hence, the aggregate expected integration output may outweigh the increase in the aggregate expected non-integration output. Consequently, $Q(P)$ may decrease.

Proposition 4 *For low product market prices the organizationally augmented industry supply $Q(P)$ is increasing in P. However, $Q(P)$ may be backward-bending for high prices. A necessary condition for the backward-bending supply curve is a negative correlation between the product market price and integration, i.e.,* $\frac{db^*(P)}{dP} > 0$.

In the following example we show that the industry supply curve may be backward-bending.

Example 2 (Backward-bending OAS) We maintain the same parameter specifications as in Example 1. Figure 6 depicts a backward-bending organizationally augmented supply curve which is S-shaped. For low and high levels of the product market prices the quantity supplied is increasing in P, whereas for intermediate price levels the industry supply is decreasing in the market price. For example, consider a price increase from $P = 0.563$ to $P' = 0.577$. In this case, the output decreases from $Q(P) = 0.16658$ to $Q(P') = 0.15288$. The arc-elasticity of supply with respect to P and P' is given by -3.49205, i.e., a 1% increase in the product market price implies an approximate decrease in the industry supply $Q(P)$ by about 3.5% on average.

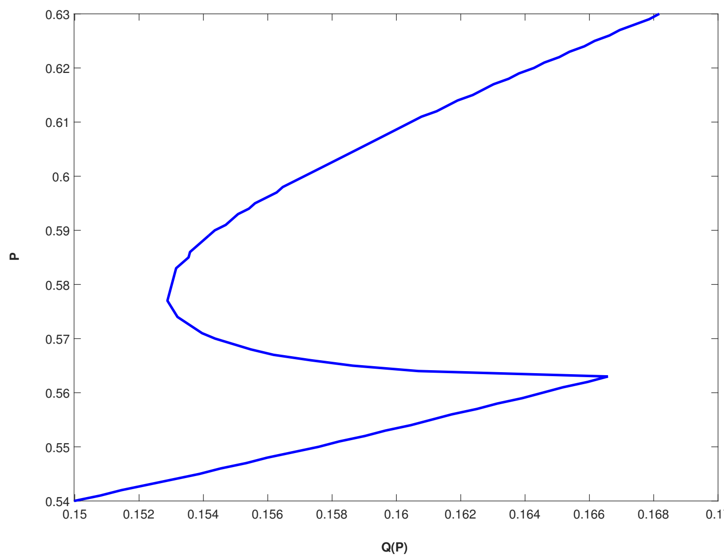

Figure 6: A backward-bending industry supply. The OAS is increasing in P for low and high levels of market price, whereas it is decreasing for the intermediate price levels.

The organizationally augmented supply curve $Q(P)$ is positively correlated with the threshold productivity $b^*(P)$. For intermediate price levels, when an increase in P implies a decrease in the measure of integrated enterprises in equilibrium, we have a downward-sloping industry supply curve. On the other hand, there are more integration and increasing supply following an increase in P for extreme values of the product market price. To close our model, the equilibrium product market price P^* is determined by the intersection of the OAS, $Q(P)$ and the demand curve, $D(P)$.

5 Empirical implications

The results obtained in Section 3 would help understand some recent empirical findings on the determinants of ownership structure in particular industries. On the other hand, the results of Section 4 have interesting testable implications for cross-sectional variation in modes of organization and aggregate output due to changes in the price level. Proposition 2 implies that, in a given industry, firms are *ex post* heterogeneous because heterogeneous input suppliers form enterprises following a positively assortative matching pattern. Such matching pattern is robust irrespective of the differences in the chosen ownership structure. Moreover, input suppliers at the upper tails of the type distribution integrate (Proposition 3). These predictions conform to the empirical findings of Atalay et al. (2014) who, using data on shipments originating from mining, manufacturing, wholesale, and catalog and mail-order retail establishments, spanning approximately 600 four-digit SIC industries, find that high-productivity firms are more likely to integrate and their partners are also likely to be high-productivity firms.

We have shown that the price level in the product market may be positively or negatively correlated with the incidence of integration and aggregate output may decrease with price. The empirical evidence on the relationship between price and the incidence of integration has also been mixed – Hastings (2004) and Alfaro et al. (2016) find a positive correlation, whereas the findings of Hortaçsu and Syverson (2007) show a negative correlation.

Recall that Proposition 4 suggests three possibilities: (a) a backward-bending OAS accompanied by less integration, (b) an upward-sloping OAS and less integration and (c) an upward-sloping OAS along with more integration. With this taxonomy in mind our model is amenable to analyze cross-sectional implications of exogenous changes in the product market price. To this end, we would take adverse preference shocks as an exogenous variation in the price level, which shifts the industry demand curve in.

If we are in case (a), then we would observe (i) a lower product price, (ii) a higher aggregate output and (iii) a greater number of integrated firms. This prediction is consistent with the findings of Hortaçsu and Syverson (2007). Legros and Newman (2013) consider two distinct levels (high and low) of exogenously given enterprise revenues and introduce heterogeneous technology shock that increases the measure of the high-revenue firms. This shifts the OAS out and given a fixed downward-sloping demand, results in a lower price, higher output and more integration. Thus, our model offers an explanation of the same phenomenon which is complementary to that of Legros and Newman (2013), in the sense that we rely on a demand rather than a technology shock.

A lower price can be associated with lower aggregate output but more integration even if the OAS is not backward-bending, which would be implied by case (b) in the above taxonomy. This case is more likely to occur when the initial product market price is high enough. On the other hand, when the initial market price is low, case (c) is more likely to occur, which implies that lower market price and output would be associated with less integration. This prediction conforms to the finding of Alfaro et al. (2016) who establish a positive correlation between market price and integration, by restricting their analysis to highly competitive sectors, which presumably are characterized by low market prices.

6 Conclusion

We analyze the determinants of firm boundaries when firms interact in a perfectly competitive product market. The choice of ownership structure in a given enterprise depends on the trade-off between coordination and private benefits: non-integration, a mode based on contingent revenue shares, puts too much weight on the private costs of managerial actions and hinders coordination; integration, which is based on delegation of decision rights to an outsider, facilitates coordination but ignores private benefits. Neither mode of organization thus achieves efficiency. Unbalanced revenue shares between the two units induce the managers of the (input) supplier units to opt for integration because coordination if they remain separate is poor. Balanced revenue shares, on the other hand, make non-integration more likely to dominate.

When supplier units are vertically differentiated with respect to productivity, competition for high-quality units arises naturally in the supplier market. We model such competition as a two-sided matching game, which endogenizes revenue share or utility allocation in each enterprise. In an equilibrium allocation, the matching is positive assortative, i.e., more productive suppliers match together to form enterprises. Thus, ex-ante differences in input productivity imply ex-post differences in firm revenue. Moreover, high-revenue firms opt to integrate because incentive to coordinate is high in such firms, whereas low-revenue enterprises stay separate.

Our paper contributes to the extant literature pertaining to OIO by embedding a competitive supplier market [arising due to two-sided heterogeneity] into a perfectly competitive product market. Therefore, an external change in the price level in the product market implies an internal reorganization of the supplier market. Within each firm, a price change modifies the utility allocation at which the supplier units are indifferent between the two organizational modes. This is the direct price effect. On the other hand, a change in price also alters the endogenously determined revenue shares or utility of each market participant. These two forces may pull in opposite directions, and consequently, the product market price may be positively or negatively correlated with the decision to integrate. Moreover, the possible negative correlation between price and integration may give rise to a backward-bending industry supply curve. These findings generate interesting testable implications.

The present model yields interesting normative implications with respect to *managerial firms*. When managers are partial revenue claimants, they tend to underweight enterprise revenues in favor of private benefits because the perceived price is lower than the actual market price. Essentially, the impact of partial revenue claims by the managers on ownership structures is similar to the impact market price has when firms are non-managerial. The presence of managerial firms thus yields an equilibrium that is organizationally inefficient (e.g. Leibenstein, 1966) because the true market price is unchanged. Thus, under an endogenous utility effect, when juxtaposed against the price effect, the presence of managerial firms may imply either 'too little' or 'too much' integration in equilibrium relative to the social optimum with non-managerial firms. This can have interesting policy implications related to corporate governance. Furthermore, taxes that affect the market price can also have similar implications on the efficiency of organizational choice.

Appendix: Proofs

Proof of Lemma 1. Substituting for e_A and e_B from the incentive compatibility constraints (IC_A) and (IC_B), the optimal contracting problem in an arbitrary enterprise (a, b) reduces to:

$$\max_{s\in[0,1]} \; V_A(s;R) \equiv \frac{R^2}{4c}(1-s^2), \tag{$\mathscr{M}'$}$$

$$\text{subject to } \; U_B(s;R) \equiv \frac{R^2}{4c}s(2-s) = u. \tag{PC_B'}$$

From (PC_B') it follows that

$$s(a,b,u;P) = 1 - \frac{\sqrt{R(a,b)^2 - 4cu}}{R(a,b)}.$$

We ignore the other root since it is strictly larger than 1. The bargaining frontier under N is given by:

$$\phi^N(a,b,u;P) = \frac{R(a,b)^2\left(1-s^2\right)}{4c} = \frac{1}{4c}\left[2R(a,b)\sqrt{R(a,b)^2-4cu} - \left\{R(a,b)^2-4cu\right\}\right]. \tag{A1}$$

This completes the proof of the lemma.

Proof of Proposition 1. First, consider the case when non-integration completely dominates integration in a given enterprise (a,b). In this case the minimum non-integration surplus must be strictly higher than the maximum surplus under integration, i.e.,

$$\frac{R^2}{4c} > R - \frac{c}{2} \iff \left[R - c\left(2-\sqrt{2}\right)\right]\left[R - c\left(2+\sqrt{2}\right)\right] > 0$$

Since $R \le 2c$, the above holds for $R < \left(2-\sqrt{2}\right)c \equiv R^-$. Next, consider the case when integration completely dominates non-integration in a given enterprise (a,b). In this case the minimum non-integration surplus must be strictly higher than the maximum surplus under non-integration, i.e.,

$$R - \frac{c}{2} > \frac{3R^2}{8c} \iff \left(R - \frac{2c}{3}\right)(R-2c) < 0$$

Since $R \le 2c$, the above holds for $R > \frac{2c}{3} \equiv R^+$. This completes the proofs of Parts (a) and (c). To show Part (b), note first that $\phi^N(a,b,u;P)$ intersects the linear function $\phi^I(a,b,u;P)$ exactly twice since both are symmetric with respect to the 45^0-line and the non-linear frontier is strictly concave. The two intersection points are given by:

$$u_L(a,b) = \frac{1}{8}\left[4R(a,b) - 2c - \frac{R(a,b)}{c}\cdot\sqrt{(2c-R(a,b))(2c-3R(a,b))}\right], \tag{A2}$$

$$u_H(a,b) = \frac{1}{8}\left[4R(a,b) - 2c + \frac{R(a,b)}{c}\cdot\sqrt{(2c-R(a,b))(2c-3R(a,b))}\right]. \tag{A3}$$

Note that $u_H(a,b) = R(a,b) - \frac{c}{2} - u_L(a,b)$. Thus, for the existence of the two intersection points it suffices to show that $u_L(a,b) \ge 0$, which holds if $R(a,b) \le -\frac{(2+\sqrt{10})c}{3}$ or $R^- \le R(a,b) \le R^+$ or $2c \le R(a,b) \le (2+\sqrt{2})c$. Given that $0 \le R(a,b) \le 2c$, this case occurs when $R^- \le R(a,b) \le R^+$. This completes the proof of the proposition.

Some properties of the bargaining frontier. We analyze some useful properties of the bargaining frontier in the following lemma. For the time being, we suppress the argument P from the bargaining frontiers.

Lemma 5 *Let $\phi^d(a,b,u)$ be the bargaining frontier associated with a given enterprise (a,b) when the B-supplier must receive u and the enterprise chooses ownership structure $d \in \{N, I\}$. Then,*

(a) *the frontier $\phi^d(a,b,u)$ satisfies the following single-crossing property (SCP):*

$$\frac{\partial}{\partial a}\left[-\frac{\phi_2^d(a,b,u)}{\phi_3^d(a,b,u)}\right] > 0 \quad \textit{for } d = N, I; \tag{SCP}$$

(b) *There are threshold values of the optimal share [of a type b B-supplier] $\bar{s}$ and $\hat{s}$ with $0 < \bar{s} < \hat{s} < 1$ such that*

(a) *if $s \geq \hat{s}$, then*

$$R_{ab}(a,b) - \left\{ \phi_{21}^N(a,b,u) - \frac{\phi_2^N(a,b,u)}{\phi_3^N(a,b,u)} \cdot \phi_{31}^N(a,b,u) \right\} \leq 0; \tag{A4}$$

(b) *if $\bar{s} \leq s \leq \hat{s}$, then*

$$R_{ab}(a,b) - \left\{ \phi_{21}^N(a,b,u) + R_b(a,b)\phi_{31}^N(a,b,u) \right\} \geq 0. \tag{A5}$$

Proof We first prove Part (a). Let $d = N$. Differentiating (PC_B') with respect to $x = a, b$ and u, respectively we obtain

$$\frac{\partial s}{\partial x} = -\frac{s(2-s)R_t}{(1-s)R} \quad \text{for } x = a, b, \tag{A6}$$

$$\frac{\partial s}{\partial u} = \frac{2c}{(1-s)R^2}. \tag{A7}$$

Denote by ϕ_i^d the partial derivative of ϕ^d with respect to the i-th argument, and by ϕ_{ij}^d the cross-partial with respect to the i-th and the j-th arguments. Differentiating (A1) with respect to a, b and u, and using (A6) and (A7) we obtain

$$\phi_1^N(a,b,u) = \frac{1-s+s^2}{1-s} \cdot \frac{RR_a}{2c} > 0,$$

$$\phi_2^N(a,b,u) = \frac{1-s+s^2}{1-s} \cdot \frac{RR_b}{2c} > 0, \tag{A8}$$

$$\phi_3^N(a,b,u) = -\frac{s}{1-s} < 0. \tag{A9}$$

Note that condition (SCP) is equivalent to

$$\frac{\phi_{21}^d(a,b,u)}{\phi_2(a,b,u)} - \frac{\phi_{31}^d(a,b,u)}{\phi_3(a,b,u)} > 0.$$

Differentiating (A8) with respect to a we get

$$\frac{\phi_{21}^N}{\phi_2^N} = \frac{R_{ab}}{R_b} + \frac{R_a}{R} + \left(\frac{2s-1}{1-s+s^2} + \frac{1}{1-s} \right) s_a = \frac{R_{ab}}{R_b} + \frac{1-3s+s^3}{(1-s)^2(1-s+s^2)} \cdot \frac{R_a}{R}. \tag{A10}$$

On the other hand, Differentiating (A9) with respect to a we get

$$\frac{\phi_{31}^N}{\phi_3^N} = -\frac{2-s}{(1-s)^2} \cdot \frac{R_a}{R}. \tag{A11}$$

Therefore, equations (A10) and (A11) together imply

$$\frac{\phi_{21}^N}{\phi_2^N} - \frac{\phi_{31}^N}{\phi_3^N} = \frac{R_{ab}}{R_b} + \left[\frac{1-3s+s^3}{(1-s)^2(1-s+s^2)} + \frac{2-s}{(1-s)^2} \right] \frac{R_a}{R} = \frac{R_{ab}}{R_b} + \frac{3}{1-s+s^2} \cdot \frac{R_a}{R} > 0. \tag{A12}$$

The above is true because $R_x = Pz_x > 0$ for $x = a, b$, $R_{ab} = Pz_{ab} > 0$ and $s \in [0, 1]$. Therefore, (SCP) holds for $d = N$. Next, consider $d = I$. Note that $\phi_2^I = R_b$, $\phi_{21}^I = R_{ab}$, $\phi_3^I = -1$ and $\phi_{31}^I = 0$. Therefore,

$$\frac{\phi_{21}^I(a,b,u)}{\phi_2^I(a,b,u)} - \frac{\phi_{31}^I(a,b,u)}{\phi_3^I(a,b,u)} = \frac{R_{ab}}{R_b} > 0, \tag{A13}$$

which proves that (SCP) holds for $d = I$. At this juncture, it is worth interpreting condition (SCP). Consider the indifference curve of any type a A-supplier in the $b-u$ space under ownership structure $d = N, I$. Then, the slope of the indifference curve is given by:

$$\left.\frac{du}{db}\right|_a = -\frac{\phi_2^d(a, b, u)}{\phi_3^d(a, b, u)} > 0.$$

Now consider any two a' and a'' with $a'' > a'$. Then, condition (SCP) asserts that, under each ownership structure $d = N, I$, the indifference curve of a'' is steeper than that of a', and consequently, they intersect each other at most once.

Next, we prove Part (b). It follows from (A8) and (A12) that

$$R_{ab} - \left\{\phi_{21}^N - \frac{\phi_2^N}{\phi_3^N}\phi_{31}^N\right\} = -\left\{\frac{RR_{ab}}{2c}\left(\frac{1-s+s^2}{1-s} - \frac{2c}{R}\right) + \frac{3R_aR_b}{2c(1-s)}\right\}.$$

The above expression is negative if

$$h(s) \equiv \frac{1-s+s^2}{1-s} \geq \frac{2c}{R}.$$

Note that $h'(s) > 0$ for $s \in [0, 1]$, $h(0) = 1$ and $\lim_{s\to 1} h(s) = \infty$. Therefore, by the Intermediate Value theorem, there is a unique $\hat{s}$, which solves

$$h(s) \equiv \frac{1-s+s^2}{1-s} = \frac{2c}{R} \implies \hat{s} = \frac{\sqrt{(2c-R)(2c+3R)} - (2c-R)}{2R} \in (0.5, 1),$$

such that $s \geq \hat{s}$, or equivalently, $u \geq \hat{u}$, where $R^2\,\hat{s}(2-\hat{s}) = 4c\hat{u}$, implies condition (A4).

Finally, we prove (A5). It follows from (A8)-(A11) that

$$R_{ab} - \left\{\phi_{21}^N + R_b\phi_{31}^N\right\} = \frac{RR_{ab}}{2c}\left\{\frac{2c}{R} - h(s)\right\} - \frac{R_aR_b}{2c(1-s)^3}\left\{1 - 3s + s^3 + s(2-s)\cdot\frac{2c}{R}\right\} \quad \text{(A14)}$$

Because $2c \geq R$,

$$1 - 3s + s^3 + s(2-s)\cdot\frac{2c}{R} \geq 1 - 3s + s^3 + s(2-s) = (1-s)^2(1+s) \geq 0.$$

Thus, a necessary condition for (A5) to hold is that $h(s) \leq 2c/R$, i.e., $s \leq \hat{s}$. Condition (A5) can be written as

$$\frac{RR_{ab}}{R_aR_b} = \frac{zz_{ab}}{z_az_b} \geq \frac{(1-3s+s^3) + s(2-s)\cdot\frac{2c}{R}}{(1-s)^3\cdot\frac{2c}{R} - (1-s)^2(1-s+s^2)}. \quad \text{(A5}'\text{)}$$

Since $z(a, b)$ is log-supermodular in (a, b), we have $zz_{ab}/z_az_b \geq 1$. Therefore, a sufficient condition for (A5) to hold is that

$$\frac{1-3s+s^3+s(2-s)\cdot\frac{2c}{R}}{(1-s)^3\cdot\frac{2c}{R} - (1-s)^2(1-s+s^2)} \leq 1 \iff H(s) \equiv \frac{2-6s+4s^2-2s^3+s^4}{1-5s+4s^2-s^3} \leq \frac{2c}{R}.$$

Figure 7 depicts the sufficient conditions in Part (b). The graph of $H(s)$ has two parts. For $s \in [0, \bar{s})$, $H'(s) > 0$ with $H(0) = 2$ and $\lim_{s\to\bar{s}^-} H(s) = \infty$. On the other hand, for $s \in (\bar{s}, 1]$, $H'(s) > 0$ with $\lim_{s\to\bar{s}^+} H(s) = -\infty$ and $H(1) = 1$. Since $2c \geq R$, condition (A5′) always holds if $s \in [\bar{s}, \hat{s}]$. This completes the proof of the Lemma. ■

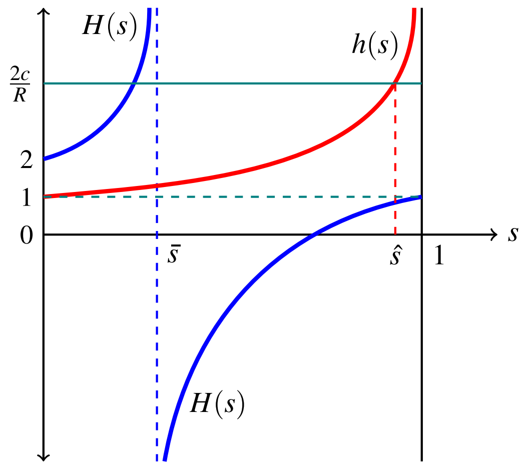

Figure 7: The blue curves represent $H(s)$, which have asymptotes at $\bar{s} \approx 0.243$, whereas the red curve represents $h(s)$. The line $2c/R$ intersects $h(s)$ at $s = \hat{s}$. Any share in $[\bar{s}, \hat{s}]$ guarantees condition (A5). On the other hand, any share in $[\hat{s}, 1]$ guarantees condition (A4).

Supermodularity of the objective function.

Lemma 6 *The bargaining frontier* $\phi(a, b, u(b)) = \max\{\phi^N(a, b, u(b)), \phi^I(a, b, u(b))\}$ *of each enterprise* (a, b) *is strictly supermodular in* (a, b)

Proof Note that

$$\begin{aligned}\phi(a, b, u(b)) &= \max\left\{\phi^N(a, b, u(b)), \phi^I(a, b, u(b))\right\} \\ &= \frac{1}{2}\left[\phi^N(a, b, u(b)) + \phi^I(a, b, u(b))\right] + \frac{1}{2}\left|\Delta(a, b)\right|,\end{aligned}$$

First we prove that each of $\phi^N(a, b, u(b))$ and $\phi^I(a, b, u(b))$ is strictly supermodular in (a, b). Note that (SCP), for $d = N, I$, implies that

$$\phi^d(a', b', u') - \phi^d(a', b, u) > \phi^d(a, b', u') - \phi^d(a, b, u) \quad \text{for any } a' > a, b' > b \text{ and } u' > u.$$

Since in equilibrium $u(b)$ is strictly increasing in b, letting $u = u(b)$ and $u' = u(b')$ for $b' > b$, the above inequality immediately implies strict supermodularity of $\phi^d(a, b, u(b))$ in (a, b). Thus, we have proved that

$$\frac{1}{2}\left[\phi^N(a, b, u(b)) + \phi^I(a, b, u(b))\right]$$

is strictly supermodular in (a, b) because the sum of two strictly supermodular functions is strictly supermodular.

Now we prove that $|\Delta(a, b)|$ is supermodular in (a, b). Recall that

$$\Delta(a, b) \equiv \phi^N(a, b, u(b)) - \phi^N(a, b, u(b)).$$

Then the supermodularity of $|\Delta(a, b)|$ is equivalent to

$$\begin{cases} \Delta_{ab}(a, b) \geq 0 & \text{whenever } \Delta(a, b) \geq 0, \\ \Delta_{ab}(a, b) \leq 0 & \text{whenever } \Delta(a, b) < 0. \end{cases}$$

Note that

$$\Delta_{ab} = \phi_{21}^N + u'(b)\phi_{31}^N - \left[\phi_{21}^I + u'(b)\phi_{31}^I\right] = \phi_{21}^N + u'(b)\phi_{31}^N - R_{ab}.$$

The above is true since $\phi_{31}^I = 0$ and $\phi_{21}^I = R_{ab}$. Since

$$u'(b) = \begin{cases} -\frac{\phi_2^N(a,b,u(b))}{\phi_3^N(a,b,u(b))} & \text{whenever } \Delta(a, b, u(b)) \geq 0, \\ -\frac{\phi_2^I(a,b,u(b))}{\phi_3^I(a,b,u(b))} & \text{whenever } \Delta(a, b, u(b)) < 0, \end{cases}$$

Therefore, it follows from Lemma 5-(b) that $\Delta(a, b)$ is supermodular in (a, b) for $s(a, b, u) \geq \bar{s}$. This completes the proof of the Lemma. ■

Proof of Proposition 3. Let $(\alpha(b^*), b^*)$ be the indifferent enterprise in an equilibrium allocation where the threshold productivity level b^* is determined by the intersection between the equilibrium utility $u(b)$ of each type b B-supplier and the indifferent locus (see Figure 2). First we prove the existence of b^*. We have assumed that $b_{min} < b^- = Z(R^-)$. Clearly, $b_{max} \equiv Z(2c/P) > Z(R^+) = b^+$. Since $u(b)$ is continuous on $[b_{min}, b_{max}]$, all enterprises $(\alpha(b), b)$ will stay separate in equilibrium if $b \in [b_{min}, b^-)$ and will integrate if $b \in (b^+, b_{max}]$. In other words, high-productivity enterprises will integrate and the low-productivity ones will stay separate. So, if there is an intersection between $u(b)$ and the indifference locus at some b^* then b^* must lie in the interval $[b^-, b^+]$. The existence of b^* is guaranteed because $u(b)$ is continuous everywhere on $[b_{min}, b_{max}]$, bounded above by $R(a_{max}, b_{max}) - c/2$ (since the enterprise $(\alpha(b_{max}), b_{max})$ always integrate) and the domain of the indifference locus is $[b^-, b^-]$ on which it is continuous and bounded above by $R(\alpha(b^-), b^-) - c/2$. This is to say that if $u(b)$ starts from b_{min} which is strictly lower than b^- and has to reach b_{max} that is strictly higher than b^+, then it must intersect the indifference locus at least once.

Note that b^* is not necessarily unique. To guarantee uniqueness, suppose that $u(b)$ intersects the $\tilde{u}_L(b)$ part of the indifference locus (as drawn in Figure 2). From the above discussion it follows that this intersection is well defined, which without a slight abuse of notation we call b^*. By construction of the indifference point, $u'(b) = R_b(\alpha(b), b)$ at any $b = b^* + \varepsilon$ with $\varepsilon > 0$ but very small. If $u(b)$ intersects the indifference locus again at some $b' \in (b^*, b^+)$, then second intersection has to be with $\tilde{u}_L(b)$. To avoid such a second intersection, a sufficient condition is that $u(b)$ is flatter than $\tilde{u}_L(b)$ everywhere on $[b^-, b^+]$. Note that

$$\tilde{u}_L'(b) = \frac{du_L}{dR}\left(R_a(\alpha(b), b)\alpha'(b) + R_b(\alpha(b), b)\right),$$

where

$$\frac{du_L}{dR}(R) = \frac{1}{2} - \frac{3R^2 - 6cR + 2c^2}{4c\sqrt{(R-2c)(3R-2c)}}.$$

It is easy to show that $u_L(R)$ is strictly convex on $[R^-, R^+]$ so that $du_L(R)/dR$ achieves its minimum value at $R^- = (2-\sqrt{2})c$, which is given by:

$$\frac{du_L}{dR}(R^-) = \frac{1}{\sqrt{2}}.$$

Recall that $\alpha'(b) = f(b)/g(\alpha(b))$. Therefore, the sufficient condition for the uniqueness of b^* in this case is given by:

$$\begin{aligned}
&\frac{1}{\sqrt{2}}\left(R_a(\alpha(b),b)\alpha'(b) + R_b(\alpha(b),b)\right) > R_b(\alpha(b),b) \\
\Leftrightarrow\ &\frac{1}{\sqrt{2}}\left(Pz_a(\alpha(b),b)\cdot\frac{f(b)}{g(\alpha(b))} + Pz_b(\alpha(b),b)\right) > Pz_b(\alpha(b),b) \\
\Leftrightarrow\ &r(b) > \sqrt{2}-1.
\end{aligned} \tag{A15}$$

Now suppose that $u(b)$ intersects the $\tilde{u}_H(b)$ or the vertical part of the indifference locus at b^*. If there is a second intersection it must be on $\tilde{u}_H(b)$. To rule out such intersection, a sufficient condition would be that $\tilde{u}'_H(b) < u'(b)$ for all $b \in [b^-, b^+]$. Since for any $b = b^* + \varepsilon$ with $\varepsilon > 0$ but very small, enterprise $(\alpha(b), b)$ integrate in equilibrium, and hence, $u'(b) = R_b(\alpha(b), b)$ for $b = b^* + \varepsilon$. On the other hand,

$$\tilde{u}'_H(b) = \frac{du_H}{dR}\left(R_a(\alpha(b),b)\alpha'(b) + R_b(\alpha(b),b)\right),$$

where

$$\frac{du_H}{dR}(R) = \frac{1}{2} + \frac{3R^2 - 6cR + 2c^2}{4c\sqrt{(R-2c)(3R-2c)}}.$$

It is easy to show that $u_H(R)$ is strictly concave on $[R^-, R^+]$ so that $du_H(R)/dR$ achieves its maximum value at $R^- = (2-\sqrt{2})c$, which is given by:

$$\frac{du_L}{dR}(R^-) = \frac{1}{2+\sqrt{2}}.$$

Therefore, the sufficient condition for the uniqueness of b^* in this case is given by:

$$\begin{aligned}
&\frac{1}{2+\sqrt{2}}\left(R_a(\alpha(b),b)\alpha'(b) + R_b(\alpha(b),b)\right) < R_b(\alpha(b),b) \\
\Leftrightarrow\ &\frac{1}{2+\sqrt{2}}\left(Pz_a(\alpha(b),b)\cdot\frac{f(b)}{g(\alpha(b))} + Pz_b(\alpha(b),b)\right) < Pz_b(\alpha(b),b) \\
\Leftrightarrow\ &r(b) < \sqrt{2}+1.
\end{aligned} \tag{A16}$$

Thus, conditions (A15) and (A16) together establish the proposition.

Proof of Lemma 3. Let $H(s) \equiv \frac{1-s+s^2}{s}$. We differentiate (10) with respect to P to obtain

$$\frac{du(b)}{dP} = \int_{b_{min}}^{b} \frac{Pzz_b}{2c}\left[PH'(s)\left\{\frac{ds}{dP} + \frac{ds}{du}\frac{du(x)}{dP}\right\} + 2H(s)\right]dx, \tag{A17}$$

where $H'(s) < 0$, $ds/dP < 0$ and $ds/du > 0$.

First, note that $du(b_{min})/dP = du_0/dP = 0$. This is intuitive since the lowest utility is given exogenously and is not a function of the price P. Second, for values of b arbitrarily close to b_{min}, $du(b)/dP$ is positive. Since in this case $du(x)/dP$ is arbitrarily close to zero, and the other terms in the integrand of (A17) are positive, $du(b)/dP$ cannot be negative for any b in the neighborhood of b_{min}.

Suppose by way of contradiction that $du(b)/dP$ becomes negative for some b. Consider the b, denoted by $\tilde{b}$, at which $du(\tilde{b})/dP = 0$. Given, as we showed above, that the integrand of (A17) is positive initially, then at $\tilde{b}$ it must be negative (so that the positive and negative areas cancel each other out in the integration). But the integrand of (A17) evaluated at $\tilde{b}$ is strictly positive, a contradiction. Given that the integrand of (A17) is always positive, the lemma follows.

References

Acemoglu, Daron, Rachel Griffith, Philippe Aghion, and Fabrizio Zilibotti (2010), "Vertical Integration and Technology: Theory and Evidence." *Journal of the European Economic Association*, 8, 989–1033.

Aghion, Philippe, Rachel Griffith, and Peter Howitt (2006), "Vertical Integration and Competition." *The American Economic Review*, 96, 97–102.

Alfaro, Laura, Paola Conconi, Harald Fadinger, and Andrew Newman (2016), "Do Prices Determine Vertical Integration?" *The Review of Economic Studies*, 83, 855–888.

Alonso, Ricardo, Wouter Dessein, and Niko Matouschek (2008), "When Does Coordination Require Centralization?" *The American Economic Review*, 98, 145–179.

Alonso-Paulí, Eduard and David Pérez-Castrillo (2012), "Codes of Best Practice in Competitive Markets for Managers." *Economic Theory*, 49, 113–141.

Atalay, Enghin, Ali Hortaçsu, and Chad Syverson (2014), "Vertical Integration and Input Flows." *The American Economic Review*, 104, 1120–1148.

Bernard, Andrew, Andreas Moxnes, and Karen Helene Ulltveit-Moe (2014), "Two-sided Heterogeneity and Trade." NBER Working Paper No. 20136.

Besley, Timothy and Maitreesh Ghatak (2005), "Competition and Incentives with Motivated Agents." *The American Economic Review*, 95, 616–636.

Birkhoff, Garret and Gian-Carlo Rota (1989), *Ordinary Difierential Equations*. John Wiley.

Bloom, Nicholas and John van Reenen (2007), "Measuring and Explaining Management Practices Across Firms and Countries." *The Quarterly Journal of Economics*, 122, 1351–1408.

Dragusanu, Raluca (2014), "Firm-To-Firm Matching Along the Global Supply Chain." Mimeo, Harvard University.

Fallick, Bruce, Charles Fleischman, and James Rebitzer (2006), "Job-Hopping in Silicon Valley: Some Evidence Concerning the Microfoundations of a High-Technology Cluster." *The Review of Economics and Statistics*, 88, 472–481.

Forbes, Silke and Mara Lederman (2009), "Adaptation and Vertical Integration in the Airline Industry." *The American Economic Review*, 99, 1831–1849.

Forbes, Silke and Mara Lederman (2010), "Does Vertical Integration Affect Firm Performance? Evidence from the Airline Industry." *RAND Journal of Economics*, 41, 765–790.

Gibbons, Robert (2010), "Inside Organizations: Pricing, Politics and Path Dependence." *Annual Review of Economics*, 2, 337–365.

Gibbons, Robert, Richard Holden, and Michael Powell (2012), "Organization and Information: Firms' Governance Choices in Rational-Expectations Equilibrium." *The Quarterly Journal of Economics*, 127, 1813–1841.

Grossman, Gene and Elhanan Helpman (2002), “Integration versus Outsourcing in Industry Equilibrium.” *Quarterly Journal of Economics*, 85–120.

Grossman, Sanford and Oliver Hart (1986), “The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration.” *The Journal of Political Economy*, 94, 691–719.

Hart, Oliver and Bengt Hölmstrom (2010), “A Theory of Firm Scope.” *The Quarterly Journal of Economics*, 125, 483–513.

Hastings, Justine (2004), “Vertical Relationships and Competition in Retail Gasoline Markets: Empirical Evidence from Contract Changes in Southern California.” *The American Economic Review*, 94, 317–328.

Hortaçsu, Ali and Chad Syverson (2007), “Cementing Relationships: Vertical Integration, Foreclosure, Productivity, and Prices.” *Journal of Political Economy*, 115, 250–301.

Lafontaine, Francine and Margaret Slade (2007), “Vertical Integration and Firm Boundaries: The Evidence.” *Journal of Economic Literature*, 45, 629–685.

Legros, Patrick and Andrew Newman (2007), “Beauty is a Beast, Frog is a Prince: Assortative Matching with Nontransferabilities.” *Econometrica*, 75, 1073–1102.

Legros, Patrick and Andrew Newman (2013), “A Price Theory of Vertical and Lateral Integration.” *The Quarterly Journal of Economics*, 128, 727–770.

Legros, Patrick and Andrew Newman (2014), “Contracts, Ownership, and Industrial Organization: Past and Future.” *Journal of Law, Economics, and Organization*, 30, i82–i117.

Leibenstein, Harvey (1966), “Allocative Efficiency vs." X-Efficiency".” *The American Economic Review*, 56, 392–415.

Perry, Martin (1989), “Vertical Integration: Determinants and Effects.” In *Handbook of Industrial Organization*, volume 1, 183–255, Elsevier.

Sørensen, Morten (2007), “How Smart is Smart Money? A Two-Sided Matching Model of Venture Capital.” *The Journal of Finance*, 62, 2725–2762.

Sugita, Yoichi, Kensuke Teshima, and Enrique Seira (2015), “Assortative Matching of Exporters and Importers.” Mimeo, Instituto Tecnológico Autónomo de México.

Syverson, Chad (2011), “What Determines Productivity?” *Journal of Economic literature*, 49, 326–365.

Terviö, Marko (2008), “The Difference that CEOs Make: An Assignment Model Approach.” *The American Economic Review*, 98, 642–668.