CHAPTER 4: Selected Topics

1 Competition and Managerial Incentives

1.1 Product Market Competition, Turnover Costs and Incentives

Consider the model of managerial incentive contract as in Schmidt (1997) where a risk neutral firm characterized by a constant marginal cost of production $\theta \in \{0, c\}$ with $c > 0$ hires a manager who undertakes cost-reducing effort $e \in [0, 1]$. The effort cost function is given by $\psi(e) = e^2/2$. Let e also denote the probability that $\theta = 0$. The product market profit of the firm at state θ is given by $\pi(\theta, m)$ where m denotes the level of competition at the product market. Assume that

A1. $\Delta(m) := \pi(0, m) - \pi(c, m) > 0$ for all m;

A2. $\pi(\theta, m) < \pi(\theta, m')$ for any $m > m'$ for $\theta \in \{0, c\}$.

In the case that the manager fails to reduce the marginal cost the firm faces a probability $l(m)$ of being liquidated with $l'(m) > 0$, i.e., the probability of liquidation increases with the degree of competition in the product market. In this case the manager faces a loss of the amount $L > 0$ which is interpreted as the turnover cost. Therefore, given the salary w, the manager's utility is given by:

$$U = \begin{cases} w - \frac{e^2}{2} & \text{if the firm stays in the market,} \\ w - \frac{e^2}{2} - L & \text{if the firm is liquidated.} \end{cases}$$

Manager's salary is state-contingent, i.e., the incentive contract is given by $w(\theta)$ which is subject to limited liability, i.e., $w(\theta) \geq 0$ for $\theta \in \{0, c\}$. The owner of the firm thus chooses $(w(0), w(c))$ to solve the following maximization problem:

$$\max_{\{w(0), w(c), e\}} \; e[\pi(0, m) - w(0)] + (1 - e)[\pi(c, m) - w(c)] \tag{M$_1$}$$

$$\text{subject to } e = \operatorname{argmax}_{e'} \left\{ e' w(0) + (1 - e')[w(c) - l(m)L] - \frac{e'^2}{2} \right\} = w(0) - w(c) + l(m)L, \tag{IC}$$

$$ew(0) + (1 - e)[w(c) - l(m)L] - \frac{e^2}{2} \geq 0, \tag{IR}$$

$$w(0) \geq 0, \tag{LL$_0$}$$

$$w(c) \geq 0, \tag{LL$_c$}$$

We will consider the contracts under binding limited liability constraint(s) so that the first best contracts are not implemented. Notice first that both (LL_0) and (LL_c) cannot bind together. Suppose on the contrary that both bind. In this case the manager's expected utility is given by:

$$-\left[\frac{e^2}{2}+(1-e)l(m)L\right]<0,$$

which implies that (IR) is violated. Next, suppose that (LL_0) binds, but (LL_c) does not. In this case, (IC) reduces to $w(c)-l(m)L=-e$, and hence the manager's expected utility is given by:

$$-\left[(1-e)e+\frac{e^2}{2}\right]<0.$$

In this case also (IRA) is violated. Therefore,

Lemma 1 *At the optimal managerial contract it must be the case that* $w(0)>w(c)=0$.

Substituting $w(c)=0$ from Lemma 1 the incentive compatibility constraint reduces to $w(0)=e-l(m)L$. Then, the expected utilities of the owner and the manager are respectively given by:

$$\begin{aligned}V(e)=&e\pi(0,m)+(1-e)\pi(c,m)-e[e-l(m)L]=e[\Delta(m)-e+l(m)L]+\pi(c,m),\\U(e)=&e[e-l(m)L]-(1-e)l(m)L-\frac{e^2}{2}=\frac{e^2}{2}-l(m)L.\end{aligned}$$

Therefore, the above maximization problem reduces to:

$$\max_e\ e[\Delta(m)-e+l(m)L]+\pi(c,m) \tag{M_1'}$$

$$\text{subject to } \frac{e^2}{2}-l(m)L\geq 0. \tag{IR$'$}$$

Managerial effort e can be interpreted as the 'power of incentives'. The optimal managerial effort is described in the following proposition.

Proposition 1 *The optimal second best contract* $w(0)$ *implements the effort level* $e^{SB}=\max\{e^0,e^*\}$ *where* e^0 *and* e^* *are respectively given by:*

$$\begin{aligned}e^0=&\frac{1}{2}[\Delta(m)+l(m)L],\\e^*=&\sqrt{2l(m)L}.\end{aligned}$$

The optimal managerial bonus is given by $w^{SB}=e^{SB}-l(m)L$.

Proof. Easy. The effort level e^0 is optimal when (IR$'$) does not bind, and when (IR$'$) binds e^* is the optimal effort. □

Now we see the effect of an increase in the degree of competition on managerial incentives. It follows from Proposition 1 that

Proposition 2 *The effect of a marginal increase in competition on the optimal managerial effort is given by:*

$$\frac{de^{SB}}{dm} = \begin{cases} \frac{1}{2}[\Delta'(m) + l'(m)L] & \text{if } (IR') \text{ does not bind,} \\ = \frac{l'(m)L}{e^*} & \text{if } (IR') \text{ binds.} \end{cases}$$

When the turnover cost L is sufficiently high the individual rationality constraint of the manager binds, and hence the optimal effort is given by e^*. In this case competition unambiguously increases managerial effort. This is called the *threat of liquidation effect*. When (IR$'$) does not bind, then effect of increased competition on the optimal effort depends on the two effects. The first is the threat of liquidation effect, i.e., the sign of $l'(m)$. The second one is the *value of cost reduction effect* which is the sign of $\Delta'(m)$. If $\Delta'(m) > 0$, i.e., the value of cost reduction increases with competition, then $de^{SB}/dm > 0$. But if $\Delta'(m) < 0$, then the sign of de^{SB}/dm depends on which of the two countervailing effects is stronger, and hence the effect of increased competition on managerial incentives is ambiguous.

1.2 Oligopolistic Competition and Managerial Incentives

Consider an industry in which n risk neutral firms produce a homogeneous product and compete in quantities. We measure the level of product market competition by the number of firms n which is exogenously given. The inverse demand function is given by $P(Q) = 1 - Q$ where $Q = \sum_{i=1}^{n} q_i$ is the aggregate industry output. Each firm has a linear cost of production function given by $C_i(q_i) = c_i q_i$. The marginal cost c_i of each firm i may take values in $\{0, c\}$. Firms are otherwise identical. The probability of low marginal cost 0 in firm i is given by e_i. We assume that the true realizations of marginal costs are independent across firms. Initially, all firms own the inefficient technology, i.e., the marginal cost of production is given by $c_i = c$ for all $i = 1, \ldots, n$. Each firm hires a risk neutral manager whose principal task is to exert R&D efforts in order to reduce the marginal cost. Without loss of generality, the probability of success e_i will be interpreted as the managerial effort in firm i. We further assume that managerial effort is not publicly verifiable, and hence cannot be contracted upon by his employer, which gives rise to a potential moral hazard problem in effort choice. The manager in firm i incurs a total cost of effort which is given by:

$$\psi(e_i) = \frac{e_i^2}{2}.$$

Each firm i hires a manager by offering a binding contract (f_i, b_i) where f_i is the base salary and b_i is the bonus offered in the event that the manager succeeds in reducing firm's marginal cost. Our focus is on the effect of product market competition on managerial efforts and incentives. It is well-known in the agency literature that under risk neutrality, incentive problem becomes important only when the manager's limited liability constraint binds, i.e., $f_i = 0$. In other words, under limited liability the efficient level of effort cannot be implemented. We therefore normalize $f_i = 0$ for all $i = 1, \ldots, n$. Thus, b_i will be interpreted as the power of managerial incentives. Since the realizations of marginal costs are independent across firms in the industry, managerial contract in one firm is not conditioned on the performance of a rival manager.

The economy lasts for four dates, $t = 0, 1, 2, 3$. At date 0, each firm i hires a manager by offering an incentive contract b_i. At $t = 1$ each manager exerts the non-verifiable effort e_i. At the end of the period, all firms observe its realized marginal cost and that of the rivals. At date 2, all firms compete in quantity in a common product market. Finally, at $t = 3$ all parties receive payments agreed upon in the contracts.

We analyze managerial efforts and incentives in a symmetric Nash equilibrium with n Cournot firms. Assume $(n+1)c < 1$ so that no firm shuts down. It is easy to show that the symmetric equilibrium is unique, although we cannot a priori discard the possible asymmetric equilibria.

Consider a representative Cournot firm i. Define by $\pi_i(c_i \mid k)$ the realized profit of firm i at marginal cost $c_i \in \{0, c\}$ when k out of the $n-1$ rival firms realize high marginal cost c, which is given by:

$$\pi_i(c_i \mid k) = \left(\frac{1 - nc_i + kc}{n+1}\right)^2.$$

In a symmetric Nash equilibrium, firm i induces an effort level e_i given that each rival firm j chooses a common effort level, i.e., $e_j = e$ for all $j \neq i$. Note that the probability that k out of $n-1$ rival firms realize a high marginal cost c when the managers exert a common effort level e is given by the following binomial density:

$$p_k(n-1, e) = \binom{n-1}{k} e^{n-1-k}(1-e)^k.$$

Therefore, the expected profit of the representative firm i at marginal cost c_i is given by:

$$\pi_i(c_i \mid e) = \sum_{k=0}^{n-1} p_k(n-1, e)\pi_i(c_i \mid k)$$

From the above it follows that the optimal managerial incentive and effort at firm i solve the following maximization problem:

$$\max_{\{e_i, b_i\}} \ \pi_i(e_i, b_i) := e_i[\pi_i(0 \mid e) - b_i] + (1-e_i)\pi_i(c \mid e)$$

$$\text{subject to} \quad U(e_i, b_i) := e_i b_i - \frac{e_i^2}{2} \geq 0, \qquad (\text{IR}_i)$$

$$e_i = \text{argmax}_{\hat{e}_i} \left\{\hat{e}_i b_i - \frac{\hat{e}_i^2}{2}\right\} = b_i. \qquad (\text{IC}_i)$$

Constraint (IR_i) is the *individual rationality* constraint of manager at firm i whose outside option is normalized to zero, and (IC_i) is the *incentive compatibility* constraint which asserts that the manager chooses effort e_i to maximize his expected utility given the bonus b_i. It is easy to show that (IR_i) always holds given (IC_i), and hence can be ignored. Further, substituting $b_i = e_i$ the objective function of firm i reduces to:

$$\pi_i(e_i, e) := e_i[\pi_i(0 \mid e) - e_i] + (1-e_i)\pi_i(c \mid e). \qquad (1)$$

Define by $\Delta\pi_i(e) := \pi_i(0 \mid e) - \pi_i(c \mid e)$ the expected marginal benefit of cost reduction of firm i, which is given by:

$$\Delta\pi_i(e) = \frac{nc(2-nc) + 2c^2 n(n-1)(1-e)}{(n+1)^2}.$$

The first order condition of the above maximization problem thus yields the following best reply in effort:

$$e_i(e) = \frac{\Delta\pi_i(e)}{2}.$$

Note that managerial efforts and incentives are strategic substitutes since $e_i'(e) < 0$. The following lemma characterizes the managerial efforts and incentives in a symmetric oligopoly equilibrium.

Lemma 2 *Under a Cournot oligopoly in the product market there is a unique symmetric Nash equilibrium in which the efforts and incentives are given by:*

$$e^*(n) = b^*(n) = \frac{nc}{2}\left[\frac{2+(n-2)c}{(n+1)^2+n(n-1)c^2}\right].$$

The proofs of the above lemma and of the subsequent results are relegated to the Appendix. Note that the equilibrium managerial effort and bonus are increasing in c. This is to say that higher values of c which correspond to larger gain from cost reduction induces each Cournot firm to offer more high-powered incentives to its manager. Consequently, the managerial effort is also higher. The following proposition analyzes the effect of an increase in the number of firms on the equilibrium managerial efforts and incentives.

Proposition 3 *Let $e^*(n)$ and $b^*(n)$ be the managerial effort and incentives in the unique symmetric Nash equilibrium under a Cournot oligopoly in the product market. Then, $e^*(n) > e^*(n+1)$ and $b^*(n) > b^*(n+1)$ for $n \geq 2$.*

The equilibrium managerial efforts and incentives are determined by the expected value of cost reduction of each firm i in a Cournot oligopoly, which is decreasing in the number of firms whenever there are more than two firms in the market. Schmidt (1997), among many others, take the degree of product market competition as parametrically given, and analyze the effect of this parameter on the incentive structure of a single representative firm. Such approach is related to the effect of competition on innovation (e.g Arrow, 1962; Belleflamme and Vergari, 2011) when only a single firm in a market innovates. In the current context, all n firms are engaged in process innovation in which action of one manager has strategic effects on that of others, and hence the marginal benefit of cost reduction is expressed in terms of its expected value.

1.3 More On Product Market Competition and Managerial Incentives

Interested readers should refer to Hart (1983), Scharfstein (1988), Hermalin (1992, 1994), Martin (1993), Raith (2003) and Golan, Parlour, and Rajan (2014); Dam and Robinson (2014) for more models that analyze the effects of product market competition on managerial incentives. There is also a plethora of empirical works (e.g. Nickell, 1996; Aggarwal and Samwick, 1999; Kole and Lehn, 1999; Palia, 2000; Karuna, 2007; Cuñat and Guadalupe, 2009) which analyze this topic.

2 The Bargaining Problem

2.1 The Axiomatic Approach to Bargaining

A bargaining problem is a surplus division problem between two players, 1 (agent) and 2 (principal) in a more general context than the principal-agent problem such as wage bargaining, bankruptcy claim, etc.

But for our purpose we would stick to the surplus division problem between a principal and an agent. Formally, a bargaining problem is defined as follows:

Definition 1 *A bargaining problem $B \in \mathscr{B}$ is a pair (S, d) where S is the set of possible agreements in terms of utilities where $u = (u_1, u_2)$ is an element of S, and $d \in S$ is the disagreement or threat point. The set $\mathscr{B}$ is the set of all bargaining problems.*

If an agreement $u \in S$ is reached in the bargaining process B, then the principal gets u_2 and the agent obtains u_1. In case the negotiations fail, the agent and the principal get d_1 and d_2, respectively. Note that in general there is a difference between the threat point $d = (d_1, d_2)$ and the outside options. Outside option of an individual is defined as the utility she/he obtains if the individual does not accept to be part of the bargaining problem, whereas d is the utility allocations if the bargaining process fails. Often we will abstract away from such distinction. Given a bargaining problem B we now require to define a solution to B.

Definition 2 *A solution to a bargaining problem $f : \mathscr{B} \to S$ is a mapping which assign to each bargaining problem $B \in \mathscr{B}$ an utility vector $u = (u_1, u_2) \in S$ such that $u_1 = f_1(B)$ and $u_2 = f_2(B)$.*

Suppose we require that the solution f must satisfy a set of "desirable properties". Can we characterize f completely? This is the basic premise of the axiomatic approach to bargaining. Thus, consider the following axioms:

PAR Pareto efficiency:
Given a bargaining problem $B = (S, d)$, the solution $f(B)$ satisfies the Pareto property if there is no $u' = (u'_1, u'_2) \in S$ and $u' \neq f(B)$ such that $u'_1 > f_1(B)$ and $u'_2 > f_2(B)$.

SYM Symmetry:
Given a bargaining problem $B = (S, d)$, if S is a symmetric set, i.e., S is symmetric around the 45^0 line and $d_1 = d_2$, then $f_1(B) = f_2(B)$.

IUO Independence of utility origins:
Let $B = (S, d)$ and $B' = (S', d')$ be two bargaining problems such that $S' = S + b$ and $d' = d + b$ for some vector $b = (b_1, b_2) \in \mathbb{R}^2$. Then $f(S', d') = f(S, d) + b$.

Note that the above implies that $f(S, d) = f(S \setminus \{d\}, 0) + d$, and hence from now on it would be convenient to normalize our problem to $d = (0, 0)$, and we denote a generic bargaining problem by $B = (S, 0)$.

IUU Independence of utility units:
Let $B = (S, 0)$ and $B' = (S', 0)$ be two bargaining problems such that $S' = \alpha S$ for some vector $\alpha = (\alpha_1, \alpha_2) \in \mathbb{R}^2_{++}$. Then $f(S', 0) = \alpha f(S, 0)$.

IIA Independence of irrelevant alternatives:
Let $B = (S, 0)$ and $B' = (S', 0)$ be two bargaining problems such that $S' \subset S$ and $f(B) \in S'$, then $f(B') = f(B)$.

Following are some solutions to a given bargaining problem B.

Example 1 (Egalitarian solution) The egalitarian solution $f^e(B)$ implies that the surplus is split equally between the principal and the agent, i.e., $f_1^e(B) = f_2^e(B)$. In other words, given $B = (S, 0)$, the solution picks a point on the frontier which lies on the 45^0 line. ■

Example 2 (Utilitarian solution) The utilitarian solution $f^u(B)$ maximizes $u + v$ on S. If S is strictly convex, then $f^u(B)$ is the unique tangency point of the line $u_1 + u_2 = c$ for some $c > 0$ to the upper boundary of S. ■

Example 3 (Nash solution) The Nash solution $f^N(B)$ maximizes $H(u_1, u_2) = u_1 u_2$ on S. If S is strictly convex, then $f^N(B)$ is the unique tangency point of the hyperbola $u_1 u_2 = c$ for some $c > 0$ to the upper boundary of S. The Nash solution has a very nice geometry. Denote by $u^* = f^N(B)$, and consider the set

$$P := \{(u_1, u_2) \in \mathbb{R}_+^2 \mid u_1 u_2 \geq c\} \quad \text{for some } \; c > 0,$$

which is the upper contour set of the hyperbola $u_1 u_2 = c$. Since (u_1^*, u_2^*) maximizes $H(u_1, u_2)$ on S, and both S and P are strictly convex sets (P is strictly convex since $H(u_1, u_2)$ is strictly quasi concave, by the separating hyperplane theorem, there is a unique line through u^* which supports both S and P and separates them. Note that this line is given by:

$$S^* = \left\{ (u_1, u_2) \in \mathbb{R}^2 : \frac{u_1}{u_1^*} + \frac{u_2}{u_2^*} = 2 \right\}.$$

Note also that the above line intersects the horizontal axis at $(2u_1^*, 0)$ and the vertical axis at $(0, 2u_2^*)$, and hence (u_1^*, u_2^*) is the midpoint of the segment $[(0, 2u_2^*), (2u_1^*, 0)]$. ■

Convince yourselves that neither f^e nor f^u satisfies IUU, and f^N satisfies all the five axioms.

2.2 The Nash Solution

Now we state a very important result due to Nash (1950) that f^N is the only solution that satisfies all the above five axioms.

Theorem 1 (Nash, 1950) *The Nash bargaining solution is the only solution that satisfies PAR, SYM, IUU and IIA.*

Proof. First we show that the Nash solution is well defined and unique. The function $H(u_1, u_2) = u_1 u_2$ which is a continuous function, and S is a compact set. Then by Weierstrass theorem, a maximizer $f^N(B)$ of H on S exists. Moreover, since S is assumed to be strictly convex, and H is strictly quasi concave, the solution is unique.

Next we show that the Nash solution satisfies PAR, SYM, IUU and IIA. The function $H(u_1, u_2)$ is a strictly increasing function in its arguments. If $f^N(B)$ does not maximize $H(u_1, u_2)$ then there would exist $u \in S$ such that $u_i > u_i^*$ for $i = 1, 2$. Therefore, f^N satisfies PAR. Next, let B be symmetric, and (u_1^*, u_2^*) maximizes $H(u_1, u_2)$ on S. Since H is a symmetric function, then (u_2^*, u_1^*) also maximizes $H(u_1, u_2)$ on S. Since the maximizer is unique, we have $u_1^* = u_2^*$, and hence f^N is symmetric. To show that f^N satisfies IUU, take B and B' such that $S' = \alpha S$ for some $\alpha \in \mathbb{R}_{++}^2$. Then, $(u_1', u_2') \in$

S' if and only if there is $(u_1, u_2) \in S$ with $(u'_1, u'_2) = (\alpha_1 u_1, \alpha_2 u_2)$. Therefore, we have $H(u'_1, u'_2) = u'_1 u'_2 = \alpha_1 \alpha_2 u_1 u_2 = \alpha_1 \alpha_2 H(u_1, u_2)$. Thus, (u^*_1, u^*_2) maximizes $H(u_1, u_2)$ on S if and only if $(\alpha_1 u^*_1, \alpha_2 u^*_2)$ maximizes $H(u'_1, u'_2)$ on $S' = \alpha S$. Finally, if $S \subset S'$ and $(u^*_1, u^*_2) \in S$ maximizes $H(u_1, u_2)$ on S', then it also maximizes $H(u_1, u_2)$ on S, and hence the Nash solution satisfies IIA.

For a formal proof of the converse statement see (Osborne and Rubinstein, 1990, pp. 14). We will do a geometric proof. Given a bargaining problem B, take a candidate solution f which PAR, SYM, IUU and IIA. We will show that $f(B) = f^N(B)$ for all B. Consider the sets

$$S' = \left\{ (u_1, u_2) \in \mathbb{R}^2 : \frac{u_1}{u^*_1} + \frac{u_2}{u^*_2} \leq 2 \right\} \quad \text{and} \quad S'' = \left\{ (u_1, u_2) \in \mathbb{R}^2 : u_1 + u_2 \leq 2 \right\}.$$

which define two new bargaining problems $B' = (S', 0)$ and $B'' = (S'', 0)$. Since B'' is symmetric, and f satisfies PAR and SYM, we have $f(B'') = (1, 1)$. By construction, $S' = u^* S''$. Therefore, we have $f(B') = (u^*_1, u^*_2)$ since f satisfies IUU. Since $S \subseteq S'$ and $f(B') \in S$, by IIA we have $f(B) = f(B') = u^* = f^N(B)$. $\square$

2.3 The Generalized Nash Solution

Given a bargaining problem B, the generalized Nash solution $f^\beta(B)$ maximizes the generalized Nash product $H(u_1, u_2) = u_1^{\beta_1} u_2^{\beta_2}$ on S. Clearly, this solution is not symmetric as long as $\beta_1 \neq \beta_2$.

Theorem 2 *The generalized Nash bargaining solution is the only solution that satisfies PAR, IUU and IIA.*

Proof. Left as an exercise. Hint: assume $\beta_1 + \beta_2 = 1$ and as in the previous theorem make use of the sets

$$S' = \left\{ (u_1, u_2) \in \mathbb{R}^2 : \frac{\beta_1 u_1}{\hat{u}_1} + \frac{\beta_2 u_2}{\hat{u}_2} \leq 2 \right\} \quad \text{and} \quad S'' = \left\{ (u_1, u_2) \in \mathbb{R}^2 : u_1 + u_2 \leq 1 \right\}$$

assuming that $f(S'') = (\beta_1, \beta_2) \neq (1/2, 1/2)$ where $f^\beta(B) = (\hat{u}_1, \hat{u}_2)$. Then apply IUU and IIA. $\square$

The parameter β_i is interpreted as the bargaining power of individual $i = 1, 2$ since as β_1/β_2 increases $\hat{u}_1$ becomes larger relative to $\hat{u}_2$. Thus, by normalizing $\beta_1 + \beta_2 = 1$ and varying β_1 between 0 and 1, one generates the entire UPF.

2.4 Relationship With the Principal-Agent Problem

Consider a simple moral hazard problem in effort choice with the agent's outside option $\bar{u}_1$. An optimal contract x, which is subject to incentive compatibility, limited liability, etc. induces the value function $\phi(\bar{u}_1)$ for the principal where $\bar{u}_1 \in [0, u_1^{max}]$. Formally,

$$\phi(\bar{u}_1) := \max_{x \in X} \{EU_2(x) \mid EU_1(x) = \bar{u}_1\}.$$

The above is the equation of the Pareto or the utility possibility frontier (UPF) on which any point (u_1, u_2) is given by $u_1 = \bar{u}_1$ and $u_2 = \phi(\bar{u}_1)$. By varying $\bar{u}_1$ on $[0, u_1^{max}]$ one thus generates the entire UPF.

Convince yourselves that under incentive problems $\phi(u_1)$ is a concave downward sloping function. Given a principal-agent relationship, the above is a standard approach to the surplus division problem where the principal assumes all the bargaining power and makes a 'take-it-or-leave-it' incentive compatible contract x to the agent. Note that the same Pareto frontier is induced by solving the following problem:

$$\psi(\bar{u}_2) := \max_{x \in X} \{EU_1(x) \mid EU_2(x) = \bar{u}_2\},$$

where $\bar{u}_2$ is the outside option of the principal. Clearly, $\bar{u}_2 = \phi(\psi(\bar{u}_2))$, i.e., $\psi \equiv \phi^{-1}$. In the above problem, the agent assumes all the bargaining power and makes a 'take-it-or-leave-it' contract offer to the principal. By varying $\bar{u}_2$ one generates the entire frontier. This immediately implies that the equilibrium utility allocations between principal and agent depends the values of $\bar{u}_1$ and $\bar{u}_2$, and who designs the contract is irrelevant.

Now consider the problem where the principal assumes all the bargaining power which generates $\phi(\bar{u}_1)$, and consider the set

$$S = \{(u_1, u_2) : u_2 \leq \phi(u_1) \text{ and } u_1 = \bar{u}_1\}.$$

The above set defines a bargaining problem. Given a value of $\bar{u}_1$, any equilibrium utility allocation $(\hat{u}_1, \hat{u}_2)$ is a point on the UPF. This point can be thought of as a maximizer of the generalized Nash product $u_1^{\beta} u_2^{1-\beta}$ for some $\beta \in [0, 1]$. The converse is also true. Therefore, there is an equivalence between the generalized Nash solution and the standard principal-agent problem.

3 Subjective Performance Evaluation

In the standard principal-agent model under moral hazard [in Chapter 1] the performance q of the relationship, which is a noisy signal of the agent's effort, has been assumed to be publicly verifiable, and hence any contract can be made contingent on q. Let q represents the quality of a good produced by the agent. Often the agent does not possess ability to assess the quality, but the principal does. On the basis of such assessment, the principal designs a contract for the agent. Such situation is called the *subjective performance evaluation* (SPE) as opposed to objective evaluation where q is publicly observable. In what follows we discuss two models of incentive contracting under SPE.

3.1 A Simple Model of Optimal Money Burning Contract

Consider a simple version of MacLeod (2003) where a risk neutral principal (P) hires a risk neutral agent (A) to work on a project. The agent spends non-verifiable effort $e \in \{0, 1\}$ by incurring a total cost $\psi(e) = \varphi e$ with $\varphi > 0$. Agent's effort choice induces a probability distribution over the stochastic return (performance) $q \in \{q_H, q_L\}$ with $q_H > q_L \geq 0$ of the project which is given by:

$$\Pr.[q = q_H \mid e = 0] = \pi_0,$$
$$\Pr.[q = q_H \mid e = 1] = \pi_1,$$

with $0 < \pi_0 < \pi_1 < 1$. Principal observes the realization of the return privately, and sends message $m \in M = \{m_H, m_L\}$ to the agent. Under SPE, a contract between the principal and the agent is contingent on the message sent by the principal. We may restrict attention to the direct revelation mechanism, i.e., $M = \{y_H, y_L\}$.

Now consider standard bonus contract $\gamma = (z, b)$ under limited liability discussed in Chapter 1, where z is the fixed salary and b is the bonus if $q = q_H$. Let $V_i(\gamma, q_j)$ be P's payoff if she observes $q = q_i$ for $i = H, L$, but reports $m = q_j$ for $j = H, L$. Then,

$$V_H(\gamma, q_H) = q_H - z - b,$$
$$V_H(\gamma, q_L) = q_H - z.$$

If $b > 0$, then $V_H(\gamma, q_H) < V_H(\gamma, q_L)$, i.e., P never has incentives to report truthfully. One way to induce principal to report truthfully is that P puts b on the table before A exerts effort, and this amount goes to A if P reports q_H, and is destroyed otherwise. In this case $V_H(\gamma, q_L) = q_H - z - b$ which implies that $V_H(\gamma, q_H) = V_H(\gamma, q_L)$. Thus, the organization must burn the amount b in order to incentivize the principal to report truthfully.

We now analyze the optimal money burning contract for such organization. Assume that P has a fixed budget W. A contingent money burning contract $\gamma = (w_H, w_L, b_H, b_L)$ is such that for $i = H, L$

$$w_i = w(q_i), \quad b_i = b(q_i),$$
$$w_i + b_i = W$$

Thus, a contract can be alternatively represented by $\gamma = (W, b_H, b_L)$. We assume that $q_H - q_L$ is high enough so that P always wants to implement the high effort $e = 1$.

Under a money burning contract A's expected payoffs are given by:

$$U(e = k \mid \gamma) = W - [\pi_k b_H + (1 - \pi_k) b_L] - \varphi k \quad \text{for } k = 0, 1.$$

Therefore, the agent's incentive compatibility implies

$$U(e = 1 \mid \gamma) \geq U(e = 0 \mid z) \iff b_L - b_H \geq \frac{\varphi}{\pi_1 - \pi_0}. \tag{ICA}$$

The agent is protected by limited liability, i.e., his state-contingent income must be non-negative, which is given by:

$$b_L \leq W, \quad b_H \leq W. \tag{LL}$$

Feasibility of money burning, on the other hand, requires

$$b_L \geq 0, \quad b_H \geq 0. \tag{F}$$

Agent's outside option is normalized to 0, and hence his individual rationality is given by:

$$W - [\pi_1 b_H + (1 - \pi_1) b_L] - \varphi \geq 0. \tag{IRA}$$

It is easy to verify that given constraints (ICA) and (LL), the individual rationality constraint is automatically satisfied. On the other hand, (ICA) implies that $b_L \geq b_H$, and hence the constraints $b_H \leq W$

and $b_L \geq 0$ can be ignored. The optimal money burning contract $\gamma^* = (W^*, b_H^*, b_L^*)$ solves the following minimization problem:

$$\min_{\{W, b_H, b_L\}} \; W$$
$$\text{subject to } (ICA), \; b_L \leq W \text{ and } b_H \geq 0.$$

The following lemma characterizes the optimal money burning contract.

Lemma 3 *The optimal money burning contract $z^* = (w_H^*, w_L^*, b_H^*, b_L^*)$ is given by:*

$$w_H^* = \frac{\varphi}{\pi_1 - \pi_0} > 0, \quad w_L^* = 0,$$
$$b_L^* = \frac{\varphi}{\pi_1 - \pi_0} > 0, \quad b_H^* = 0.$$

The above result is fairly intuitive. The state-contingent wages paid to the agent must be different in order to elicit effort. The wage paid in state H must be destroyed if the principal reports a low performance, i.e., $w_H^* = b_L^*$ so that she does not have incentives to misreport. Thus, the organization incurs a deadweight loss which is equal to $\pi_1 \varphi / (\pi_1 - \pi_0)$, the expected money burning.

Clearly, it is necessary for an organization destroy resources in order to ensure incentive compatibility. What happens if money burning is not feasible? Then things are even worse in this simple model. Note that when $b_H = b_L = 0$, with a fixed budget the principal must commit a fixed wage schedule $w_H = w_L = \bar{w}$ to the agent. This destroys A's incentive to exert effort since

$$U(e = 1 \mid \bar{w}) = \bar{w} - \varphi < \bar{w} = U(e = 0 \mid \bar{w}).$$

Therefore,

Lemma 4 *If money burning is not feasible under SPE, then there is no contract that implements high effort $e = 1$.*

Thus, in order to implement the high effort some degree of inefficiency via money burning is necessary for an organization. Several attempts (e.g. Levin, 2003; MacLeod, 2003; Fuchs, 2007; Chan and Zheng, 2011; Lang, 2014) have been made to search for contractual instruments to minimize expected money burning in an organization. In the next subsection we will discuss the model prescribed by Bester and Münster (2013).

3.2 Termination Contracts under SPE and Public Information

Consider a principal-agent relationship where a risk neutral principal (P) hires a risk neutral agent (A) to accomplish a task which yields a stochastic output $q \in \{q_H, q_L\}$ with $q_H > q_L \geq 0$ and $\Pr.[q = q_H \mid e] = e$ where $e \in [0, 1]$ is the effort chosen by A. The agent incurs a cost of effort given by $\psi(e) = e^2/2$. Effort is not observable, and hence there is the standard moral hazard problem in effort choice. The principal privately observes the final realization of output q. In a standard principal-agent model as in Chapter 1 this output is verifiable by a third party, and hence all contracts can be made contingent on

q. Here, we assume that the third party observes an imprecise signal $s \in \{s_H, s_L\}$ of the output q where $\Pr.[s_i \mid q_i] = \sigma > 0.5$ for $i = H, L$ and $\Pr.[s_i \mid q_j] = 1 - \sigma < 0.5$ for $i, j = 1, 2$ and $i \neq j$, i.e., the signal is correct with probability σ. Thus, for $\sigma = 1$ we have the standard principal-agent model under moral hazard.

A contract can be made contingent on the public signal s and the 'report' $m \in M$ by the principal. By the revelation principle there is no loss of generality in restricting attention to a message space $M = \{q_H, q_L\}$. Suppose that Let $w_{ij} = w(\sigma_i, q_j)$ be the wage received by the agent when the public signal is s_i and P sends a message $m = q_j$ for $i, j = H, L$, and $b_{ij} = b(s_i, q_j)$ be the associated amount of money burning. Moreover, a contract specifies a probability θ_{ij} of firing the agent before project completion in which case P looses $\alpha \in (0, 1]$ fraction of the output. Denote by $\gamma = (w, b, \theta)$ a contract which must satisfy $b \geq 0$.

The economy lasts for four dates $t = 0, 1, 2, 3$. At $t = 0$, P designs the contract. At date 1, A accepts or rejects the contract. If the contract is accepted, A exerts effort. At $t = 2$, the principal observes output and sends message to the agent. Finally, at $t = 3$ the public signal realizes and the contract is executed.

Let $V_L(\gamma, q_j)$ and $V_H(\gamma, q_j)$ be the expected payoffs of the principal when she observes outputs q_L and q_H, respectively, which are given by:

$$V_L(\gamma, q_j) = \sigma[(1-\alpha\theta_{Lj})q_L - w_{Lj} - b_{Lj}] + (1-\sigma)[(1-\alpha\theta_{Hj})q_L - w_{Hj} - b_{Hj}], \tag{2}$$

$$V_H(\gamma, q_j) = \sigma[(1-\alpha\theta_{Hj})q_H - w_{Hj} - b_{Hj}] + (1-\sigma)[(1-\alpha\theta_{Lj})q_H - w_{Lj} - b_{Lj}]. \tag{3}$$

Thus, P's incentive compatibility constraints [for truthful revelation] are given by:

$$V_L(\gamma, q_L) \geq V_L(\gamma, q_H), \tag{ICPL}$$

$$V_H(\gamma, q_H) \geq V_H(\gamma, q_L). \tag{ICPH}$$

Since P reports truthfully, her ex-ante expected payoff at $t = 2$ is given by:

$$V(\gamma, e) = eV_H(\gamma, q_H) + (1-e)V_L(\gamma, q_L). \tag{4}$$

Truthful reporting by the principal also implies that the agent's expected payoffs are given by:

$$U_L(\gamma) = \sigma w_{LL} + (1-\sigma)w_{HL}, \tag{5}$$

$$U_H(\gamma) = \sigma w_{HH} + (1-\sigma)w_{LH}, \tag{6}$$

when P observes q_L and q_H, respectively. Therefore, the agent's ex-ante expected payoff at $t = 2$ is given by:

$$U(\gamma, e) = eU_H(\gamma) + (1-e)U_L(\gamma) - \frac{e^2}{2}. \tag{7}$$

At $t = 1$, the incentive compatibility constraint of A is thus given by:

$$U_H(\gamma) - U_L(\gamma) = e. \tag{ICA}$$

Finally, the participation constraint of the agent is given by:

$$U(\gamma, e) \geq 0. \tag{PC}$$

The maximization problem of the principal is given by:

$$\max_{\{\gamma,e\}} V(\gamma,e)$$

$$\begin{aligned}
\text{subject to}\quad & V_L(\gamma,q_L) \geq V_L(\gamma,q_H), && \text{(ICPL)}\\
& V_H(\gamma,q_H) \geq V_H(\gamma,q_L), && \text{(ICPH)}\\
& U_H(\gamma) - U_L(\gamma) = e, && \text{(ICA)}\\
& U(\gamma,e) \geq 0, && \text{(PC)}\\
& w_{ij} \geq 0 \ \text{ for all } i,\, j = H, L, && \text{(LL)}\\
& b_{ij} \geq 0 \ \text{ for all } i,\, j = H, L, && \text{(MB)}\\
& 0 \leq \theta_{ij} \leq 1 \ \text{ for all } i,\, j = H, L, && \text{(F)}
\end{aligned}$$

In a series of results we characterize the optimal contract for the organization. See Bester and Münster (2013) for omitted proofs. The following lemma states some important properties of the optimal contract.

Lemma 5 *Let (γ,e) solves the above maximization problem. Then*

(a) *The participation constraint (PC) of the agent does not bind;*

(b) *The optimal contract γ satisfies $w_{HL} = w_{LL} = 0$, $b_{HH} = b_{LH} = b_{LL} = 0$, and $\theta_{HH} = \theta_{LH} = 0$;*

(c) *$b_{HL} > 0$ implies $\theta_{HL} = 1$, and $\theta_{LL} > 0$ implies $\theta_{HL} = 1$;*

(d) *Out of the two incentive compatibility constraints of the principal, (ICPH) binds but (ICPL) does not bind;*

(e) *(γ,e) satisfies*

$$e = \sigma(\alpha\theta_{HL}q_H + b_{HL}) + (1-\sigma)\alpha\theta_{LL}q_H; \tag{8}$$

(f) *The wages w_{HH} and w_{LH} are determined (not uniquely) from*

$$e = \sigma w_{HH} + (1-\sigma)w_{LH}.$$

Part (a) is a standard result for optimal contract under limited liability where the agent earns efficiency wage. By part (b), the wage payments can be positive only if the output is high. If w_{HL} and w_{LL} were positive, the principal could decrease these payments while increasing b_{HL} and b_{LL} by the same amount. This would relax (ICA) while the other constraints remain unaffected. Moreover, part (b) also implies that firing and money burning can only occur if output is low. In fact, only (ICPH) will bind at the optimum since P has incentives to *underreport*. Lowering any of θ_{HH}, θ_{LH}, b_{HH} or b_{LH} increases $V_H(\cdot,q_H)$ thereby making underreporting less tempting for the principal while leaving the other constraints unaffected and increasing P's payoff, and hence these variables must be set at zero.

The logic behind $b_{LL} = 0$ is a bit less intuitive. Note that money burning in the case when P reports low output is an instrument to deter her from underreporting. Suppose $b_{LL} > 0$. Then, b_{LL} can be lowered and b_{HL} increased which will increase P's payoff. If b_{LL} lowered at the margin, $V_H(\cdot,q_L)$ increases by $1-\sigma$, while it decreases by σ if b_{HL} is increased marginally. Since $\sigma > 1-\sigma$, b_{HL} has a stronger incentive effect than b_{LL}. Moreover, b_{HL} affects the principal's payoff less adversely than b_{LL}, since b_{HL} has to be paid only when output is low but the public signal is high (which occurs with probability $(1-e)(1-\sigma)$), whereas b_{LL} has to be paid in the more likely event that output is low and the public signal is low as well (which occurs with probability $(1-e)\sigma$).

At this juncture, you must have realized that there are two instruments, namely firing and money burning, to deter P from underreporting. If $\theta_{LL} > 0$ then it can be lowered marginally which would increase $V_H(\cdot, q_L)$ by $\alpha(1-\sigma)q_H$ while a marginal increase in θ_{HL} decreases this payoff from misreporting by $\alpha\sigma q_H$, and hence θ_{HL} appears to be a stronger incentive device. Thus, one must resort to θ_{HL} before using θ_{LL}. A similar logic applies for the first statement of part (c). Firing when output is high is more costly for P than money burning since burning one dollar always costs one dollar. Using θ_{HL} has the advantage that firing only occurs when output is low, but it deters P from underreporting when output is high.

Part (d) carries the same intuition as any adverse selection model where only the 'downward incentive constraint' matters. By part (b), we have

$$\begin{aligned}
&U_L(\gamma) = 0,\\
&U_H(\gamma) = \sigma w_{HH} + (1-\sigma)w_{LH},\\
&V_H(\gamma, q_H) = q_H - [\sigma w_{HH} + (1-\sigma)w_{LH}],\\
&V_H(\gamma, q_L) = q_H - [\sigma(\alpha\theta_{HL}q_H + b_{HL}) + (1-\sigma)\alpha\theta_{LL}q_H].
\end{aligned}$$

By (ICA), $e = \sigma w_{HH} + (1-\sigma)w_{LH}$. Since $V_H(\gamma, q_H) = V_H(\gamma, q_L)$ by part (d), condition (8) follows. Finally part (f) is just (ICA) rewritten. This also allows P to set $w_{HH} = w_{LH} = w_H = e$, i.e., A's wage w_H can be made dependent only on P's message, and not on the public signal.

From the above lemma it follows that the only variables remain to be determined are the firing probabilities θ_{HL} and θ_{LL}, and money burning b_{HL} since e can is uniquely determined from (8).

Lemma 6 *There exists a unique* $\widehat{\sigma} \in (1/2, 1)$ *such that*

(a) $\theta_{LL} = 0$ *if* $\sigma > \widehat{\sigma}$*;*

(b) *If* $\sigma < \widehat{\sigma}$*,* $b_{HL} > 0$ *implies* $\theta_{LL} = 1$.

The above lemma asserts that the public signal is sufficiently precise, firing should not be used as an incentive device when both public signal and output are low. It is more attractive to use b_{HL} to deter P from underreporting for two reasons. There is a small chance that the event $\{s_H \mid q_L\}$ occurs, and hence the likelihood of actual money burning is low as well. Second, when σ is sufficiently large, given that output is high, the public signal is also likely to be high. Therefore, if P underreports she will pay b_{HL} with high probability. When $\sigma < \widehat{\sigma}$, the countervailing consideration [that money burning is less effective than firing] makes θ_{LL} a more attractive instrument than b_{HL}. The following proposition analyzes the main result.

Proposition 4 *Suppose that* $\sigma > \widehat{\sigma}$*. Then there is a unique cutoff* $\widehat{\alpha} \in (0, 1)$ *of the loss from project termination such that*

(a) $b_{HL} > 0$ *and* $\theta_{HL} = 1$ *if* $\alpha < \widehat{\alpha}$*;*

(b) $b_{HL} = 0$ *and* $\theta_{HL} \in (0, 1)$ *if* $\alpha \geq \widehat{\alpha}$.

The above proposition shows that project termination and money burning are clearly ranked as incentive devices for truthful reporting: Money burning occurs as a secondary instrument when the other instrument is exhausted in the sense that θ_{HL} cannot be increased further as it is bounded above by 1. Indeed, money burning is completely eliminated if the loss from project termination is relatively high.

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