CHAPTER 3: Adverse Selection*

1 Examples of adverse selection

By "adverse selection" we refer to a situation where the principal cannot observe the characteristics of the agent she is contracting with. An agent may have private information about his "type", e.g. ability, productivity, efficiency, etc. which he may use in order to extract "rent" for the principal. The principal's objective is to design a contract, often called *mechanism*, that offers adequate incentives to the agent so that he reveals such private information. Following are some typical examples of adverse selection.

Example 1 Suppose a seller (principal) sells wines of high and low quality. The buyer may be of two types: sophisticated who is willing to pay a high price for vintage wine, and with moderate taste who is content with low-quality wine. If the seller could observe perfectly the type of buyer she is dealing with, she could have used discriminatory pricing: (a) a wine of high quality for a high price, and (b) a wine of low quality for a low price with the objective that the sophisticated agent would choose the first contract and the other type would settle for the other contract. One main objective of this chapter is to analyze under what conditions such allocation rules are possible to implement. ■

Example 2 In a labor market, the workers differ in their abilities which are private information to them. The firms which hire them do not know to which type of workers the jobs are being offered. The firms then must screen the workers so as to lure the high-ability workers and discard the others. ■

Example 3 In a regulated market, the firms often possess more information about their technology than the regulator. Such informational advantages can be used in the firms' favor to increase their profits. ■

We will study particular economic situations consisting of adverse selection problems, although the methodology of solving such problems is quite general.

2 Timing of events

The principal-agent relationship lasts for four dates, $t = 0, 1, 2, 3$. At date 0, the agent discovers his type θ. At $t = 1$, the principal offers a contract. At $t = 2$, the agent accepts or rejects the contract. Finally, at date 3 the contract is executed.

*These notes are heavily borrowed from Bolton and Dewatripont (2005).

3 A model of adverse selection

3.1 Monopolistic screening with two types

Consider an economy as in Maskin and Riley (1984) where a monopolist produces and sells a private good to a consumer. The total cost of production is cq where q is the quantity produced and $c > 0$ is the constant marginal cost of production. The consumer has utility function $U(t, q; \theta_i) = \theta_i V(q) - t$ with $V(0) = 0$, $V' > 0 > V''$, where t is the total price paid to the monopolist, and $\theta_i \in \{\theta_H, \theta_L\}$ represents his marginal valuation or "type". The consumer may have "high" or "low" valuation, i.e., $\theta_H > \theta_L$ which is his private information. The seller only knows that the consumer is of type θ_i with probability λ_i. This situation is equivalent, due to the law of large numbers, to the one in which the seller faces a continuum of consumer, a λ_H proportion of which are of high type. The profit of the monopolist is given by $\Pi = t - cq$. A contract is a type-contingent allocation rule (q_i, t_i) for $i = H, L$ where q_i is the quantity sold to, and t_i is the transfer received by the seller from type i.

3.1.1 Symmetric information: perfect price discrimination

Suppose first that the monopolist can observe perfectly the types of the consumer. The monopolist chooses (q_i, t_i) for $i = H, L$ to solve the following maximization problem:

$$\max_{\{q_i, t_i\}} \; t_i - cq_i, \tag{P$_1$}$$

$$\text{subject to } \; \theta_i V(q_i) - t_i \geq 0, \;\text{ for }\; i = H, L. \tag{IR$_i$}$$

The constraint (IR$_i$) is the individual rationality constraint of a consumer of type θ_i under the assumption that the consumer obtains zero if he does not consume anything. The monopolist will extract the entire consumer surplus in order to maximize profit, and hence will set $t_i = \theta_i V(q_i)$ for $i = H, L$. Then the maximization problem reduces to

$$\max_{q_i} \; \theta_i V(q_i) - cq_i. \tag{P$_1'$}$$

The first order conditions give $\theta_i V'(q_i^*) = c$ for $i = H, L$. Therefore,

$$V'(q_L^*) = \frac{c}{\theta_L} > \frac{c}{\theta_H} = V'(q_H^*) \implies q_L^* < q_H^*, \text{ since } V'' < 0.$$

Hence, $t_i^* = \theta_i V(q_i^*)$ for $i = H, L$. Assume that the surplus function is given by:

$$V(q) = \frac{1}{2}\left[1 - (1-q)^2\right].$$

The demand function of type θ_i will be

$$D_i(p) = 1 - \frac{p}{\theta_i}.$$

And the consumer surplus of type θ_i is given by

$$S_i(p) = \frac{(\theta_i - p)^2}{2\theta_i}.$$

The optimal contracts are given by:

$$q_i^* = 1 - \frac{c}{\theta_i} \iff p^* = c,$$
$$t_i^* = \theta_i V(q_i^*) = \frac{\theta_i^2 - c^2}{2\theta_i}.$$

Notice that

$$t_i^* = S_i(c) + cq_i^* \equiv A_i + cq_i^*.$$

Thus, optimality is achieved by a *type-specific two-part tariff* in which a consumer of type θ_i can buy any quantity by paying a per-unit price c and a type-contingent fixed fee equal to $S_i(c)$. This is perfect price discrimination. The entire consumer surplus is extracted, and the monopolist sets a price equal to marginal cost. The total surplus is maximized, and it is divided between the buyer and the seller according to the participation constraint.

3.1.2 Asymmetric information: non-linear pricing

Now suppose that the monopolist cannot observe the consumer's type, but only knows its probability distribution λ_H and λ_L. Notice that

$$U(t_L^*, q_L^*; \theta_H) = \theta_H V(q_L^*) - t_L^* = \theta_H V(q_L^*) - \theta_L V(q_L^*) + \theta_L V(q_L^*) - t_L^* = (\theta_H - \theta_L) V(q_L^*) > 0,$$
$$U(t_H^*, q_H^*; \theta_L) = \theta_L V(q_H^*) - t_H^* = \theta_L V(q_H^*) - \theta_H V(q_H^*) + \theta_H V(q_H^*) - t_H^* = -(\theta_H - \theta_L) V(q_H^*) < 0.$$

From the above it is clear that, if the symmetric information contracts $((q_L^*, t_L^*), (q_H^*, t_H^*))$ are offered, then the high-type consumer will choose the contract for the low-type. The low-type consumer although will continue choosing the contract (q_L^*, t_L^*) since by choosing the other contract, he looses. Hence, the first-best contracts will not be optimal under asymmetric information.

When the types of the consumers are private information the monopolists will have to find contracts (q_L, t_L) and (q_H, t_H) which are incentive compatible as well. Such mechanism is a direct mechanism where the consumer announces his type, and based on his announcement he is offered the contracts $(q(\theta_i), t(\theta_i))$ for $i \in \{H, L\}$. Alternatively, the monopolist just quotes a menu (q_L, t_L) and (q_H, t_H), and the consumer selects his allocation. Let the utility of a consumer of type θ_i announces that his type is θ_j be $U(\theta_j; \theta_i) = \theta_i V(q(\theta_j)) - t(q(\theta_j))$. Incentive compatibility requires that $U(\theta_i) \equiv U(\theta_i; \theta_i) \geq U(\theta_j; \theta_i)$

for $\theta_i,\, \theta_j \in \{\theta_H,\, \theta_L\}$. Hence, the monopolist's problem is to

$$\begin{aligned}
\max_{\{(q_L,t_L),(q_H,t_H)\}} \quad & \lambda_L[t_L - cq_L] + \lambda_H[t_H - cq_H], && \text{(P}_2\text{)}\\
\text{subject to} \quad & U(\theta_L) \geq U(\theta_H,\, \theta_L), && \text{(IC}_L\text{)}\\
& U(\theta_H) \geq U(\theta_L,\, \theta_H), && \text{(IC}_H\text{)}\\
& U(\theta_L) \geq 0, && \text{(IR}_L\text{)}\\
& U(\theta_H) \geq 0, && \text{(IR}_2\text{)}
\end{aligned}$$

Notice that (IC$_L$) and (IC$_H$) together imply

$$\begin{aligned}
& (\theta_H - \theta_L)V(q_L) \leq U(\theta_H) - U(\theta_L) \leq (\theta_H - \theta_L)V(q_H) && \text{(A)}\\
\implies & (\theta_H - \theta_L)(V(q_H) - V(q_L)) \geq 0\\
\implies & V(q_H) - V(q_L) \geq 0\\
\implies & q_H > q_L, \;\text{ since }\; V' > 0.
\end{aligned}$$

The first and the second inequalities in (A) follow from (IC$_H$) and (IC$_L$), respectively. From (A) it also follows that $U(\theta_H) \geq U(\theta_L)$. This implies that $U(\theta_L)$ will be set as low as possible. Given (IR$_L$) we get $U(\theta_L) = 0$, and hence

$$\theta_L V(q_L) = t_L. \tag{B}$$

We impose (IC$_H$) to be binding. This gives

$$\begin{aligned}
& U(\theta_H) = U(\theta_L,\, \theta_H)\\
\implies & \theta_H V(q_H) - t_H = \theta_H V(q_L) - t_L\\
\implies & \theta_H V(q_H) - t_H = (\theta_H - \theta_L)V(q_L) \qquad \text{[using (B)]}\\
\implies & t_H = \theta_H V(q_H) - (\theta_H - \theta_L)V(q_L). && \text{(C)}
\end{aligned}$$

Substituting (B) and (C) into the objective function one reduces the principal's maximization problem as

$$\max_{\{q_L,q_H\}} \lambda_L[\theta_L V(q_L) - cq_L] + \lambda_H[\theta_H V(q_H) - cq_H - (\theta_H - \theta_L)V(q_L)] \tag{P$'_2$}$$

Let the optimal solutions be $(\hat{q}_L,\, \hat{q}_H)$. The first order conditions give

$$\theta_L V'(\hat{q}_L)\left[1 - \frac{\lambda_H}{\lambda_L}\frac{\theta_H - \theta_L}{\theta_L}\right] = c, \tag{1}$$

$$\theta_H V'(\hat{q}_H) = c. \tag{2}$$

For an interior solution to exist we need $\lambda_L > \tilde{\lambda}_L \equiv (\theta_H - \theta_L)/\theta_H$, which means that the proportions of the low type must be sufficiently high for price discrimination to be feasible. Denote by z the term in the square bracket in equation (1), which is strictly less than 1. Thus we have

$$\theta_L V'(\hat{q}_L) = \frac{c}{z}. \tag{1$'$}$$

The optimal transfers are given by:

$$\hat{t}_L = \theta_L V(\hat{q}_L), \tag{3}$$
$$\hat{t}_H = \theta_H V(\hat{q}_H) - (\theta_H - \theta_L)V(\hat{q}_L). \tag{4}$$

Finally, we need to check that, under the above solutions, (IR_2) and (IC_L) are satisfied. Notice that (IR_2) requires that $U(\theta_H) \geq 0$. Since $U(\theta_H) = \theta_H V(\hat{q}_H) - \hat{t}_H = (\theta_H - \theta_L)V(\hat{q}_L)$, it is strictly positive given that $\theta_H > \theta_L$ and $\hat{q}_L$. Next, notice that $\hat{q}_L < q_L^* < q_H^* = \hat{q}_H$. The constraint ($IC_L$) requires that $0 = U(\theta_L) \geq U(\theta_H, \theta_L)$. Now,

$$\begin{aligned} U(\theta_H, \theta_L) =& \theta_L V(\hat{q}_H) - \hat{t}_H \\ =& \theta_L V(\hat{q}_H) - \theta_H V(\hat{q}_H) - (\theta_H - \theta_L)V(\hat{q}_L) \\ =& -(\theta_H - \theta_L)[V(\hat{q}_H) - V(\hat{q}_L)] \leq 0, \end{aligned}$$

and hence (IC_L) is satisfied. Note a few important properties of the optimal contract under asymmetric information.

1. Since $z < 1$, we have
$$\theta_L V'(\hat{q}_L) = \frac{c}{z} > c = \theta_L V'(q_L^*).$$
The above implies $\hat{q}_L < q_L^*$ since $V'' < 0$, i.e., as compared with the symmetric information case, the low type consumer always consumes a lower quantity under asymmetric information. On the other hand, from (2) we have $\hat{q}_H = q_H^*$, i.e., the high type consumer receives the efficient quantity. This is called the "no distortion at the top" property. The monopolist offers the efficient quantity to the high-valuation consumer, and distorts the consumption of the low type so as to make the contract for the low type consumer less attractive for the high type.

2. Under perfect price discrimination, the principal, who has all the bargaining power, is able to extract the entire rent from both types of the consumer. Since $U(\theta_H) = (\theta_H - \theta_L)V(\hat{q}_L) > 0$ under asymmetric information, such rent extraction from the high type is not possible any more, at least when the seller wants both types to participate in the mechanism. The monopolist has to give up some rent to the high-type consumer because the consumer possesses informational advantages. The term $(\theta_H - \theta_L)V(\hat{q}_L)$ is referred to as the "informational rent" of type θ_H. Clearly, the higher the marginal valuation of this type relative to θ_L, the more is his informational rent. The low-type consumer gets zero informational rent.

3. Define $S(\theta_i) := [\theta_i V(q_i) - t_i] + [t_i - cq_i]$ which is the aggregate surplus of the trade between the monopolist and the consumer of type θ_i. Using this, the seller's objective function can be written as
$$\underbrace{\lambda_H S(\theta_H) + \lambda_L S(\theta_L)}_{\text{Expected allocative efficiency}} - \underbrace{\lambda_H U(\theta_H) + \lambda_L U(\theta_L)}_{\text{Expected informational rent}}.$$
The seller thus faces a clear trade-off between efficiency and rent extraction. Rewrite the first order condition (1) with respect to q_L as
$$\lambda_L[\theta_L V'(\hat{q}_L) - c] = \lambda_H(\theta_H - \theta_L)V'(\hat{q}_L). \tag{5}$$
The above equation represents the optimal trade-off between efficiency and rent extraction under

asymmetric information. The expected marginal efficiency gain [the left-hand-side of (5)] and the expected marginal cost of the rent [the right-hand-side of (5)] brought about by an infinitesimal increase of consumption of the low type are equated.

The above properties that we have obtained in the simplest possible model of adverse selection with two types in general carry through under more general specifications, which we will see in the next section.

3.2 Monopolistic screening with a continuum of types

We extend the model in Section 3.1 to a case with a continuum of types. The agent has a type $\theta \in \Theta := [\underline{\theta}, \bar{\theta}]$ that constitutes his private information. Let $W(q,t) = t - cq$ and $U(q,t,\theta) = \theta V(q) - t$ denote the utilities of the principal and the agent of type θ, respectively. We assume that $V(\cdot)$ is strictly increasing and strictly concave in q. The additively separable form of $U(q,t,\theta)$ implies that the agent's utility is quasilinear in consumption and money. The principal entertains an a priori belief on the agent's type that is embodied in a cumulative distribution function $F(\theta)$ with density $f(\theta) > 0$ for all $\theta \in \Theta$.

3.2.1 Incentive compatibility

We restrict our attention to a *direct mechanism*, $g(\tilde{\theta}) = (q(\tilde{\theta}), t(\tilde{\theta}))$ where $\tilde{\theta}$ is the announced type of the consumer. We use the following notations:

$$\begin{aligned} U(\tilde{\theta},\theta) &\equiv U(q(\tilde{\theta}), t(\tilde{\theta}), \theta) = \theta V(q(\tilde{\theta})) - t(\tilde{\theta}), \\ U(\theta) &\equiv U(q(\theta), t(\theta), \theta) = \theta V(q(\theta)) - t(\theta). \end{aligned}$$

The first of the above two expressions is the utility of the agent of type θ when he announces his type to be $\tilde{\theta}$, while the second one is the utility of a type θ from telling the truth. A mechanism $g(\theta)$ must satisfy the following individual rationality and incentive compatibility constraints:

$$U(\theta) \geq 0 \;\text{ for all }\; \theta \in \Theta, \tag{IR$_\theta$}$$

$$U(\theta) \geq U(\tilde{\theta},\theta) \;\text{ for all }\; (\theta,\tilde{\theta}) \in \Theta \times \Theta,. \tag{IC$_\theta$}$$

Note that (IC$_\theta$) can equivalently be written as

$$\theta = \underset{\tilde{\theta}}{\text{argmax}} \; \left\{\theta V(q(\tilde{\theta})) - t(\tilde{\theta})\right\}. \tag{IC$'_\theta$}$$

We first prove a very important result.

Proposition 1 *A direct mechanism $g(\cdot)$ is incentive compatible, i.e., satisfies (IC$_\theta$) if and only if the following two conditions hold for all $\theta \in \Theta$:*

$$\theta V'(q(\theta))q'(\theta) - t'(\theta) = 0, \tag{FOC}$$

$$q'(\theta) \geq 0. \tag{MON}$$

Proof. We first prove the necessity of (FOC) and (MON). For any two θ and θ' with $\theta \neq \theta'$, (IC_θ) implies that

$$\theta V(q(\theta)) - t(\theta) \geq \theta V(q(\theta')) - t(\theta'),$$
$$\theta' V(q(\theta')) - t(\theta') \geq \theta' V(q(\theta)) - t(\theta).$$

Summing the above two we obtain

$$(\theta' - \theta)[V(q(\theta')) - V(q(\theta))] \geq 0.$$

If $\theta' > \theta$, then the above inequality implies that $q(\theta') \geq q(\theta)$ since $V'(\cdot) > 0$. As $q(\cdot)$ must be non-decreasing, it is differentiable almost everywhere (a. e.). Therefore, $t(\cdot)$ is also differentiable with the same points of non-differentiablity. The local first-order necessary condition associated with (IC'_θ) at $\tilde{\theta} = \theta$ (truth-telling) is given by:

$$\theta V'(q(\theta))q'(\theta) - t'(\theta) = 0. \tag{FOC}$$

The local second-order necessary condition at $\tilde{\theta} = \theta$ is given by:

$$\theta V''(q(\theta))[q'(\theta)]^2 + \theta V'(q(\theta))q''(\theta) - t''(\theta) \leq 0. \tag{SOC}$$

On the other hand, differentiating (FOC) with respect to θ, we obtain:

$$V'(q(\theta))q'(\theta) + \underbrace{\theta V''(q(\theta))[q'(\theta)]^2 + \theta V'(q(\theta))q''(\theta) - t''(\theta)}_{\leq 0 \text{ by (SOC)}} = 0,$$

and hence, $q'(\theta) \geq 0$ since $V'(\cdot) > 0$.

To show the sufficiency of (FOC) and (MON), suppose that these two conditions hold for all $\theta \in \Theta$, but by contradiction, for at least one type θ the buyer's incentive compatibility is violated, i.e.,

$$\theta V(q(\tilde{\theta})) - t(\tilde{\theta}) > \theta V(q(\theta)) - t(\theta)$$

for at least one $\tilde{\theta} \neq \theta$. Integrating the above we get

$$\int_\theta^{\tilde{\theta}} \left[\theta V'(q(x))q'(x) - t'(x)\right] dx > 0. \tag{6}$$

Suppose that $\tilde{\theta} > \theta$. Since $q(\cdot)$ is non-decreasing by assumption, we have $q(x) \leq q(\tilde{\theta})$ for any $x \in [\theta, \tilde{\theta}]$. Since $V'(\cdot) > 0$, the last inequality implies that

$$\theta V'(q(x)) < xV'(q(x)).$$

Therefore,

$$\int_\theta^{\tilde{\theta}} \left[\theta V'(q(x))q'(x) - t'(x)\right] dx < \underbrace{\int_\theta^{\tilde{\theta}} \left[xV'(q(x))q'(x) - t'(x)\right] dx = 0}_{\text{because (FOC) holds for } x}. \tag{7}$$

Thus, (6) and (7) contradict each other. If $\tilde{\theta} < \theta$, the same logic leads to similar contradiction. This completes the proof. $\square$

The above proposition permits us to replace an infinity of constraints in (IC_θ) by only two constraints – namely, (FOC) and (MON).

3.2.2 *Solving the model*

Now consider the individual rationality constraint (IR_θ) of the buyer of type θ, which holds in particular for $\theta = \underline{\theta}$, i.e.,

$$U(\underline{\theta}) \equiv \underline{\theta}V(q(\underline{\theta})) - t(\underline{\theta}) \geq 0. \qquad (\text{IR}_{\underline{\theta}})$$

Given that the incentive compatibility holds for any type $\theta > \underline{\theta}$, this implies

$$U(\theta) \equiv \theta V(q(\theta)) - t(\theta) \geq \theta V(q(\underline{\theta})) - t(\underline{\theta}) > \underline{\theta}V(q(\underline{\theta})) - t(\underline{\theta}) \geq 0.$$

Therefore, the only relevant individual rationality constraint, among the infinitely many ones in (IR_θ), we have to take into account is the constraint ($\text{IR}_{\underline{\theta}}$). Therefore, the monopolist's maximization problem can be written as

$$\max_{\{q(\theta),t(\theta)\}} \int_{\underline{\theta}}^{\bar{\theta}} [t(\theta) - cq(\theta)]d\theta,$$

subject to ($\text{IR}_{\underline{\theta}}$), (MON) and (FOC).

Note first that ($\text{IR}_{\underline{\theta}}$) must bind at the optimum, otherwise the seller can increase the transfer $t(\underline{\theta})$ a little bit by holding the quantity $q(\underline{\theta})$ constant, the type $\underline{\theta}$ buyer will still accept the contract and the seller would increase her expected profits. Therefore, we will have $U(\underline{\theta}) = 0$ or, $t(\underline{\theta}) = \underline{\theta}V(q(\underline{\theta}))$. We will ignore (MON) for the time being, and solve the relaxed problem with binding ($\text{IR}_{\underline{\theta}}$). Recall that

$$U(\theta) = \theta V(q(\theta)) - t(\theta) = \max_{\tilde{\theta}} \{\tilde{\theta}V(q(\tilde{\theta})) - t(\tilde{\theta})\}.$$

Applying the Envelope theorem to the above we get

$$U'(\theta) = V(q(\theta)$$

or, integrating

$$U(\theta) - U(\underline{\theta}) = U(\theta) = \int_{\underline{\theta}}^{\theta} V(q(x))dx \equiv R(\theta). \qquad (\text{R}_\theta)$$

The above expression is the *informational rent* that accrues to a type $\theta > \underline{\theta}$ buyer, which is similar to that in the two-type case. From the above we have

$$t(\theta) = \theta V(q(\theta)) - R(\theta).$$

Thus, the seller's objective function reduces to:

$$W = \int_{\underline{\theta}}^{\bar{\theta}} \left[\theta V(q(\theta)) - \int_{\underline{\theta}}^{\theta} V(q(x))dx - cq(\theta)\right] f(\theta)d\theta. \qquad (8)$$

Note that

$$\underbrace{\int_{\underline{\theta}}^{\bar{\theta}} \underbrace{\left[\int_{\underline{\theta}}^{\theta} V(q(x))dx\right]}_{R(\theta)} f(\theta)d\theta}_{\text{expected informational rent}} = \left[\left\{\int_{\underline{\theta}}^{\theta} V(q(x))dx\right\} F(\theta)\right]_{\underline{\theta}}^{\bar{\theta}} - \int_{\underline{\theta}}^{\theta} V(q(\theta))F(\theta)d\theta$$

$$= \int_{\underline{\theta}}^{\bar{\theta}} V(q(\theta))d\theta - \int_{\underline{\theta}}^{\bar{\theta}} V(q(\theta))F(\theta)d\theta$$

$$= [1 - F(\theta)] \int_{\underline{\theta}}^{\bar{\theta}} V(q(\theta))d\theta$$

$$= \int_{\underline{\theta}}^{\bar{\theta}} \left[\frac{1-F(\theta)}{f(\theta)}\right] V(q(\theta))f(\theta)d\theta.$$

Thus, the principal's objective function is given by:

$$\begin{aligned}
&\int_{\underline{\theta}}^{\bar{\theta}} [t(\theta) - cq(\theta)]f(\theta)d\theta \\
&= \underbrace{\int_{\underline{\theta}}^{\bar{\theta}} [\theta V(q(\theta)) - cq(\theta)]f(\theta)d\theta}_{\text{expected allocative efficiency}} - \underbrace{\int_{\underline{\theta}}^{\bar{\theta}} R(\theta)f(\theta)d\theta}_{\text{expected informational rent}} \\
&= \int_{\underline{\theta}}^{\bar{\theta}} [z(\theta)V(q(\theta)) - cq(\theta)] f(\theta)d\theta, \qquad (\mathrm{P}_3)
\end{aligned}$$

where

$$z(\theta) := \theta - \frac{1-F(\theta)}{f(\theta)}$$

is the virtual valuation of the agent. Notice that the principal's objective function is a function of $q(\theta)$ alone. Call $\hat{q}(\theta)$ the optimal solution to the maximization problem whose first order condition is given by:

$$z(\theta)V'(\hat{q}(\theta)) = c, \quad \text{for all } \ \theta \in \left[\underline{\theta}, \bar{\theta}\right].$$

Notice that so far we have suppressed the monotonicity constraint, i.e., $q'(\theta) \geq 0$ for all $\theta \in \Theta$. Differentiating the above optimality condition, we get

$$\hat{q}'(\theta) = -\frac{z'(\theta)V'(\hat{q}(\theta))}{z(\theta)V''(\hat{q}(\theta))},$$

which implies that

$$\mathrm{sign}[\hat{q}'(\theta)] = \mathrm{sign}[z'(\theta)] = \mathrm{sign}\left[1 - \frac{d}{d\theta}\left(\frac{1-F(\theta)}{f(\theta)}\right)\right].$$

Hence, $\hat{q}(\theta)$ is non-decreasing if and only if

$$\frac{d}{d\theta}\left[\frac{1-F(\theta)}{f(\theta)}\right] \leq 1. \qquad (9)$$

Now define by

$$h(\theta) := \frac{f(\theta)}{1 - F(\theta)}$$

the *hazard rate* associated with the distribution function $F(\theta)$. Notice that

$$h'(\theta) = \frac{d}{d\theta}\left[\frac{f(\theta)}{1 - F(\theta)}\right] > 0 \tag{MHR}$$

implies that the condition (9) is satisfied. The condition (MHR) is called the *monotone hazard rate property* of the distribution function $F(\theta)$, which is a sufficient condition for the optimum consumption allocation is non-decreasing in types. Notice that $\hat{q}(\theta)$ non-decreasing implies that the types are separated and truthfully revealed. This is a *separating* contract. Finally, notice that

$$V'(q^*(\theta)) = \frac{c}{\theta} < \frac{c}{z(\theta)} = V'(\hat{q}(\theta)) \implies \hat{q}(\theta) < q^*(\theta) \text{ for all } \theta \in \left[\underline{\theta}, \bar{\theta}\right),$$

where $q^*(\theta)$ is the first best consumption allocation. Since $z(\bar{\theta}) = \bar{\theta}$, we have $\hat{q}(\bar{\theta}) = q^*(\bar{\theta})$, i.e., the "no-distortion-at-the-top" property is satisfied.

3.2.3 Pooling or bunching mechanism

What happens if the monotone hazard rate property does not hold? Since (MHR) is only a sufficient condition for $\hat{q}(\theta)$ to be non-decreasing on Θ, if this property is not satisfied $\hat{q}(\theta)$ may be strictly decreasing over a subset of Θ. Therefore, in solving the principal's maximization problem we cannot ignore anymore the monotonicity constraint. Call the solution to the principal's constraint maximization problem $\bar{q}(\theta)$, rewrite the maximization problem as the following:

$$\max_{q(\theta)} \int_{\underline{\theta}}^{\bar{\theta}} \left[z(\theta)V(q(\theta)) - cq(\theta)\right] f(\theta) d\theta \tag{P$_3'$}$$

$$\text{subject to } \; q'(\theta) = \mu(\theta), \tag{10}$$

$$\mu(\theta) \geq 0. \tag{11}$$

The above maximization is an optimal control problem, the Hamiltonian of which is given by:

$$H(\theta, q, \mu, \lambda) = \left[z(\theta)V(q(\theta)) - cq(\theta)\right] f(\theta) + \lambda(\theta)\mu(\theta).$$

Assume that $H(\theta, q, \mu, \lambda)$ is concave in q. Then by Pontryagin's maximum principle, the necessary and sufficient conditions for an optimum $(\bar{q}(\theta), \bar{\mu}(\theta))$ are given by:

$$H(\theta, \bar{q}(\theta), \bar{\mu}(\theta), \bar{\lambda}(\theta)) \geq H(\theta, \bar{q}(\theta), \mu(\theta), \lambda(\theta)); \tag{12}$$

$$\left[z(\theta)V'(\bar{q}(\theta)) - c\right] f(\theta) + \lambda'(\theta) = 0 \text{ for almost all } \theta \in \Theta; \tag{13}$$

$$\text{and the transversality conditions } \lambda(\underline{\theta}) = \lambda(\bar{\theta}) = 0 \text{ are satisfied.} \tag{14}$$

Integrating (13), we get

$$\lambda(\theta) = -\int_{\underline{\theta}}^{\theta} \left[z(\theta)V'(\bar{q}(\theta)) - c\right] f(\theta) d\theta,$$

and using the transversality conditions, we then have

$$0 = \lambda(\bar{\theta}) = -\int_{\underline{\theta}}^{\bar{\theta}} \left[z(\theta)V'(\bar{q}(\theta)) - c\right] f(\theta)d\theta.$$

Next, since (12) requires that $\mu(\theta)$ must maximiza H subject to $\mu(\theta) \geq 0$, wh must have $\lambda(\theta) \leq 0$, i.e.,

$$\int_{\underline{\theta}}^{\theta} \left[z(\theta)V'(\bar{q}(\theta)) - c\right] f(\theta)d\theta \geq 0.$$

Whenever $\lambda(\theta) < 0$, we get

$$\bar{\mu}(\theta) = \bar{q}'(\theta) = 0.$$

Thus we get the following complementary slackness condition:

$$\bar{q}'(\theta) \cdot \int_{\underline{\theta}}^{\theta} \left[z(\theta)V'(\bar{q}(\theta)) - c\right] f(\theta)d\theta = 0.$$

Now let $\bar{q}(\theta)$ is constant over the interval $[\theta_1, \theta_2] \subset \Theta$, and strictly increasing otherwise. Clearly over $\Theta \setminus [\theta_1, \theta_2]$, $\bar{q}(\theta)$ coincides with $\hat{q}(\theta)$ since $\bar{q}'(\theta) > 0$ implies that $\lambda(\theta) = 0$ which in turn implies that

$$\lambda'(\theta) = 0 \implies z(\theta)V'(\bar{q}(\theta)) = c.$$

A constant $q(\theta)$ implies that the principal offers the same allocations $(\bar{q}(\theta), \bar{t}(\theta))$ to all types $\theta \in [\theta_1, \theta_2]$. Such contract is called a *bunching* or *pooling* contract. Notice that, by continuity of $\lambda(\theta)$ we must have $\lambda(\theta_1) = \lambda(\theta_2) = 0$, so that

$$\int_{\underline{\theta}_1}^{\theta_2} \left[z(\theta)V'(\bar{q}(\theta)) - c\right] f(\theta)d\theta = 0. \tag{15}$$

On the other hand, by continuity of $\bar{q}(\theta)$ we have

$$\hat{q}(\theta_1) = \hat{q}(\theta_2). \tag{16}$$

Therefore, solving the equations (15) and (16) we get the optimal bunching interval $[\theta_1, \theta_2]$. For further discussions on the bunching contracts, see (Bolton and Dewatripont, 2005, Chapter 2).

4 A general model of adverse selection: non-quasilinear utility

We assume that the buyer's utility is given by $U(q, t, \theta)$ with $U_q > 0$, $U_t < 0$ and $U_\theta \geq 0$. In order to have a proposition similar to Proposition 1, we require to impose more structure of the buyer's utility function – namely, the Spence-Mirrlees condition or the single-crossing condition, which is given by:

$$\frac{\partial}{\partial \theta}\left[-\frac{U_q(q, t, \theta)}{U_t(q, t, \theta)}\right] > 0 \quad \text{for all } (q, t, \theta). \tag{SM}$$

Note that the slope of the indifference curve of any type θ buyer, $U(q, t, \theta) = \bar{u}$ is given by $-(U_q/U_t) > 0$. Thus, condition (SM) implies that, given any two types θ and θ' with $\theta' > \theta$, the indifference curve of θ' is everywhere steeper than that of θ, and hence, they cross only once. This condition means that if type

θ is indifferent between two consumption-transfer pairs (q,t) and (q',t') with $(q,t)<(q',t')$, then the higher type θ' is willing to pay more than t' to receive consumption q'. Thus, in an incentive compatible mechanism, the principal is able to separate the different types of the agent by offering larger allocations q to higher types and making them pay for the privilege. Therefore, the Spence-Mirrlees condition is also called the sorting condition. The following result is a generalization of Proposition 1 to a non-quasilinear environment.

Proposition 2 *Suppose the buyer's utility function $U(q,t,\theta)$ satisfies condition (SM). Then, a direct mechanism $g(\cdot)$ is incentive compatible if and only if the following two conditions hold for all $\theta\in\Theta$:*

$$U_q(q(\theta),t(\theta),\theta)q'(\theta)+U_t(q(\theta),t(\theta),\theta)t'(\theta)=0, \tag{FOC}$$
$$q'(\theta)\geq 0. \tag{MON}$$

Proof. We restrict attention to differentiable mechanisms $(q(\tilde{\theta}),t(\tilde{\theta}))$. Incentive compatibility is given by:

$$U(q(\theta),t(\theta),\theta)=\max_{\tilde{\theta}}\{U(q(\tilde{\theta}),t(\tilde{\theta}),\theta)\}. \tag{IC$_\theta$}$$

We first prove the necessity of (FOC) and (MON). The local first-order necessary condition associated with (IC$_\theta$) at $\tilde{\theta}=\theta$ (truth-telling) is given by:

$$U_q(q(\theta),t(\theta),\theta)q'(\theta)+U_t(q(\theta),t(\theta),\theta)t'(\theta)=0. \tag{FOC}$$

The local second-order necessary condition at $\tilde{\theta}=\theta$ is given by:

$$\begin{aligned}\Omega(\theta)\equiv&[U_{qq}(q(\theta),t(\theta),\theta)q'(\theta)+U_{qt}(q(\theta),t(\theta),\theta)t'(\theta)]q'(\theta)+U_q(q(\theta),t(\theta),\theta)q''(\theta)\\&+[U_{tq}(q(\theta),t(\theta),\theta)q'(\theta)+U_{tt}(q(\theta),t(\theta),\theta)t'(\theta)]t'(\theta)+U_t(q(\theta),t(\theta),\theta)t''(\theta)\leq 0.\end{aligned} \tag{SOC}$$

On the other hand, differentiating (FOC) with respect to θ, we obtain:

$$U_{q\theta}(q(\theta),t(\theta),\theta)q'(\theta)+U_{t\theta}(q(\theta),t(\theta),\theta)t'(\theta)+\underbrace{\Omega(\theta)}_{\leq 0\text{ by (SOC)}}=0,$$

which implies that

$$\begin{aligned}&U_{q\theta}(q(\theta),t(\theta),\theta)q'(\theta)+U_{t\theta}(q(\theta),t(\theta),\theta)t'(\theta)\geq 0\\ \Longleftrightarrow\ & q'(\theta)\left[U_{q\theta}(q(\theta),t(\theta),\theta)-U_{t\theta}(q(\theta),t(\theta),\theta)\cdot\frac{U_q(q(\theta),t(\theta),\theta)}{U_t(q(\theta),t(\theta),\theta)}\right]\geq 0\\ \Longleftrightarrow\ & q'(\theta)\cdot -U_t(q(\theta),t(\theta),\theta)\cdot\frac{\partial}{\partial\theta}\left[-\frac{U_q(q,t,\theta)}{U_t(q,t,\theta)}\right]\geq 0.\end{aligned}$$

Thus, under (SM), the above implies that $q'(\theta)\geq 0$.

Next, we prove the sufficiency of (FOC) and (MON) for incentive compatibility. Global incentive compatibility requires

$$
\begin{aligned}
& U(q(\theta), t(\theta), \theta) \geq U(q(\tilde{\theta}), t(\tilde{\theta}), \tilde{\theta}) \text{ for all } (\theta, \tilde{\theta}) \in \Theta \times \Theta \\
\iff & \int_{\tilde{\theta}}^{\theta} \left[U_q(q(x), t(x), \theta) q'(x) + U_t(q(x), t(x), \theta) t'(x)\right] dx \geq 0 \\
\iff & \int_{\tilde{\theta}}^{\theta} q'(x) \cdot -U_t(q(x), t(x), \theta) \left[-\frac{U_q(q(x), t(x), \theta)}{U_t(q(x), t(x), \theta)} - \left\{-\frac{U_q(q(x), t(x), x)}{U_t(q(x), t(x), x)}\right\}\right] dx \geq 0.
\end{aligned} \tag{17}
$$

The last inequality arises because (FOC) holds for any $x \in [\tilde{\theta}, \theta]$. Because $q'(x) \geq 0$, $U_t < 0$, and using (SM) we can conclude that

$$
\begin{aligned}
& \int_{\tilde{\theta}}^{\theta} q'(x) \cdot -U_t(q(x), t(x), \theta) \left[-\frac{U_q(q(x), t(x), \theta)}{U_t(q(x), t(x), \theta)} - \left\{-\frac{U_q(q(x), t(x), x)}{U_t(q(x), t(x), x)}\right\}\right] dx \\
\geq & \int_{\tilde{\theta}}^{\theta} q'(x) \cdot -U_t(q(x), t(x), \theta) \left[-\frac{U_q(q(x), t(x), x)}{U_t(q(x), t(x), x)} - \left\{-\frac{U_q(q(x), t(x), x)}{U_t(q(x), t(x), x)}\right\}\right] dx = 0,
\end{aligned}
$$

which proves (17). This completes the proof of the Proposition. □

To solve the model explicitly we of course require specific form of the buyer's utility function $U(q, t, \theta)$. Once that is known, we can follow procedure similar to that in Section 3.2.2.

5 Direct mechanism and the revelation principle

In the previous section we have seen that the principal, to maximize her expected utility, offers a direct incentive compatible mechanism $g(\theta) = (q(\theta), t(\theta))$ to the agent without knowing his true type, and the agent truthfully reveals his type. A direct mechanism, which we will define formally below, is a mechanism in which the agent is asked to report his type directly to the principal on the basis of which the principal proposes an allocation. A direct mechanism is the simplest form of mechanisms that can be offered to the agent. In general a mechanism may be much more complex than just asking the agent to announce his type.

Definition 1 (Mechanism) *A mechanism is a game form $\Gamma = (\mathscr{M}, \tilde{g})$ that consists of a message or strategy space $\mathscr{M}$ of the agent, and a mapping $\tilde{g} : \mathscr{M} \to \mathscr{A}$, writes $\tilde{g}(m) = (\tilde{q}(m), \tilde{t}(m))$ for all $m \in \mathscr{M}$, where*

$$\mathscr{A} = \{(q, t) \mid q \in \mathbb{R}_+ \text{ and } t \in \mathbb{R}\}$$

is the set of possible allocations.

When facing such a mechanism, the agent with type θ chooses a best response $m^*(\theta)$ which is implicitly defined as

$$V(\tilde{q}(m^*(\theta)), \theta) - \tilde{t}(m^*(\theta)) \geq V(\tilde{q}(m), \theta) - \tilde{t}(m) \text{ for all } m \in \mathscr{M}. \tag{18}$$

The mechanism $(\mathscr{M}, \tilde{g}(\cdot))$ induces therefore an allocation rule $a(\theta) = (\tilde{q}(m^*(\theta)), \tilde{t}(m^*(\theta)))$ mapping the set of types Θ into the set of allocations $\mathscr{A}$.

Definition 2 (Direct mechanism) *A direct revelation mechanism is a mapping $g : \Theta \to \mathscr{A}$ which writes as $g(\theta) = (q(\theta), t(\theta))$ for all $\theta \in \Theta$. The principal commits to offer the transfer $t(\theta')$ and the production* *level $q(\theta')$ if the agent announces the value θ' for any $\theta' \in \Theta$. Moreover, a direct mechanism $g(\cdot)$ is 'truthful' if it is incentive compatible for the agent to announce his true type, for any type, i.e.,*

$$V(q(\theta), \theta) - t(\theta) \geq V(q(\theta'), \theta) - t(\theta') \;\; \textit{for all} \;\; \theta, \theta' \in \Theta. \tag{19}$$

Notice that the message space $\mathscr{M}$ of the agent may be very complex which may consist of a whole gamut of messages including report of own types. First, one may wonder if a better outcome could be achieved with a more complex contract allowing the agent to possibly choose among more options. Second, one may also wonder whether some sort of communication device between the agent and the principal could be used to transmit information to the principal so that the latter can recommend outputs and payments as a function of transmitted information. This is not the case. Indeed, the *Revelation Principle* ensures that there is no loss of generality in restricting the principal to offer simple menus having at most as many options as the cardinality of the type space Θ.

Proposition 3 (The Revelation Principle) *Any allocation rule $a(\theta)$ that can be implemented by a mechanism $(\mathscr{M}, \tilde{g}(\cdot))$ can also be implemented by a direct incentive compatible mechanism $g(\theta)$.*

Proof. The indirect mechanism $(\mathscr{M}, \tilde{g}(\cdot))$ induces an allocation rule $a(\theta) = (\tilde{q}(m^*(\theta)), \tilde{t}(m^*(\theta)))$. Now construct a direct mechanism $g(\theta)$ such that $g := \tilde{g} \circ m^* : \Theta \to \mathscr{A}$. To prove the proposition we have to check that the direct mechanism that we have constructed is incentive compatible. Since condition (18) holds for all $m \in \mathscr{M}$, it holds in particular for $m^*(\theta')$ for all $\theta' \in \Theta$, i.e.,

$$V(\tilde{q}(m^*(\theta)), \theta) - \tilde{t}(m^*(\theta)) \geq V(\tilde{q}(m^*(\theta')), \theta) - \tilde{t}(m^*(\theta'))\theta \;\; \text{for all} \;\; \theta, \theta' \in \Theta. \tag{20}$$

Then, using the definition of g, we get the incentive compatibility condition (19) for the direct mechanism $g(\theta)$, which completes the proof. □

The above result is special case of the general result proven by Myerson (1981) in the context of a mechanism design problem with many agents. Analysis of mechanism design with many agents is beyond the scope of this course.

6 Regulation of natural monopolies

In their classic papers Hotelling (1938) and Dupuit (1952) considered pricing policies for a bridge that had a fixed cost of construction and zero marginal cost. They demonstrate that the pricing policy that maximizes consumer well-being sets price equal to marginal cost and provides a transfer (or subsidy) to the supplier equal to the fixed cost, so that a firm would be willing to provide the bridge. This is a classical solution to the problem of regulation which was extended later by Baron and Myerson (1982) and Laffont and Tirole (1986) in the context of asymmetric information.

We first describe a general regulatory mechanism. Consider the contracting problem between a utilitarian regulator and a monopolist who produces a private good in quantity q by incurring a total cost $C(q, \theta)$ which is given by:

$$C(q, \theta) = \theta q,$$

where the constant marginal cost θ is private information of the firm. The parameter θ is distributed according to the cumulative distribution function $F(\theta)$ on $\Theta := [\underline{\theta}, \bar{\theta}]$ with the corresponding density $f(\theta) > 0$, and has monotone hazard rate $d[F(\theta)/f(\theta)]/d\theta > 0$. Suppose the monopolist faces the inverse demand function $p = P(q)$ with $P'(q) < 0$. Then the monopolist's total profit is given by:

$$U = t + qP(q) - \theta q, \tag{21}$$

where t is the transfer from the regulator to the monopolist. The net consumer surplus is given by $S(q) - qP(q)$. Notice that $S'(q) = P(q)$. The regulator raises t through consumption taxes. We first analyze the full information benchmark, i.e., when the regulator can observe the firm's cost parameter θ.

6.1 Full information benchmark

We will case two distinct cases. First, when the taxes are distortionary with rate $\lambda > 0$, i.e., the shadow cost of public fund is $(1+\lambda)$. In other words, if the regulator wants to raise \$1, the loss in consumer surplus is $\$(1+\lambda)$. This model is due to Laffont and Tirole (1986). The other case is where $\lambda = 0$, but the regulator attaches a weight $\alpha < 1$ to the producer surplus, which is analyzed by Baron and Myerson (1982).

6.1.1 Ramsey pricing under full information

When taxes are distortionary, i.e., $\lambda > 0$, then the social welfare is given by:

$$\begin{aligned} W &= S(q) - qP(q) - (1+\lambda)t + U \\ &= S(q) + \lambda qP(q) - (1+\lambda)\theta q - \lambda U. \end{aligned} \tag{22}$$

The last expression is obtained by substituting for t from (21). The regulator, by choosing q maximizes the above expression subject to $U \geq 0$, which is the individual rationality constraint of the firm. This constraint will bind at the optimum, and hence the regulator's maximization problem reduces to:

$$\max_q S(q) + \lambda qP(q) - (1+\lambda)\theta q. \tag{23}$$

The first order condition of the above maximization problem implies that

$$L := \frac{P(q^*) - \theta}{P(q^*)} = \frac{\lambda}{1+\lambda}\frac{1}{\eta}, \tag{24}$$

where η is the price elasticity of demand of the monopolist's product. The above is the *Ramsey pricing* formula under full information, which asserts that the Lerner index of the firm is inversely proportional to the price elasticity. Under the Ramsey pricing rule, as long as $\lambda > 0$, the optimal price is strictly greater than the marginal cost, i.e., $P(q) > \theta$. The optimal transfer is determined from the binding individual rationality constraint.

6.1.2 Marginal cost pricing under full information

Now suppose that the taxes are not distortionary, i.e., $\lambda = 0$. The the social welfare is given by:

$$\begin{aligned} W &= S(q) - qP(q) - t + \alpha U \\ &= S(q) - \theta q - (1-\alpha)U. \end{aligned} \tag{25}$$

The regulator, by choosing q maximizes the above expression subject to $U \geq 0$, which is the individual rationality constraint of the firm. This constraint will bind at the optimum, and hence the regulator's maximization problem reduces to:

$$\max_q S(q) - \theta q. \tag{26}$$

The first order condition of the above maximization problem implies that

$$P(q^*) = \theta. \tag{27}$$

When there is no shadow cost of public fund, the product is priced at its marginal cost.

6.2 Optimal regulation under asymmetric information

First we describe some general feature of a direct incentive compatible mechanism $g(\theta) = (q(t), t(\theta))$. As usual, define by

$$U(\theta', \theta) = t(\theta') + q(\theta')P(q(\theta')) - \theta q(\theta')$$

the profit of a type θ monopolist when he announces to be of type θ. By defining $U(\theta) := U(\theta, \theta)$, we have

$$U(\theta', \theta) = U(\theta') + (\theta' - \theta)q(\theta').$$

Then the incentive compatibility constraint of type θ is given by:

$$U(\theta) = \max_{\theta'}\{U(\theta') + (\theta' - \theta)q(\theta')\}. \tag{IC$_\theta$}$$

And the individual rationality constraint is given by:

$$U(\theta) \geq 0 \;\text{ for all }\; \theta \in \Theta. \tag{IR$_\theta$}$$

A mechanism $g(\theta)$ is feasible if it satisfies (IR$_\theta$) and (IC$_\theta$) for all θ. Then, as Proposition **??**, we can write the following characterization result.

Proposition 4 *A regulatory mechanism $g(\theta) = (q(\theta), t(\theta))$ is feasible if and only if it satisfies the following conditions for all $\theta \in \Theta$:*

(a) $U(\bar{\theta}) \geq 0$;

(b) *$q(\theta)$ is non-increasing in θ;*

(c) *Local incentive compatibility constraint:*

$$U'(\theta) + q(\theta) = 0. \tag{LIC$_\theta$}$$

Proof. Similar to the proof of Proposition 1. □

From the local incentive compatibility and binding individual rationality of type $\bar{\theta}$, i.e., $U(\bar{\theta}) = 0$, we get

$$U(\theta) = \int_{\theta}^{\bar{\theta}} q(x)dx \equiv R(\theta), \tag{28}$$

where $R(\theta)$ is the informational rent of type θ. From the above it also follows that the average informational rent is given by:

$$\int_{\underline{\theta}}^{\bar{\theta}} R(\theta)f(\theta)d\theta = \int_{\underline{\theta}}^{\bar{\theta}} q(\theta)L(\theta)f(\theta)d\theta,$$

where $L(\theta) := F(\theta)/f(\theta)$. Also,

$$t(\theta) = -[P(q(\theta)) - \theta]q(\theta) + U(\theta) = -[P(q(\theta)) - \theta]q(\theta) + R(\theta).$$

6.2.1 Ramsey pricing under asymmetric information

When the taxes are distortionary, using the expressions for $t(\theta)$ and the expected informational rent, the regulator's objective function reduces to:

$$\int_{\underline{\theta}}^{\bar{\theta}} [S(q(\theta)) + \lambda q(\theta)P(q(\theta)) - (1+\lambda)z(\theta)q(\theta)]\, f(\theta)d\theta,$$

where

$$z(\theta) := \theta + \frac{\lambda}{1+\lambda}L(\theta)$$

is the virtual marginal cost of the monopolist. The regulator chooses $q(\theta)$ in order to maximize the expected welfare such that $q'(\theta) \leq 0$. We ignore for the time being the monotonicity constraint. Let the optimal solution be $q(\theta)$ and $P(\theta) := P(q(\theta))$. Then the first order condition of the above maximization problem implies that

$$L(\theta, z) := \frac{P(\theta) - z(\theta)}{P(\theta)} = \frac{\lambda}{1+\lambda}\frac{1}{\eta(\theta)}.$$

The above is the virtual Lerner index of the regulated firm. When the regulated firm has private information regarding its marginal cost of production, the regulator perceives that its marginal cost is equal to its virtual value $z(\theta)$ which is higher than θ. Hence, as compared with the full information case, the efficient mechanism is offered to $z(\theta)$ instead of θ. The true Lerner index is given by:

$$L(\theta) := \frac{P(\theta) - \theta}{P(\theta)} = \underbrace{\frac{\lambda}{1+\lambda}\frac{1}{\eta(\theta)}}_{\text{Ramsey markup}} + \underbrace{\frac{\lambda}{1+\lambda}\frac{L(\theta)}{P(\theta)}}_{\text{Incentive correction}}.$$

Notice that, under asymmetric information, the true Lerner index of the regulated firm consists of a Ramsey markup term and an incentive correction term. The second term was absent in the full information case since no incentive correction via distortion was necessary. Note that, the no-distortion-at-the-top property holds here too since $z(\underline{\theta}) = \underline{\theta}$, i.e, the most efficient type of the monopolist receives the efficient contract. Finally, under the assumption of monotone hazard rate, at the optimum we have $q(\theta)$ a non-increasing function of the firm's type.

6.2.2 Marginal cost pricing under asymmetric information

In the model of Baron and Myerson (1982), using the above procedure the regulator's objective function boils down to:

$$\int_{\underline{\theta}}^{\bar{\theta}} [S(q(\theta)) - z(\theta)q(\theta)]\, f(\theta) d\theta,$$

where

$$z(\theta) := \theta + (1-\alpha)L(\theta)$$

is the virtual marginal cost of the monopolist. Notice that the virtual type in this case is different from that in the model of Laffont and Tirole (1986). From the first order condition of the regulator's maximization problem it follows that

$$P(\theta) = z(\theta) = \underbrace{\theta}_{\text{Marginal cost}} + \underbrace{(1-\alpha)L(\theta)}_{\text{Incentive correction}}.$$

The above pricing rule adheres to the marginal cost pricing where the price charged by the regulated firm is equated with her virtual marginal cost of production, which is the sum of the true marginal cost and an incentive correction term.

References

Baron, David and Roger Myerson (1982), "Regulating a Monopolist with Unknown Cost." *Econometrica*, 50, 911–930.

Bolton, Patrick and Mathias Dewatripont (2005), *Contract Theory*. The MIT Press.

Dupuit, Jules (1952), "On the Measurement of the Utility of Public Works." *International Economic Papers*, 2, 83–110.

Hotelling, Harold (1938), "The General Welfare in Relation to Problems of Taxation and of Railway and Utility Rates." *Econometrica*, 6, 242–269.

Laffont, Jean-Jacques and Jean Tirole (1986), "Using Cost Observation to Regulate Firms." *Journal of Political Economy*, 94.

Maskin, Eric and John Riley (1984), "Monopoly with Incomplete Information." *RAND Journal of Economics*, 15, 171–196.

Myerson, Roger (1981), "Optimal Auction Design." *Mathematics of Operations Research*, 6, 58–63.