Bank Competition and Risk-Taking under Market Integration*

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Abstract

Linkages between bank competition and risk-taking are analyzed in a general setting where market integration is the principal driver of increased competition. Risk implications of across-market competition under banking market integration are significantly different from that of within-market competition. While both modes of competition increase the number of competitor banks, across-market competition yields a bank-customer effect that can potentially reverse any relation that prevails between within-market competition and risk-taking. Robust to various settings, this result suggests that the lack of consensus in the bank competition-financial stability literature is not an anomaly but an inherent feature of the analysis.

JEL codes: D82, G21, L13.
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1. Introduction

The Global Financial Crisis has rekindled interest in the much-studied and yet unresolved relation between bank competition and financial stability.[1] Over the past five decades, *market integration* has transformed banking from a within-market local phenomenon to span multiple erstwhile segmented markets.[2] Empirical studies have analyzed this progressive evolution of bank competition through geographic deregulation and the expansion of banking *across* markets. Meanwhile, theory on bank compe-

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[1]The lack of consensus is pervasive and reflected even in surveys of this literature. For example, Vives (2016) and Corbae and Levine (2018) argue that there is a significant trade-off between competition and financial stability and the OECD report OECD (2011, p. 10), finds that "changes in business models and activities in response to competition ... proved not to be conducive to financial stability." In contrast, Carletti and Hartmann (2003) and Beck, Coyle, Dewatripont, Freixas, and Seabright (2010) argue that competition is important for financial stability and the aforementioned tradeoff does not generally hold.

[2]The term "market integration" is used as a shorthand for the evolution of bank competition through deregulation episodes that removed geographic restrictions on banking. For example, branching restrictions both within and across state borders in the United States until the 1970s created local monopolies in banking (Kroszner and Strahan, 1999). Thereafter, deregulation within states allowed state-wide branching while deregulation across state lines occurred through bilateral, regional, and even national reciprocal arrangements (Amel, 1993; Strahan, 2003). Reciprocity in these agreements integrated banking markets, enabling geographic expansion of bank operations to span multiple local markets (Radecki, 1998; Dick, 2006). Vives (2016) describes how the European experience has followed a similar trajectory.

tition and risk-taking has largely defined increased competition as increases in the number of competitor banks *within* an individual market. While both modes of analyzing competition increase the number of competitor banks, does across-market competition under banking market integration have the same implications for risk-taking as within-market increases in the number of competitor banks?

In this paper, we model how increased banking competition affects risk-taking when market integration is the principal driver of increased competition. Previous theory has analyzed the effect of increased competition on risk-taking by modeling competition as a within-market increase (or threat of increase) in the number of competitor banks (Besanko and Thakor, 1992; Allen and Gale, 2004; Boyd and De Nicoló, 2005) In practice, however, market integration has been the key driver of increased bank competition, and understanding its effect on risk-taking is critical to understanding its financial stability implications. Market integration not only increases the number of banks, but also entrepreneurs (borrowers) and depositors—the potential customer base of each bank—in the integrated market. We present a generalized theoretical framework for analyzing bank competition and stability linkages when banking markets integrate.

Our results show that risk implications of across-market competition under banking market integration are significantly different from that of increases in the number of within-market competitor banks. We find that any relation between competition and risk-taking that prevails under within-market competition can be reversed with across-market competition under market integration. This result is robust to a variety of settings, namely, allowing for bank mergers, the presence of an interbank market, and the benefits of geographic diversification from integrating segmented markets. Moreover, the richer set of results can also help understand why the weight of empirical evidence regarding banking market structure and risk-taking has been mixed, with no clear consensus.

This paper distinguishes between two risk-incentive mechanisms from increased competition under market integration that operate through *loan rates*. First, market integration increases competition by raising the number of competitor banks, and this tends to reduce loan rates. This traditional negative relationship, that applies to both within-market and across-market competition, is termed as the *bank-competitor effect* of increased competition on loan rates. Second, market integration also increases competition by increasing the number of entrepreneurs and depositors—customers of banks—in the integrated market. Competition increases with the expansion in market size because deposit supply and loan demand become more elastic, individual banks become small relative to the market, and behave more like price takers (Novshek, 1980).[3] Increasing the number of depositors makes the deposit supply schedule in the integrated market more elastic than that in segmented markets prior to integration. This reduces the (per unit) cost of loanable funds which tends to lower the equilibrium loan rate. Similarly, increasing the number of borrowers also makes the loan demand schedule more elastic. This increases loan demand at any given loan rate and tends to raise the equilibrium loan rate in the integrated market. As long as the integrating markets do not have the same composition of customers, or more precisely, *ratio of borrowers to depositors*, these changes in loan demand and deposit supply generate a *bank-customer effect* of increased competition on loan rates. Unlike the negative bank-competitor effect, the bank-customer effect can be positive or negative, depending on the relative changes of the deposit supply

[3] As Novshek (1980, p. 473) observes, "Firms (banks) may become small relative to the market in two ways: through changes in technology, absolute firm size (the smallest output at which minimum average cost is attained) may become small, or, through shifts in demand, the absolute size of the market (the market demand at competitive price) may become large."

and loan demand schedules from integration. The effect of increased across-market competition under market integration on loan rates comprises the bank-competitor and bank-customer effects.[4]

Initially, markets where borrowers are in relatively shorter supply compared to depositors will, prior to integration, have lower loan rates than those where borrowers are in relatively larger supply compared to entrepreneurs. *Ceteris paribus*, what happens to loan rates when the markets are integrated depends on what happens to the relative balance between depositors and entrepreneurs in the new, integrated market. Loan rates will be lower than in the segmented market where depositors were scarce, but they may be higher than in the segmented market where depositors were plentiful, depending on the relative changes in the numbers of depositors and entrepreneurs. Therefore, the composition of the banks' customer base changes when markets are integrated—and the effect of such changes on loan rates is the bank-customer effect. When the bank-customer effect is positive (because integration increases the number of borrowers relative to depositors) and sufficiently strong to outweigh the negative bank-competitor effect, market integration reverses the traditional negative association between competition and loan rates. As a result, loan rates could go up for a relatively large fraction of entrepreneurs when markets are integrated and because loan rates have adverse effects on the volume of borrowing and on risk-taking, overall welfare could fall with integration.

Our results underscore the importance of incorporating market integration into any examination of the competition-stability linkages. We demonstrate that within- and across-market competition have different implications for risk-taking as long as the integrating markets are heterogeneous. Heterogeneity in the ratio of entrepreneurs to depositors yields the nonzero bank-customer effect under across-market competition that can potentially reverse the relation between within-market competition and risk-taking. Evidently, the bank-customer effect highlighted here is novel and, by construction, absent under within-market competition. The empirical relevance and implications of these results are discussed in Section 7.

Main results. We begin with an examination of the association between loan rates and risk-shifting.[5] Banks lend to entrepreneurs (borrowers) who invest in risky projects but have limited liability. Banks face entrepreneurial moral hazard because the borrowers' choice of project risk is unverifiable and cannot be contracted upon. Typically, models of moral hazard in banking yield a positive relation between loan rates and risk-taking. Raising the loan rate decreases the net return on successful projects, incentivizing borrowers to seek projects less likely to succeed but with higher returns when successful (Stiglitz and Weiss, 1981). In a generalized version of this basic model, where the project output exhibits diminishing marginal productivity of investment, we find that the relation between loan rates and risk-taking can also be negative. Under decreasing marginal productivity, we show that risk-taking increases (decreases) with loan rates according as the output elasticity decreases (increases) with investment.

Prior research has shown that the bank-competitor effect is negative (Boyd and De Nicoló, 2005).

[4]Throughout, the bank-competitor and bank-customer effects are defined as the impact of increased competition on loan rates charged to borrowers. Accordingly, existing theories of within-market competition focus exclusively on the bank-competitor effect. The impact of increased competition on risk-taking follows in turn from the relation between loan rates and risk-taking by borrowers and is described below.

[5]We use the terms risk-taking and risk-shifting interchangeably. In debt-financed firms, owners and managers have incentives to take excessive risks because they benefit from the upside potential while debt-holders bear the downside risks. This well-known risk-shifting problem is particularly acute in banks where a substantial share of liabilities is insured deposits.

Although formulated in the context of competition within an individual market, we show that this result extends to competition across markets under integration. As more markets integrate and each bank faces more competitors, the increased competition tends to lower loan rates charged by banks. Combining the effect of competition on loan rates with that of loan rates on risk-taking generates implications for risk-taking in our framework. Our baseline model assumes that project risks are perfectly correlated across borrowers so that risk-taking by borrowers coincides with risk-taking by banks (Boyd and De Nicoló, 2005). In the *segmented market equilibrium* (SME), the negative bank-competitor effect combines with the effect of decreased loan rates on risk-taking to yield the effect of increased competition on risk-taking. However, unlike previous studies that have shown this effect to be unambiguously negative, we find that increased competition in the SME can also increase risk-taking. In an environment where risk-taking increases with loan rates, the bank-competitor effect of increased competition leads to lower risk-taking. The opposite is true when risk-taking decreases with loan rates.

As discussed above, changes in loan rates and risk-shifting depend on the relative strength of the bank-competitor and bank-customer effects. Heterogeneity in the customer composition—ratio of borrowers to depositors—of integrating markets leads to differences in customer composition between the *integrated market equilibrium* (IME) and the SME. Transitioning from a lower (higher) to higher (lower) ratio of borrowers to depositors implies an increase (decrease) in loan demand relative to the supply of loanable funds—a positive (negative) bank-customer effect—that tends to increase (decrease) loan rates in the IME. A negative bank-customer effect reinforces the negative bank-competitor effect and the effect of market integration on loan rates is unambiguously negative. In contrast, borrowers face higher loan rates in the IME relative to the SME when the bank-customer effect is positive and sufficiently large to outweigh the negative bank-competitor effect.

We show that any association between competition and risk-taking in the SME can be reversed by a positive and sufficiently strong bank-customer effect in the IME that outweighs the negative bank-competitor effect. If risk-taking increases with the loan rate, within-market increases in the number of banks lower loan rates and risk-taking. However, the (positive and strong) bank-customer effect under across-market competition can reverse this negative association by increasing loan rates and risk-taking in the IME. On the other hand, if risk-taking decreases with loan rates, increased within-market competition lowers loan rates but increases risk-taking. Again, the bank-customer effect of increased across-market competition that outweighs the bank-competitor effect increases loan rates and reduces risk-taking, thereby reversing the association under within-market competition. In sum, the results demonstrate how increased competition under market integration affects risk-shifting in ways beyond a simple increase in the number of rival banks.

Extensions. Consolidation in the banking industry that accompanied market integration over the decades motivates the first extension. In particular, we relax our baseline assumption of no entry and exit of banks to allow for bank mergers.[6] Importantly, the dominant reason behind exits under market integration was not bank failures but across-market merger activity, as modeled in Section 6.1 (Wheelock and Wilson,

[6]The simplifying no entry and exit assumption in the baseline model ensures that the number of competitor banks increases with the number of integrating markets. An alternative formulation would allow for free entry of banks as described in Section 4.2.5.

2000; DeYoung, 2019).[7] Regulatory barriers to competition can generate heterogeneity so that banks across different segmented markets operate at different efficiencies.[8] We model this heterogeneity in terms of differences in bank-specific operating costs (non-interest expenses) that vary by market. We use an extensive-form game wherein banks decide whether to merge and a planner or antitrust authority decides whether to allow mergers. Focusing attention on pairwise mergers, we find that horizontal mergers between like banks (a pair of high-cost banks or a pair of low-cost banks) are almost never profitable.[9] In contrast, market-extension mergers between pairs of unlike banks (a high-cost bank and a low-cost bank) are always profitable due to efficiency gains. We model how market integration incentivizes market-extension mergers between efficient and relatively inefficient banks as has been the dominant pattern of bank merger activity documented in numerous empirical studies (Berger, Demsetz, and Strahan, 1999; DeYoung, Evanoff, and Molyneux, 2009).[10] We find that even when they are privately optimal, bank mergers may not be socially optimal, thereby presenting a rationale for merger reviews. On the other hand, although mergers can increase aggregate welfare in this setting, they can also be accompanied by higher risk-taking. Accordingly, our framework presents scenarios in which the implications for welfare and risk-taking generates conflicting recommendations for merger reviews. The financial stability implications of this trade-off present a rationale behind the inclusion of the *financial stability factor* for merger reviews.[11]

In a second extension, we allow for the imperfect correlation of risks across the integrating markets (Section 6.2). The integration of markets with default risks that are imperfectly correlated across markets introduces diversification benefits for banks.[12] As the correlation decreases, the gains from diversification increase allowing banks to diversify risks at lower cost and set lower loan rates. In this way, we find that greater diversification benefits tend to reinforce the negative bank-competitor effect, requiring a stronger bank-customer effect from integration to reverse any association between competition and risk-taking. From a policy standpoint, aggregate welfare under integration also increases with diversification

[7] Although bank failures are noteworthy, the dominant reason behind exits during this period was not bank failures but bank mergers (Wheelock and Wilson, 2000; DeYoung, 2019). In their study of bank exits, Wheelock and Wilson (2000, p. 127) use the terms mergers and acquisitions interchangeably and find that “Since 1984, the number of acquisitions has exceeded the number of failures four-fold, even when acquisitions of insolvent banks are counted only as failures.” The contrast is stronger for the period (post-1993) after their study (for example, see Figure 31-3 in DeYoung, 2019).

[8] Market segmentation and the ensuing lack of competitive pressures have been viewed as the source of inefficiencies among banks (Koetter, Kolari, and Spierdijk, 2012). For example, Kroszner (2001, p. 38) argues that, “. . . branching restrictions tend to reduce the efficiency and consumer convenience of the banking system, and small banks tend to be particularly inefficient in states where branching restrictions offer them the most protection.”

[9] The result is similar to Salant, Switzer, and Reynolds (1983). We find that mergers between like banks are profitable only if the market consist of the same type of banks (all high-cost or all low-cost banks) and if the resulting post-merger market structure is a monopoly.

[10] Berger et al. (1999, p. 150) presents empirical evidence in support of such merger activity: “The prior geographic restrictions on competition may have allowed some inefficient banks to survive. The removal of these constraints allowed some previously prohibited M&As to occur, which may have forced inefficient banks to become more efficient by acquiring other institutions, by being acquired, or by improving management practices internally.”

[11] Regulatory reviews of bank merger applications in the United States have avoided consolidation where excessive increases in risks would be expected. More recently, the inclusion of the financial stability factor in Section 604(d) of the Dodd-Frank Act has replaced Section 3(c) of the Bank Holding Company Act of 1956 (Congress of the United States of America, 2010). “[T]he addition of a financial stability factor . . . contrasts with an antitrust pre-merger review, in which the focus is solely on whether the transaction would substantially lessen competition” (Tarullo, 2014). When evaluating a proposed bank acquisition or merger, the Federal Reserve Board is now required to consider “the extent to which [the] proposed acquisition, merger, or consolidation would result in greater or more concentrated risks to the stability of the United States banking or financial system”.

[12] Indeed, geographic risk diversification has been one of the stated goals of banking deregulation in the United States (Aguirregabiria, Clark, and Wang, 2016).

benefits, making the planner more likely to integrate banking markets.

In the third and final extension, we introduce interbank lending (Section 6.3). Banks take the interbank rate, the policy rate set by the central bank, as given. Unlike the baseline model, the bank-competitor effect is no longer unambiguously negative with interbank lending. In addition to the negative direct effect on loan rates from an increase in the number of banks, there is a positive indirect effect that operates through deposit rates (higher deposit rates from reduced market power tends to raise loan rates as well).[13] Although the negative direct effect is strengthened at low interbank rates, the positive indirect effect dominates at high interbank rates. Ceteris paribus, changes in the stance of monetary policy can reverse the observed association between competition and risk-taking and this can happen even without the bank-customer effect.

Related literature. The theoretical literature on the effect of increased competition on risk-taking can be viewed as comprising two segments. The first set of studies emphasize the effect of increased competition on interest rates (Allen and Gale, 2004; Repullo, 2004; Boyd and De Nicoló, 2005), while the second set examines the effect of interest rates on risk-taking incentives of banks and borrowers (Stiglitz and Weiss, 1981; Martinez-Miera and Repullo, 2010; Wagner, 2010; Dell'Ariccia and Marquez, 2013; González-Aguado and Suárez, 2015). This paper contributes to both segments of the literature.

Our contribution to the first set of studies lies in the bank-customer effect, a previously unexplored risk-incentive mechanism, which is distinct from the bank-competitor effect explored in prior studies (Boyd and De Nicoló, 2005; Martinez-Miera and Repullo, 2010). We demonstrate that any relationship between competition and risk-taking in the SME can be reversed by a positive and sufficiently strong bank-customer effect in the IME. This has a couple of implications. First, unlike increased competition from an increase in the number of banks, increased competition under market integration is not necessarily rate-reducing.[14] Second, the effect of increased competition from market integration depends on the underlying conditions, namely, loan demand and deposit supply in the integrating markets. Therefore, as long as the integrating markets are asymmetric, no two deregulation (market integration) episodes necessarily yield the same outcome in terms of loan rates (see discussion in Section 7).

With respect to the second set of studies, our contribution lies in extending the effect of interest rates on risk-taking to a more generalized version, whereby risk-taking can increase or decrease with loan rates. Models of borrower moral hazard and limited liability have shown that risk-taking increases with loan rates (Stiglitz and Weiss, 1981; Boyd and De Nicoló, 2005). However, risk-taking can decrease with loan rates in moral hazard settings where banks monitor borrower actions (Besanko and Kanatas, 1993; Dell'Ariccia and Marquez, 2013; Martinez-Miera and Repullo, 2017). We show that both outcomes are possible, even in the absence of two-sided moral hazard in borrower and bank actions (such as bank monitoring), as long as borrowers' project returns exhibit diminishing marginal productivity.

Our model produces a richer set of results that can help explain some observed patterns in the evolution of competition. For example, empirical studies have documented that market integration yields

[13] With interbank lending, loan and deposit rates are typically independent as long as the probability of default is exogenous. With endogenous default rates, changes in deposit rates also affect loan rates in our setting.

[14] Under strategic default and non-exclusive contracts, wherein lenders cannot prevent borrowers from taking multiple loans, increased competition among lenders can also lead to higher loan rates relative to the competitive level (Parlour and Rajan, 2001).

pro-competitive gains, such as lower loan rates, even without a concomitant increase in the number of competitor banks (Jayaratne and Strahan, 1996; Dick, 2006). In the model, the negative association between market integration and loan rates can originate from the bank-customer effect that alters price-taking behavior, or from efficiency gains (Section 6.1) and diversification benefits (Section 6.2). Importantly, a more parsimonious setting that defines increased competition in terms of the number of banks within an individual market cannot explain this pattern in the data. In addition to the much explored diversification benefits and efficiency gains from deregulation episodes, our model offers a third mechanism by which integration yields competitive outcomes.[15] Notably, this mechanism is not only independent of efficiency gains and diversification benefits but also of market concentration.

In addition to competition and risk-taking, our paper also contributes to theories examining the integration of banking markets. Morgan, Rime, and Strahan (2004) extend the model of monitored financing and borrower moral hazard in Holmström and Tirole (1997) to show how integration made "state business cycles smaller, but more alike." In the same vein, we compare equilibria where banks are immobile across markets (intrastate banking in Morgan et al., 2004) with those where banks are mobile across markets (interstate banking).[16] However, while they focus on the convergence of business cycles, we study how banking market integration affects risk-taking incentives.

Lastly, we demonstrate how predictions of previous theory prevail as special cases of the general model. In doing so, we isolate the key mechanism by which the financial stability implications of within-market bank competition can be different from across-market competition. The notable special case is one where the integrating markets are *homogenous* in that they have the same ratio of borrowers to depositors. As a result, the bank-customer effect is zero and the overall effect of increased competition is unambiguously negative and comprised entirely of the bank-competitor effect. In this way, isolating the bank-competitor effect yields the results in Boyd and De Nicoló (2005). Under the stronger assumption that the integrating markets are identical in all respects, market integration is equivalent to *replication* of the same banking market (Novshek, 1980).

The rest of the paper is organized as follows. Section 2 explains the key results of the paper using a simple two-market illustrative example. The general (baseline) model is presented in Sections 3 and 4. Section 3 provides the microfoundations, while Section 4 describes market equilibria for the general model. Section 5 presents the welfare implications of market integration using a linear model, which is also used in Section 6 to analyze the three extensions mentioned above. Section 7 presents empirical implications of the results and Section 8 concludes. Proofs of all results are given in Appendices A and B.[17]

[15] Diversification benefits from integration have been known to lower the cost of making loans (Demsetz and Strahan, 1997; Aguirregabiria, Clark, and Wang, 2016; Goetz, Laeven, and Levine, 2016; Levine, Lin, and Xie, 2021). Additionally, integration has also resulted in efficiency gains in the banking industry (Berger et al., 1999; Berger and Mester, 2003; Koetter et al., 2012). Both factors contribute to competitive outcomes such as reduced loan rates in the absence of decreased market concentration.

[16] The model also extends to market integration *within* state borders in the United States. For example, the SME in our model extends to situations where banking markets within a state are segmented, such as those under unit banking laws (Kroszner and Strahan, 1999). Relative to unit banking, statewide (intrastate) banking would be captured by the IME in our model. In this way, the present model captures integration of banking markets both within and across state lines.

[17] Appendix A contains the proofs of the main results, while proofs of all other results are presented in online Appendix B.

2. A simple linear model: within- and across-market competition

Before presenting the general model in Sections 3 and 4, we provide a simple exposition of the main results here. Our goal is to demonstrate that the risk implications of increased within-market competition are different from those of increased across-market competition. An increase in within-market competition can be brought about, for example, by promoting *de novo* entry. Even when markets are segmented, lowering set-up costs (e.g. charter fees and paid-in capital requirements) can increase competition within a given market (Carlson, Correia, and Luck, 2022). On the other hand, deregulation of entry barriers between geographic markets can also increase the number of competitor banks by allowing banks outside the region to enter the market. If this deregulation occurs on a reciprocal basis (as was the case in the United States), banking markets become integrated in that banks have access to customers across erstwhile segmented markets.

The basic linear model adopts a framework widely used in theoretical studies of bank competition and risk-taking. Following Allen and Gale (2004) and Boyd and De Nicoló (2005), we assume that banks are fully deposit financed (they have no equity and do not transact in the interbank market) and default risks of investments are perfectly correlated. We introduce two additional features to this widely used framework. First, we consider the integration of two or more *heterogenous* banking markets. Markets are heterogeneous in terms of customer composition, or more precisely, the *ratio of borrowers to depositors*. As a result, market integration not only increases the number of competitor banks—the traditional source of increased competition—but also expands deposit supply and loan demand asymmetrically. We show that the latter alters the composition of borrowers and depositors in the integrated market in a way that is fundamentally different from a simple increase in the number of banks. Second, we assume that the deposit supply function is *upward-sloping*. The importance of these assumptions is explained below.

Consider two banking markets or regions indexed by $j = 1, 2$.[18] Region j has $a_j > 0$ depositors, $b_j > 0$ entrepreneurs or borrowers so that $a_1 \neq a_2$, and $b_1 \neq b_2$, and the same number of $n \geq 1$ banks.[19] All agents are risk-neutral. Each entrepreneur has a risky project whose initial outlay is \$1 that must be borrowed from a bank (the general model below considers endogenous investment decisions). A dollar invested in the project yields θ with probability $p(\theta) \equiv 1 - \theta/\lambda$ where $\lambda > 1$, and 0 with probability $1 - p(\theta)$, where $\theta \in [0, \lambda]$ is the "riskiness" of the project and is not verifiable, and therefore, non-contractible. Given the loan rate in region j, r_j, each borrower solves

$$\theta_j \equiv \tilde{\theta}(r_j) = \underset{\theta'}{\text{argmax}} \{(1 - \theta'/\lambda)(\theta' - r_j)\} = \frac{1}{2}(\lambda + r_j). \tag{1}$$

As a result, borrower risk-shifting in this example is strictly increasing in the loan rate (risk-shifting can also decrease with loan rates, as shown in the general model below).

[18] The terms markets, regions, and economies are used interchangeably. We consider markets segmented due to institutional, non-economic barriers such as geography, legislation, or regulation.

[19] Below, we show the results hold when integrating markets do not have the same number of banks.

When markets are segmented. The inverse deposit supply and loan demand (see Section 3 for microfoundations) in each segmented market j are given by

$$R_j(D_j/a_j) = \frac{D_j}{a_j} \quad \text{and} \quad r_j(L_j/b_j) = \lambda - \frac{L_j}{b_j}\,, \tag{2}$$

respectively, where D_j and L_j are region j's aggregate deposit and loan volumes, respectively. Note that the inverse deposit supply is a function of supply-of-funds per depositor, D_j/a_j, and the inverse loan demand is a function of loan volume per borrower, L_j/b_j. Microfoundations of this model and other results are presented in Section 5.

In the absence of bank equity and an interbank market, all of bank i's deposits are invested in loans so that $L_{ij} = D_{ij}$. It follows that $L_j = D_j$ for $j = 1, 2$. Each bank i in region j chooses the amount to lend, L_{ij}, to maximize expected profit

$$(1 - \theta(r_j(L_j/b_j))/\lambda)(r_j(L_j/b_j) - R_j(L_j/a_j))L_{ij} = \frac{L_j}{2\lambda b_j}\left(\lambda - \frac{L_j}{b_j} - \frac{L_j}{a_j}\right)L_{ij}.$$

There is a unique symmetric Cournot equilibrium where $L_j = nL_{ij}$ for all i. Equilibrium loan and deposit rates depend on the number of banks, n, and the ratio of borrowers to depositors, $\xi_j \equiv b_j/a_j$, and are given by

$$r_j = \lambda\left(1 - \frac{n+1}{n+2}\cdot\frac{1}{1+\xi_j}\right) \quad \text{and} \quad R_j = \lambda\cdot\frac{n+1}{n+2}\cdot\frac{\xi_j}{1+\xi_j}\,. \tag{3}$$

As one might expect, the equilibrium loan rate decreases and deposit rate increases with the number of banks. Interest margins, $r_j - R_j = \lambda/(n+2)$, depend on the number of banks, n, but not on the ratio of borrowers to depositors, ξ_j.

We assume, without loss of generality, that $\xi_1 < \xi_2$, that is, there are more borrowers relative to depositors in region 2 than region 1. It follows from (3) that $r_1 < r_2$ and $R_1 < R_2$, and in turn, from (1) that $\theta_1 < \theta_2$. Segmented markets where depositors are relatively scarce (region 2) will initially have higher loan rates, higher deposit rates, and more risk-shifting than in those where depositors are relatively plentiful (region 1). Given that both markets have the same number of banks, interest margins are the same in either market, that is, $r_1 - R_1 = r_2 - R_2$.

When markets are integrated. Now consider the case where regions 1 and 2 are integrated to form a single banking market. We assume that there is no entry or exit of banks due to market integration.[20] As a result, there are $2n$ banks, $a_1 + a_2$ depositors, and $b_1 + b_2$ borrowers in the integrated market. The inverse deposit supply and loan demand in the integrated market are given by

$$R(D/(a_1+a_2)) = \frac{D}{a_1+a_2} \quad \text{and} \quad r(L/(b_1+b_2)) = \lambda - \frac{L}{b_1+b_2}\,, \tag{4}$$

respectively, where D and L are the aggregate deposit and loan volumes in the integrated market, respectively. Both deposit supply and loan demand in the integrated market are horizontal sums of the respective deposit supply and loan demand in the segmented markets (see Figure 1).

[20]The assumption is relaxed in Section 4.2 to include free entry and in Section 6.1 to include mergers.

Following the same analysis as above, we obtain the equilibrium loan and deposit rates in the integrated market as

$$r^* = \lambda\left(1 - \frac{2n+1}{2n+2} \cdot \frac{1}{1+\xi^*}\right) \quad \text{and} \quad R^* = \lambda \cdot \frac{2n+1}{2n+2} \cdot \frac{\xi^*}{1+\xi^*}\,, \tag{5}$$

respectively, where $\xi^* \equiv \frac{b_1+b_2}{a_1+a_2}$ is the ratio of borrowers to depositors in the integrated market. Interest margins in the integrated market, $r^* - R^* = \lambda/(2n+2)$, depend on the number of banks, $2n$, but not on the ratio of borrowers to depositors, ξ^*.

We can isolate the bank-competitor effect—the effect of an increase in the number of competitor banks due to market integration on loan rates—if we consider the case where integrating markets are *homogeneous* (i.e., $\xi_1 = \xi_2$). With homogenous markets, the integrated market also has the same ratio of borrowers to depositors as each of the segmented markets (i.e., $\xi^* = \xi_j$ for $j = 1,\,2$). For the same ξ, the difference between (3) and (5) is only due to the difference in the number of banks, and we get $r^* < r_j$ and $R^* > R_j$, for $j = 1,\,2$. This reduces interest margins from $\lambda/(n+2)$ to $\lambda/(2n+2)$. All else equal, increasing the number of banks lowers loan rates and risk-shifting—the traditional negative bank-competitor effect modeled in existing theories of bank-competition and risk-taking.

Returning to the model with heterogenous markets (i.e., $\xi_1 \neq \xi_2$), we show that market integration creates an additional risk-shifting effect that stems from the changes in the composition of borrowers and depositors. Accordingly, we define the bank-customer effect of market integration as the effect of a change in the ratio of borrowers to depositors, ξ, on loan rates. An increase in ξ increases the borrower pool relative to depositors, prompting banks to not only charge higher loan rates but also pay more on deposits. Moreover, changes in loan rates from changes in ξ are accompanied by commensurate changes in the deposit rate so that interest margins are unchanged. As changes in loan rates are directly linked to risk-taking by borrowers, changes in the customer composition affects borrower risk-taking without changing bank interest margins. The greater the change in ξ, the bigger is its impact on loan rates. Therefore, we often use the magnitude of the change in customer composition, namely $\xi^* - \xi_j$ for market $j = 1,\,2$, as a shorthand for the size of the bank-customer effect.

Importance of the bank-customer effect. In what follows, we focus the analysis on loan rates and exclude the analysis on deposit rates for brevity. From our earlier assumption that $\xi_1 < \xi_2$, we obtain $\xi_1 < \xi^* < \xi_2$. In other words, the ratio of borrowers to depositors declines (increases) upon integration for the market with the relatively higher (lower) ratio prior to integration. With $\xi^* < \xi_2$, transitioning from a higher to a lower ratio implies increased deposit supply relative to loan demand for banks in market 2 and this tends to reduce loan rates. Therefore, the bank-customer effect with respect to market 2 is negative and it reinforces the negative bank-competitor effect. As a result, $r^* < r_2$ and it follows that $\theta^* \equiv \tilde{\theta}(r^*) < \theta_2$.

With $\xi_1 < \xi^*$, the converse holds for market 1. Transitioning from a lower to a higher ratio implies increased loan demand relative to deposit supply for banks in market 1 and this tends to increase loan rates. Therefore, the bank-customer effect with respect to market 1 is positive. With a positive bank-customer effect and a negative bank-competitor effect, the final outcome depends on the relative strengths of these effects. If $\xi^* - \xi_1$ is sufficiently large, we get $r^* > r_1$ because the positive bank-customer effect outweighs the negative bank-competitor effect, and the loan rate for market 1 borrowers increases upon integration. As a result, borrowers in market 1 increase risk-shifting in the integrated market and $\theta^* > \theta_1$. However, if $\xi^* - \xi_1$ is not sufficiently large, we get $r^* \leq r_1$ because the negative bank-competitor effect dominates and market integration reduces loan rates and risk-shifting in both markets.

Given its implications for competition and risk-taking, some aspects of the bank-customer effect – unmodeled in prior studies – are worth highlighting. First, market heterogeneity, defined as differences in the measure of borrowers to depositors is a necessary condition for a non-zero bank-customer effect. As explained above, setting $\xi_1 = \xi_2$ yields $\xi^* = \xi_j$ for $j = 1, 2$ and the bank-customer effect is zero. When integrating markets are homogenous (have the same ξ), the overall effect of increased competition is comprised entirely of the bank-competitor effect and is unambiguously negative. In this way, the special case yields predictions of traditional theories on (within-market) competition and risk-taking (Boyd and De Nicoló, 2005). Another notable special case is one where the integrating markets are identical (i.e., $a_1 = a_2$ and $b_1 = b_2$). In this case, market integration is equivalent to *replication* of the same banking market (Novshek, 1980).

Second, the *reference market* is important to the notion of the bank-customer effect. We obtain

$$\xi^* - \xi_1 = \frac{a_2}{a_1 + a_2} \cdot (\xi_2 - \xi_1) \quad \text{and} \quad \xi^* - \xi_2 = -\frac{a_1}{a_1 + a_2} \cdot (\xi_2 - \xi_1). \tag{6}$$

It follows that the bank-customer effects with respect to markets 1 and 2 are of opposite sign.[21]

Lastly, the bank-customer effect can be positive or negative because of the inherent asymmetry in integrating heterogeneous banking markets, each with a different ratio of borrowers to depositors prior to integration. In contrast, the bank-competitor effect is unambiguously negative because increases in the number of banks affect both deposit and loan markets symmetrically. When the bank-customer effect is sufficiently strong to outweigh the bank-competitor effect, we obtain $r_1 < r^* < r_2$ and $R_1 < R^* < R_2$. Market integration helps borrowers in market 2 by lowering loan rates but hurts borrowers in market 1 by raising them. The converse holds for depositors. Depositors in market 1 (market 2) get higher (lower) rates on their deposits. In sum, market integration affects borrowers and depositors asymmetrically: in markets where borrowers are relatively scarce (plentiful) prior to integration, market integration makes borrowers pay higher (lower) loan rates whereas depositors receive higher (lower) rates on their deposits. Such disparate impact between borrowers and depositors across markets is not captured in models of within-market competition where the effect of increased competition is comprised entirely of the negative bank-competitor effect that benefits all customers.

Before concluding this illustrative example, it is important to discuss the second critical assumption of the model, namely, that the deposit supply function is upward-sloping. To understand its importance, let us assume instead that deposit supply is perfectly elastic, so that $R_j(D_j) = \bar{R} \geq 0$ is the banks' *constant marginal cost* of acquiring deposits in any market j. Banks can collect any amount of deposits at the flat rate to meet loan demand. Therefore, bank i's expected profit in market j is

$$\frac{L_j}{2\lambda b_j} \left(\lambda - \frac{L_j}{b_j} - \bar{R} \right) L_{ij}.$$

[21] Evidently, $\text{sign}[\xi^* - \xi_1] = \text{sign}[\xi_2 - \xi_1]$ and $\text{sign}[\xi^* - \xi_2] = -\,\text{sign}[\xi_2 - \xi_1]$.

The symmetric equilibrium loan rate in (the segmented) market j is derived as

$$r_j = \bar{R} + \frac{\lambda - \bar{R}}{n+2}.$$

In contrast to (3), $r_1 = r_2$ even if $b_1 \neq b_2$. Assuming a perfectly elastic deposit supply implies that loan rates are identical across segmented markets even when these markets have different loan demand. Moreover, the equilibrium loan rate r_j is a constant mark-up over marginal cost that is decreasing in the number of banks. Upon integration, the loan rate in the integrated market becomes

$$r^* = \bar{R} + \frac{\lambda - \bar{R}}{2n+2},$$

which is lower than r_j for $j = 1, 2$ because there are more banks in the integrated market. Even with market heterogeneity, the effect of integration on loan rates is comprised entirely of the bank-competitor effect. Therefore, with perfectly elastic deposit supply, the composition of bank customers does not affect loan rates and risk-taking under market integration.

3. The general model: microfoundations

There are $J = \{1, 2, \ldots\}$ segmented banking markets. Each market $j \in J$ consists of three distinct groups of risk-neutral agents—a continuum of depositors of measure $a_j > 0$, a continuum of borrowers or entrepreneurs of measure $b_j > 0$, and $n_j \geq 1$ banks. Within each group, all agents are identical—we assume that borrowers, depositors, and banks are homogenous both within and across markets. However, markets vary in size so that no two markets necessarily have the same number of agents in each group.

3.1. Deposit supply

Let $R_j \geq 1$ be the deposit rate offered by banks in market j. Each depositor in market j solves (dropping the subscript for the individual depositor)

$$d(R_j) = \underset{d}{\text{argmax}}\ R_j d - V(d), \tag{7}$$

where $V(\cdot)$ is the foregone utility associated with making deposits d, which is assumed to be strictly increasing and strictly convex. The first-order condition associated with (7) is given by $V'(d(R_j)) = R_j$ from which we obtain that individual deposit supply $d(R_j)$ is strictly increasing in the deposit rate R_j because $V''(d) > 0$. Given identical depositors, aggregate deposit supply in market j is $D_j = a_j d(R_j)$, and the inverse deposit supply is

$$R_j = d^{-1}(D_j/a_j) \equiv R(D_j/a_j) \quad \text{with} \;\; R'(D_j/a_j) > 0.$$

The above maximization problem of identical borrowers allows us to write the inverse deposit supply as a function of supply-of-funds per depositor, D_j/a_j.

3.2. Loan demand under borrower moral hazard

3.2.1. Optimal investment and risk-taking

Loan demand in market j is obtained from a simple model of lending under borrower moral hazard. We assume a contractual environment where entrepreneurs have access to a set of risky projects indexed by θ whose returns are random and perfectly correlated.[22] Entrepreneurs with zero net worth must borrow to invest in the project. If k dollars are invested in a given project, it yields

$$Y(\theta,\,k) = \begin{cases} y(\theta)f(k) & \text{with probability } p(\theta), \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

We assume that (i) the return, $y(\theta)$, is strictly increasing and strictly concave on $[0,\,\bar{\theta}]$, (ii) the probability of success, $p(\theta)$, is strictly decreasing and strictly concave on $[0,\,\bar{\theta}]$ with $p(0) = 1$ and $p(\bar{\theta}) = 0$, and (iii) the production function, $f(k)$, is strictly increasing and strictly concave on $[0,\,\bar{k}]$ with $f(0) \geq 0$. The variable θ represents the "riskiness" of the project—for a given k, the higher the θ, the higher is the return $y(\theta)$, but the lower is the probability of success, $p(\theta)$. Borrowers' choice of risk is not publicly verifiable, and therefore, not contractible.

There are two decision stages. In stage 1, each borrower chooses the level of investment, k, which generates individual demand for loans as a function of the loan rate $r_j \geq 1$.[23] In stage 2, taking the stage-1 borrowing decision as given, the borrower privately chooses project riskiness θ after which, project returns are realized, and payoffs are made. Borrowers have *limited liability* and the conjunction of moral hazard and limited liability affects terms of the loan contract. In granting loans, banks cannot write contracts that are contingent on project riskiness θ because this is private information of the borrower. However, banks correctly anticipate the risk-shifting incentives of borrowers, imposing a sequential rationality constraint on the equilibrium (Brander and Spencer, 1989). In stage 2, given k and r_j, each borrower in market j solves

$$\theta(k;\,r_j) = \underset{\theta}{\operatorname{argmax}}\; p(\theta)\left\{y(\theta)f(k) - r_j k\right\}. \tag{9}$$

The first-order condition associated with (9) is

$$h(\theta) = \frac{r_j}{f(k)/k}\,, \tag{10}$$

where $h(\theta) \equiv y(\theta) + y'(\theta)(p(\theta)/p'(\theta))$ is strictly increasing in θ. The above condition is the equality between the expected marginal revenue of risk-shifting and its expected marginal cost.[24] The borrower's

[22] Section 6.2 extends the model to imperfectly correlated risks. The assumption that entrepreneurs' returns are perfectly correlated follows from Boyd and De Nicoló (2005) and is equivalent to the assumption of bank portfolios comprising perfectly correlated risks (Allen and Gale, 2004). The risk associated with each project can in general be decomposed into a systemic and idiosyncratic component. With a large number of projects, the idiosyncratic component can be perfectly diversified away. This assumption helps focus the baseline analysis on the common component representing systemic risks.

[23] More formally, borrower i in market j chooses k_{ij}. Again, we drop the individual subscripts for ease of notation.

[24] Condition (10) can be written as

$$[p'(\theta)y(\theta) + p(\theta)y'(\theta)]f(k) = p'(\theta)r_j k.$$

The left-hand-side and the right-hand side of this equation are the expected marginal revenue and the expected marginal cost of risk-shifting, respectively.

risk-shifting choice, θ, in (10) depends on the average product of capital, $f(k)/k$, and the loan rate, r_j . We obtain $\theta_r(k;\, r_j) > 0$ because $h'(\theta) > 0$, so that risk-shifting is increasing in the loan rate r_j. In other words, higher cost of investment increases risk-shifting. We also obtain $\theta_k(k;\, r_j) > 0$ because $f(k)/k$ decreases with k as $f(\cdot)$ is concave and $f(0) \geq 0$, so that risk-shifting is also increasing in investment k. Therefore, when investment k increases, capital is less productive, and this incentivizes the borrower to take on more risk. In short, those who invest more are more liable to moral hazard (Banerjee, 2003).

Individual loan demand is determined at stage 1 where the borrower chooses investment k so that

$$k(r_j) = \underset{k}{\text{argmax}}\ p(\theta(k;\, r_j)) \left\{ y(\theta(k;\, r_j)) f(k) - r_j k \right\}. \tag{11}$$

It follows that $k'(r_j) \leq 0$, so that individual borrower's loan demand, $k(r_j)$, is downward sloping. Given identical borrowers, aggregate loan demand in market j is $L_j = b_j k(r_j)$, and the inverse loan demand is

$$r_j = k^{-1}(L_j/b_j) \equiv r(L_j/b_j) \quad \text{with} \;\; r'(L_j/b_j) < 0.$$

As in the case of deposit supply, the maximization problem of identical borrowers allows us to express the inverse demand for loans as a function of the loan volume per borrower, L_j/b_j.

3.2.2. *Loan rates and risk-taking*

There are two ways in which loan rates affect risk-taking in this framework. In addition to the direct effect on entrepreneurs' optimal choice of risk, loan rates also affect risk-taking through its effect on borrower's loan demand, so that $\tilde{\theta}(r_j) \equiv \theta(k(r_j);\, r_j)$. Taken together,

$$\frac{d\tilde{\theta}}{dr_j} = \theta_r(k;\, r_j) + \theta_k(k;\, r_j) k'(r_j) \tag{12}$$

From (10), the *direct effect*, denoted by the first term on the right-hand side of (12), is positive. However, with $\theta_k(k;\, r_j) > 0$ from (10) and $k'(r_j) \leq 0$ from (11), the *indirect effect* of the loan rate on risk-taking – denoted by the second term on the right-hand side of (12) – is negative. Overall, the relation between loan rates and risk-taking is summarized by the following proposition.

Proposition 1 *Borrowers' optimal risk choice in market j, $\tilde{\theta}(r_j) \equiv \theta(k(r_j);\, r_j)$, depends on the loan rate, r_j, and borrower investment, $k(r_j)$ which is a decreasing function of the loan rate. If the output elasticity of investment, $\varepsilon(k) \equiv kf'(k)/f(k)$ is decreasing (increasing) in k, then optimal risk-taking $\tilde{\theta}(r_j)$ is increasing (decreasing) in r_j.*

The proof of this proposition is provided in Appendix B. The effect of loan rate changes on risk-taking, $d\tilde{\theta}/dr_j$, depends on the relative magnitudes of the direct and indirect effects because they point in opposite directions. We show that the average productivity, $f(k)/k$ is more (less) responsive to a change in investment caused by a change in the loan rate according as the output elasticity of investment, $\varepsilon(k)$, is decreasing (increasing) in k. When average productivity is more responsive to changes in investment due to a change in r_j, the indirect effect in (12) outweigh the direct effect. The converse occurs when the average productivity is less responsive. In sum, the relation between loan rate and risk-taking depends on the behavior of the output elasticity with respect to investment.

The possibility of a negative relation between loan rates and risk-taking deviates from conventional models of moral hazard with limited liability which obtain an unambiguously positive relation between loan rates and risk-taking (Stiglitz and Weiss, 1981). We show that risk-taking can increase or decrease with loan rates in settings where the production technology exhibits diminishing returns (see Appendix B for details and related results). Notably, for constant elasticity production functions (e.g. $f(k) = k^{\eta}$ for $0 < \eta < 1$), borrower risk-taking is independent of the loan rate because the direct and indirect effects cancel each other.

Before concluding this section, it is important to point out that, in our framework with perfectly correlated risks, entrepreneurial risk-taking is synonymous with risk-taking by banks. In this context, banks' optimal asset allocation is determined by an optimal contracting problem as discussed in Boyd and De Nicoló (2005) instead of a portfolio choice problem as modeled in Allen and Gale (2004).

4. The general model: market equilibrium

In any market, segmented or integrated, banks compete à la Cournot by choosing deposit and loan volumes. We begin by characterizing equilibrium loan rates and risk-shifting for each segmented market. We refer to this equilibrium as the *segmented market equilibrium* (SME) and show how competition affects loan rates and risk-taking in the SME. Next, we solve for the equilibrium when $m \, (\geq 2)$ segmented markets are integrated into a single banking market. We refer to this equilibrium as the *integrated market equilibrium* (IME) and analyze the effects of competition on loan rates and risk-shifting in the IME. Finally, we compare the effect of competition on loan rates and risk-shifting between the SME and the IME.

In solving the model, we make several simplifying assumptions. First, there is no exit or entry of banks. We relax this assumption in Sections 4.2.5 and 6.1 where we allow for free entry of banks and bank mergers, respectively. Second, banks are funded entirely by customer deposits—banks have no equity and there is no interbank market for deposits. Again, we relax this assumption when we introduce interbank markets in Section 6.3. Third, all bank deposits are insured for which each bank pays a flat premium that is normalized to zero. Lastly, all customers (borrowers and depositors) can switch costlessly between banks. Given our assumptions that agents are homogenous within each group, this is a fairly innocuous (zero transaction cost) assumption.[25]

4.1. The segmented market equilibrium

4.1.1. The equilibrium

With no equity and no interbank market, the balance sheet identity of bank i in market j implies $L_{ij} = D_{ij}$. Consequently, aggregate loan demand equals aggregate deposit supply in market j so that $L_j = \sum_{i=1}^{n_j} L_{ij} = \sum_{i=1}^{n_j} D_{ij} = D_j$. Bank i in market j chooses the volume of loans, L_{ij}, to maximize

[25] As banks obtain information on borrower quality during the course of a lending relationship, switching between banks is beset with problems of information asymmetry. A large literature examines how competition in credit markets affects the screening problem banks face in granting loans (Broecker, 1990; Sharpe, 1990). It is important to mention that, in such models, switching costs are generally bank-specific (they apply to customers switching from one bank to another, even in the absence of market integration) and not necessarily market-specific (they do not generally apply to customers switching from local to non-local banks when markets integrate). We abstract from these considerations both for the SME and the IME.

expected profits, taking into account choices made by its competitors and the entrepreneurs' choice of risk. In the segmented market, each bank in market j solves

$$\max_{L_{ij}} P(L_j/b_j)[r(L_j/b_j) - R(L_j/a_j)]L_{ij}, \tag{13}$$

where $P(L_j/b_j) \equiv p(\tilde{\theta}(r(L_j/b_j)))$, $R(0) \geq 0$, $R'(D_j/a_j) > 0$, $R''(D_j/a_j) \geq 0$, $r(0) \geq R(0)$, $r'(L_j/b_j) < 0$ and $r''(L_j/b_j) \leq 0$.

Given that all banks are identical, and that they face the same aggregate deposit supply and loan demand schedules, in the segmented market, there are no asymmetric equilibria (see the proof of Lemma 2 in the Appendix). Moreover, the symmetric equilibrium is unique. In the symmetric Cournot equilibrium, we have $L_{ij} = L_j/n_j$ for all i. The following lemma characterizes the (symmetric) SME in market j.

Lemma 1 *The symmetric SME is characterized by loan rate, $r_j = r(L_j/b_j)$, deposit rate, $R_j = R(L_j/a_j)$, risk-shifting, $\theta_j = \tilde{\theta}(r(L_j/b_j))$, and intermediation margin*

$$r(L_j/b_j) - R(L_j/a_j) = \frac{[b_j R'(L_j/a_j) - a_j r'(L_j/b_j)]\, P(L_j/b_j)L_j}{a_j\, [n_j b_j P(L_j/b_j) + P'(L_j/b_j)L_j]}\,, \tag{14}$$

where L_j denotes the aggregate loans in market j.

Equation (14) determines bank loan volumes in the symmetric Cournot equilibrium. We obtain the loan rate in the SME from the inverse loan demand function, $r(L_j/b_j)$. Likewise, we obtain aggregate deposits and the deposit rate in the SME from $D_j = L_j$, and the inverse deposit supply function, $R(D_j/a_j)$, respectively.

4.1.2. Competition and risk-taking in the SME

Increased competition in the SME is defined as an increase in the number of banks. Competition can increase in any of the segmented markets if local banking authorities lower fixed set-up costs (Mankiw and Whinston, 1986). Such costs include charter fees or capital requirements at the founding of the bank as was required during the National Banking Era (Carlson et al., 2022).[26] From Lemma 1, comparative statics reveal that the SME loan rate, r_j, decreases as the number of banks, n_j, increases. It follows from Proposition 1 that entrepreneurial risk-shifting in the SME may increase or decrease with n_j.

Proposition 2 *In the SME of each market j, loan rate r_j is strictly decreasing in n_j. As a result, risk-shifting θ_j decreases (increases) according as the output elasticity of investment, $\varepsilon(k)$ is increasing (decreasing) in k.*

While increased competition in the SME unambiguously decreases loan rates, the effect on risk-taking is not unambiguous. Increased competition from an increase in the number of banks decreases

[26] In this context, lowering set-up costs captures within-market competition as distinct from across-market competition. As Carlson et al. (2022, p. 464) point out, "The National Banking Era constitutes a close-to-ideal empirical laboratory to study the causal effects of banking competition . . . the prevalence of unit banking ensures that banking markets are local and well defined, which allows us to compare different, arguably independent markets."

risk-taking if and only if risk-taking increases with loan rates. Proposition 2 asserts that this risk-incentive mechanism, first shown in Boyd and De Nicoló (2005), is obtained in the SME when the production technology exhibits decreasing elasticity of investment. However, in situations where the production technology exhibits increasing elasticity of investment, we find that increased competition can increase risk-taking by borrowers. Using a natural experiment that has similarities to the SME in the model, Carlson et al. (2022) find that banks operating in markets with lower entry barriers in the National Banking Era increased riskiness in lending and were more likely to default. This empirical finding lends support for our result that increased competition within segmented markets can also increase risk-taking.

4.2. The integrated market equilibrium

4.2.1. Deposit supply and loan demand in the integrated market

Now, suppose that a subset of segmented markets, $J_m \subset J$, are integrated to form a single banking market, where $J_m = \{1, 2, \ldots, m\}$ and $|J_m| = m < |J|$. Given the assumption that there is no entry or exit of banks, the number of banks in the integrated market is equal to the total number of banks across the m markets combined, so that

$$n(m) = \sum_{j=1}^{m} n_j \tag{15}$$

Although the aggregate number of banks remains unaltered after market integration, each bank faces new rivals in the integrated market. In a similar vein, the measure of depositors and borrowers in the integrated market equals the aggregate of the measures of depositors and borrowers in each of the m individual markets, respectively. Therefore

$$a(m) \equiv \sum_{j=1}^{m} a_j \quad \text{and} \quad b(m) \equiv \sum_{j=1}^{m} b_j. \tag{16}$$

The feature that the integrated market (by construction) is the sum of the individual segmented markets in terms of banks, borrowers, and depositors is conventional and largely for simplicity of exposition. As we show below, the key underlying mechanisms of the model remain robust to different outcomes. For example, the bank-competitor effect (discussed in Section 2) is negative irrespective of whether (15) holds or whether $n(m)$ is greater than, equal to, or less than n_j for each j.

Under market integration, depositors and borrowers solve the same maximization problems as given in (7) and (11), respectively. This follows directly from our assumption that depositors and borrowers are homogenous across the m integrating markets. Customer homogeneity before and after integration implies that, for given deposit and loan rates, deposit supply, $d(R)$, and loan demand, $k(r)$, in the segmented and the integrated markets are the same.

From (16), aggregate deposit supply and loan demand in the integrated market are $D = a(m)d(R)$ and $L = b(m)k(r)$, respectively. It follows that the inverse deposit supply and loan demand functions in the integrated market are

$$R = R(D/a(m)) \quad \text{where} \quad R'(D/a(m)) > 0, \tag{17}$$

$$r = r(L/b(m)) \quad \text{where} \quad r'(L/b(m)) < 0, \tag{18}$$

respectively. The inverse deposit supply of the integrated market, $R(D/a(m))$, is the horizontal sum of the m individual inverse deposit supply schedules, and consequently, more elastic than $R(D_j)$ for any $j = 1, \ldots, m$. Likewise, the inverse loan demand function, $r(L/b(m))$, which is the horizontal sum of the m individual inverse loan demand schedules, is more elastic than $r(L_j)$ for any $j = 1, \ldots, m$. Being the horizontal sum of convex deposit supply (concave loan demand) schedules, aggregate deposit supply (loan demand) is also convex (concave), that is, $R''(D/a(m)) \geq 0$ ($r''(L/b(m)) \leq 0$). Figure 1 shows the inverse loan demand function for $m = 2$ and $b_1 < b_2$.

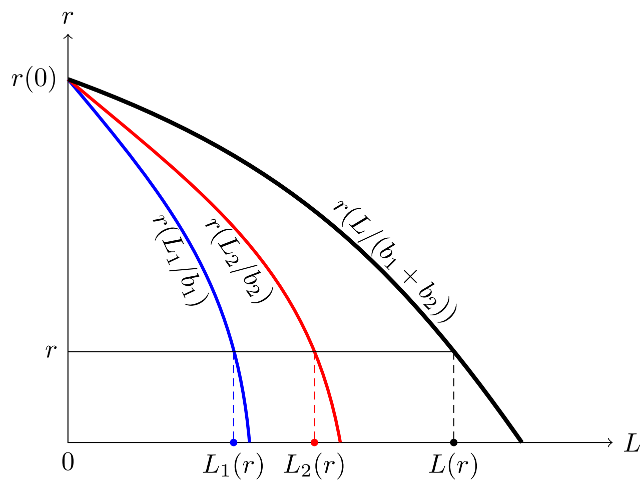

Figure 1: *Loan demand when two markets integrate. Loan demand in the segmented market i is $L_j(r) = b_j k(r)$, $j = 1, 2$. If $b_1 < b_2$, $r_1(L_1)$ lies below $r_2(L_2)$. Loan demand in the integrated market, $L(r) = L_1(r) + L_2(r) = (b_1 + b_2)k(r)$, is more elastic than that in any of the segmented markets.*

4.2.2. The equilibrium

In the integrated market, J_m, all $n(m)$ banks are identical and they face the same inverse deposit supply (17) and inverse loan demand (18). Given that banks are exclusively deposit-financed, $L_i = D_i$ holds for all i. This yields the market clearing condition, $L = D$. Bank i solves

$$\max_{L_i} P(L/b(m))[r(L/b(m)) - R(L/a(m))]L_i. \tag{19}$$

It follows that $L_i = L/n(m)$ for all i, and the symmetric IME is described by the following proposition.

Lemma 2 *The symmetric IME is characterized by loan rate, $r^* = r(L/b(m))$, deposit rate, $R^* = R(L/a(m))$, optimal risk-shifting $\theta^* = \tilde{\theta}(r(L/b(m)))$, and the intermediation margin*

$$r(L/b(m)) - R(L/a(m)) = \frac{[b(m)R'(L/a(m)) - a(m)r'(L/b(m))]\, P(L/b(m))L}{a(m)\,[n(m)b(m)P(L/b(m)) + P'(L/b(m))L]}, \tag{20}$$

where L denotes the aggregate loans in the integrated market and $a(m)$ and $b(m)$ are given by (16).

Evidently, the expressions for equilibrium loan rate and risk-taking for the IME in Lemma 2 are isomorphic to those obtained for the SME in Lemma 1. In spite of this similarity, we show below how the integration of heterogenous markets can alter the relation between competition and loan rates, and consequently, competition and risk-taking in the SME.

4.2.3. Effect of increased competition on loan rates

Increased competition in the IME is defined as an increase in the number of integrating markets. Given that integrating markets are heterogenous in our setting, we take an increase in m to imply that additional markets are integrated. Formally, if the set of integrated markets expands from J_m to $J_{m'}$, where $|J_m| = m$, $|J_{m'}| = m'$, then $J_m \subseteq J_{m'} \subset J$. In other words, we compare loan rates and risk-shifting between the IME in market J_m (shorthand for the smaller integrated set of m markets) and that in market $J_{m'}$ (shorthand for the larger integrated set of m' markets). Agents (banks and customers) are homogenous across the two sets of markets. But, their measures are different in each market, and with integration, deposit supply and loan demand schedules are more elastic in the larger (integrated) market.

Market integration intensifies bank competition in two ways. First, an increase in m increases the number of competitor banks so that each bank faces competition from more rivals in $J_{m'}$ relative to J_m. Second, integration of additional markets increases competition by expanding the size of deposit and loan markets. The deposit supply schedules for J_m and $J_{m'}$ are given by $D(R, a(m)) \equiv a(m)d(R)$ and $D(R, a(m')) \equiv a(m')d(R)$, respectively. With $a(m) < a(m')$, the inverse supply function for deposits in the larger market, $J_{m'}$, is more elastic than that in J_m. In this way, market integration makes the deposit market, and by the same logic, the loan market, more competitive as individual banks become small relative to these markets.[27]

The bank-competitor and the bank-customer effects. We explore the comparative static properties of the IME described in Lemma 2 to disentangle the effects of $a(m)$, $b(m)$, and $n(m)$ on loan rates.[28] Formally,

$$\frac{dr^*}{dm} = \underbrace{\frac{\partial r}{\partial n} \cdot n'(m)}_{\text{bank-competitor effect}} + \underbrace{\frac{\partial r}{\partial a} \cdot a'(m) + \frac{\partial r}{\partial b} \cdot b'(m)}_{\text{bank-customer effect}}. \tag{21}$$

Increasing m increases the number of competitor banks in the integrated market $n(m)$ and so, the first term on the right-hand-side of (21) denotes the *bank-competitor effect*. The second and third terms together constitute the *bank-customer effect*—the effect of an increase in the measure of customers (depositors and borrowers) under market integration. Because $n(m)$, $a(m)$, and $b(m)$ are all strictly increasing in m, the sign of each term on the right-hand side of (21) is determined by the sign of the partial derivative in each term. We summarize our findings in terms of the following proposition.

[27] Integration of economies in models of intra-industry trade not only increases the measure of consumers but also expands their choice (Krugman, 1979). Although each consumer can potentially transact with more firms upon integration, consumers spend less on each variety because products are horizontally differentiated. This prompts some firms to exit the integrated market. Such exits do not occur in our setting because we assume that products of banks, namely deposits and loans, are not differentiated.

[28] We treat m as a continuous variable. Clearly, $n(m)$, $a(m)$, and $b(m)$ are all strictly increasing functions of m.

Proposition 3 *In the IME, the effect on an increase in m on the loan rate r^* is given by*

$$\frac{dr^*}{dm} = \underbrace{Z_0(m) \cdot \hat{n}(m)}_{\text{bank-competitor effect}} + \underbrace{Z_1(m) \cdot (\hat{b}(m) - \hat{a}(m))}_{\text{bank-customer effect}} \equiv Z_0(m) \cdot \hat{n}(m) + Z_1(m) \cdot \hat{\xi}(m), \tag{22}$$

where $Z_0(m) < 0$ and $Z_1(m) > 0$ for all m, $\xi(m) \equiv b(m)/a(m)$ denotes the ratio of borrowers to depositors in market m, and $\hat{z}(m) \equiv z'(m)/z(m)$ denotes the rate of expansion of any arbitrary function $z(m)$.

Equation (22) is isomorphic to (21). Just as in (21), the first term on the right-hand-side of (22) captures the bank-competitor effect. Moreover, just as in the SME, the bank-competitor effect is negative (because $Z_0(m) < 0$ for all m) in the IME. All else equal, an increase in the number of competitor banks with more integration reduces the market power of the banks in the loan market by reducing their market share, which in turn lowers the loan rate.

We also find that $Z_1(m) > 0$ for all m in (22). Consequently, the bank-customer effect is positive, zero, or negative according as $\hat{b}(m) \gtreqless \hat{a}(m)$ or $\hat{\xi}(m) \gtreqless 0$. When $\hat{b}(m) < \hat{a}(m)$, increasing m expands the deposit supply more than the loan demand. As shown in Section 2, this implies a transition to a lower ratio of borrowers to depositors. As a result, the bank-customer effect is negative, which tends to lower loan rates. Conversely, when $\hat{b}(m) > \hat{a}(m)$, increasing m expands the loan demand more than deposit supply. Therefore, market integration implies transitioning to a higher ratio of borrowers to depositors. In this case, the bank-customer effect is positive, which tends to raise loan rates.

The overall effect of market integration on the equilibrium loan rate comprises the bank-competitor and bank-customer effects. A negative bank-customer effect reinforces the negative bank-competitor effect and the final outcome is a lower equilibrium loan rate in the integrated market. Accordingly, a negative bank-customer effect is a *sufficient condition* for a negative association between competition and loan rates. In contrast, a positive bank-customer effect can increase loan rates in the integrated market but only if it is sufficiently strong to outweigh the negative bank-competitor effect (i.e., $\hat{b}(m) - \hat{a}(m) > (-Z_0(m)/Z_1(m)) \cdot \hat{n}(m)$). Therefore, a positive bank-customer effect is a *necessary condition* for a positive association between competition and loan rates. The properties of the bank-customer effect discussed in Section 2 follow.

4.2.4. Competition and risk-taking in the IME

We turn now to analyze the effect of competition on risk-taking incentives. The effect of increased competition on risk-taking in the IME comprises two effects: first, the effect of increased competition on the loan rate as shown above and second, the effect of the loan rate on risk-taking incentives of borrowers as analyzed in Section 3.2.2. Taken together, the effect on increased competition on risk-taking in the IME is non-trivial.

First, consider the case where risk-taking increases with the loan rate, that is, $\varepsilon'(k) < 0$. As described in Section 3.2.2, this is a standard result in most models of borrower moral hazard and limited liability (Stiglitz and Weiss, 1981; Boyd and De Nicoló, 2005). With the bank-competitor effect always negative, we have shown above that risk-taking decreases with competition in the SME. However, increased competition under market integration can reverse the relation between competition and risk-taking. A positive and sufficiently strong bank-customer effect can lead to increased risk-taking in the IME, through its effect of increasing loan rates.

Next, consider the case where risk-taking decreases with the loan rate. In Section 3.2.2, this result is obtained when the production technology exhibits increasing elasticity of investment, that is, $\varepsilon'(k) > 0$. In this case, risk-taking increases with competition in the SME because of the negative bank-competitor effect—an increase in the number of banks lowers loan rates, and that leads to increased risk-taking. This relationship can also change with a sufficiently strong and positive bank-customer effect in the IME. With a sufficiently large bank-customer effect, increased competition in the IME increases loan rates and thereby reduces risk-taking. Overall, these results can be summarized as follows.

Proposition 4 *Any relationship between competition and risk-taking in the SME can be reversed by a positive and sufficiently strong bank-customer effect in the IME.*

Proposition 4 summarizes the impact of the bank-customer effect on risk-taking. As the general model here shows, the bank-customer effect exists even if bank customers (borrowers and depositors) are homogenous across all markets. As long as markets vary in terms of their composition of customers (ratio of borrowers to depositors), the bank-customer effect has the potential to alter the relationship between competition and risk-taking as more markets integrate.

4.2.5. Free entry

A final consideration is to allow free entry and endogenize the number of banks in the model. In doing so, we assume that there is a fixed setup cost, f_j, in market j. Set-up costs are market specific because they are largely determined by the local authorities and the cost of doing business within the market. Let $\pi(n_j,\, a_j,\, b_j)$ denote the maximized value of individual banks' expected profits in market j obtained from the maximization problem (13). The equilibrium number of banks in a free-entry Cournot equilibrium is derived from the *zero-profit* condition $\pi(n_j,\, a_j,\, b_j) = f_j$, and given by $n_j^* \equiv n(a_j,\, b_j,\, f_j)$. For a segmented market in our setting, the measures of entrepreneurs, a_j, and depositors, b_j are given. Therefore, the free-entry equilibrium number of banks, n_j^* is determined largely by the entry cost, f_j. Because expected equilibrium profits per bank decline in the number of banks, an increase fixed cost lowers the equilibrium number of banks in market j.[29]

As shown above, n, a, b, are all functions of m in the IME. Expected profits and consequently, the free-entry equilibrium number of banks, n^*, are no longer just a function of set-up costs, f, but also $a(m)$ and $b(m)$ (e.g. Corchón and Fradera, 2002; Ghosh Dastidar and Marjit, 2022). Market integration affects the equilibrium interest rate, r, directly through measures of the customer base, a and b, but also indirectly through the number of banks, $n^* = n(a,\, b,\, f)$. It follows that

$$\frac{dr^*}{dm} = \underbrace{\frac{\partial r}{\partial n} \cdot \frac{\partial n^*}{\partial f} \cdot f'(m)}_{\text{bank-competitor effect}} + \underbrace{\left(\frac{\partial r}{\partial n} \cdot \frac{\partial n^*}{\partial a} + \frac{\partial r}{\partial a}\right) a'(m) + \left(\frac{\partial r}{\partial n} \cdot \frac{\partial n^*}{\partial b} + \frac{\partial r}{\partial b}\right) b'(m)}_{\text{bank-customer effect}}$$

The above equation is similar to (22), with the additional assumption that set-up costs, f, are now a function of the number of integrating markets, $f(m)$. The sign of the first term, the bank-competitor

[29] From $\pi(n_j,\, a_j,\, b_j) = f_j$, we get $\frac{\partial \pi}{\partial n_j}.dn_j = df_j$ so that with $\frac{\partial \pi}{\partial n_j} < 0$, we get $\frac{dn_j}{df_j} = 1/\frac{\partial \pi}{\partial n_j} < 0$.

effect, depends on how the fixed cost of entry changes with m. We could assume that $f'(m) < 0$ because market integration can prompt different regions to competitively lower set-up costs in a bid to host banks locally. Under this assumption, the bank-competitor effect would be negative. The latter two terms of the equation above comprise the bank-customer effect. When the number of banks are exogenous, we know that $\partial r/\partial a < 0$ and $\partial r/\partial b > 0$. However, integration also affects n^* through a and b, or equivalently, the ratio of borrowers to depositors, ξ. Without additional assumptions, the sign of the overall effect is not unambiguous. Nevertheless, even when the number of banks are endogenous in the model, the role of the bank-customer effect in determining how competition from market integration affects the loan rate, and consequently, risk-shifting remains robust.

5. The linear model and welfare implications

In this section, we show that the results of the general model in Sections 3 and 4 can be obtained in a simple linear setting. In doing so, we extend our two-market example in Section 2 to the general case of m integrating markets. Lemma 6 (in Appendix B) proves the existence result for linear loan demand functions for the general model.[30] Below, we present alternative microfoundations for linear deposit supply and loan demand functions, which differs from the general model in that bank customers are heterogenous in terms of their reservation utilities (as in Martinez-Miera and Repullo, 2010). This linear model is adopted above in Section 2 and below in Section 6. The linear model demonstrates that the results of the general model are also applicable to the case where bank customers are heterogenous.

5.1. Linear model: microfoundations

There are m distinct segmented markets. The endowment of each (risk-neutral) depositor is normalized to \$1. Depositors are heterogeneous in their reservation utility v. Let $G_j(v)$ denote the measure of depositors in market j that have reservation utility less than or equal to v with $G_j'(v) > 0$ for all v. We assume that the reservation utilities of depositors are distributed uniformly on the support $[0,\, 1/a_j]$. A depositor would deposit \$1 only if $R \geq v$, where R is the deposit rate.[31] Therefore, the deposit supply in market j is given by $D_j(R) = G_j(R) = a_j R$. Markets are heterogeneous in their measure of depositors, so that $a_j \neq a_{j'}$ for any $j \neq j'$. When m distinct markets integrate, the supply of deposits is $D(R,\, a(m)) = a(m)R$ where $a(m) \equiv \sum_{j=1}^{m} a_j$ and the inverse supply function is

$$R(D/a(m)) = \frac{D}{a(m)}. \tag{23}$$

In loan market j, each entrepreneur requires an investment $k \in [0,\, 1]$ which yields θk when the project succeeds with probability $p(\theta) = 1 - \theta/\lambda$, and zero when it fails with probability $1 - p(\theta)$.

[30] While a linear deposit supply is obtained from a simple version of the maximization problem in (7), obtaining a linear loan demand function using the borrowers' maximization problem in (11) can be involved, requiring that we pin down the exact functional forms of $p(\theta)$, $y(\theta)$ and $f(k)$ in (9).

[31] For a deposit of \$1, the bank pays R with probability $p(\theta)$. Assuming that deposit insurer pays the reservation utility, v, with probability $1 - p(\theta)$ when the project fails, a depositor deposits only if $p(\theta)R + (1 - p(\theta))v \geq v$ which translates to $R \geq v$ because $p(\theta) > 0$.

Given loan rate r in market j, each entrepreneur solves

$$u(r) = \max_{\{\theta,\, k\}} \ (1 - \theta/\lambda)(\theta k - rk). \tag{24}$$

The optimal risk-shifting is given by $\theta(r) = \frac{1}{2}(\lambda + r)$, which, unlike the expression for θ obtained in (10), does not depend on the level of investment, k. We obtain $k(r) = 1$ because the production function is linear in k. Also, the relation between loan rates and risk-taking is always positive as $\theta'(r) = 1/2$.

Let $H_j(u)$ denote the measure of entrepreneurs in market j that have reservation utility less than or equal to u with $H_j'(u) > 0$ for all u. An entrepreneur would participate in the loan market only if $u(r) \geq u$. Therefore, loan demand in market j is given by $L_j(r) = H_j(u(r))$. We assume $H_j(u) = 2b_j\sqrt{\lambda u}$ defined on the support $[0,\, 1/4\lambda b_j^2]$, so that $L_j(r) = H_j(u(r)) = b_j(\lambda - r)$.

Markets are heterogeneous in their measure of borrowers, so that $b_j \neq b_{j'}$ for any $j \neq j'$. When m distinct markets integrate, the demand for loans is $L(r,\, b(m)) = b(m)(\lambda - r)$ where $b(m) \equiv \sum_{j=1}^{m} b_j$ and the inverse demand function is

$$r(L/b(m)) = \lambda - \frac{L}{b(m)}. \tag{25}$$

Optimal risk-shifting equals $\theta(r(L/b(m))) = \lambda - L/2b(m)$ so that the probability of success is given by $P(L/b(m)) \equiv 1 - \theta(L/b(m))/\lambda = L/2\lambda b(m)$.

5.2. Competition and risk-taking in the linear model

In our simple linear setting, the association between loan rates and risk-taking is positive. Given that the effects are analogous, we omit the discussion on loan rates and focus exclusively on equilibrium risk-taking for the sake of brevity. We assume that $a(m) = m^{\alpha}$ and $b(m) = m^{\delta}$ with $\alpha,\, \delta \in [0,\, 1]$.[32] It follows that $\xi(m) \equiv b(m)/a(m) = m^{\delta-\alpha}$ and $\hat{\xi}(m) \equiv \xi'(m)/\xi(m) = (\delta - \alpha)/m$. Therefore, the bank-customer effect is negative (positive) according as $\delta - \alpha < (>)\, 0$. We also assume that $n_j = j$, so that $n(m) = \sum_j j = \frac{1}{2}m(m+1)$. Using (20) and $\theta(L/b(m)) = \lambda - L/2b(m)$, the symmetric equilibrium risk-shifting in the IME is obtained as

$$\theta^*(m) = \lambda \left\{ 1 - \frac{1}{2} \cdot \frac{m(m+1)+2}{m(m+1)+4} \cdot \frac{1}{1 + m^{\delta-\alpha}} \right\}. \tag{26}$$

Proposition 5 *In the IME, where risk-shifting is given by (26), the relation between competition and risk-shifting is described as follows:*

(i) *If $\delta - \alpha \leq 0$, the bank-customer effect is negative and risk-shifting is monotonically decreasing in m;*

(ii) *There is a unique $\tilde{\gamma} \in (0,\, 1)$ such that if $\delta - \alpha > \tilde{\gamma}$, the bank-customer effect is positive and sufficiently strong so that risk-shifting is monotonically increasing in m. However, if $0 < \delta - \alpha \leq$*

[32] When $a_j = b_j = 1/m$ for all j, it follows that $\alpha = \delta = 0$. On the other hand, when $a_j = b_j = 1$ for all j, $\alpha = \delta = 1$. For any other distribution of $\{a_j\}_{j=1}^m$ and $\{b_j\}_{j=1}^m$, we have $\alpha,\, \delta \in (0,\, 1)$.

$\tilde{\gamma}$, the bank-customer effect is positive but not sufficiently strong, and risk-shifting is U-shaped with respect to m.

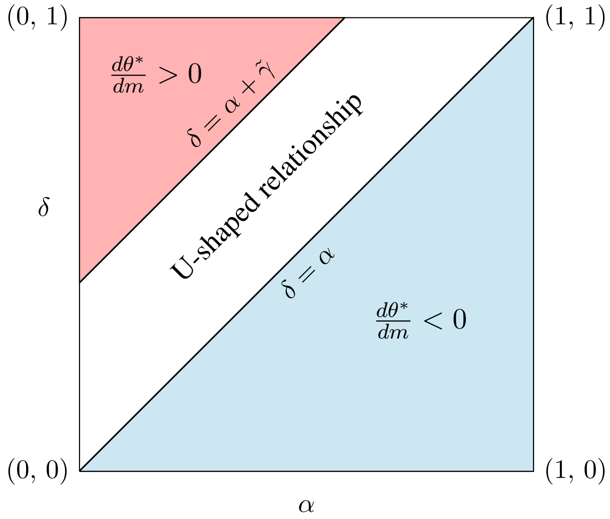

Figure 2: *Equilibrium association between risk-shifting and the number of integrated markets when* $n(m) = \frac{1}{2}m(m+1)$, $a(m) = m^{\alpha}$ *and* $b(m) = m^{\delta}$.

Figure 2 describes the effect of increased competition on risk-shifting for the IME in the α-δ space. When $\delta \leq \alpha$, the bank-customer effect is negative and sufficient to yield a negative relation between competition and risk-shifting (blue region in Figure 2). In contrast, a positive bank-customer effect ($\delta > \alpha$) does not necessarily yield a positive relation between competition and risk-shifting. For the relation to be strictly positive, we require that the positive bank-customer effect be sufficiently strong, that is, $\delta - \alpha > \tilde{\gamma}$ (pink region in Figure 2). However, if the bank-customer effect is positive and not sufficiently strong (i.e., $\delta - \alpha \leq \tilde{\gamma}$), we obtain an interesting relation between competition and risk-taking that is non-monotonic (white region in Figure 2). In this example, the negative bank-competitor effect dominates at low m, while the positive bank-customer effect dominates at higher m yielding the non-monotonic (U-shaped) relationship.[33] The above example illustrates the variety of risk implications under market integration—features that cannot be captured by focusing exclusively on the traditional within-market analyses of competition.

5.3. Welfare implications of market integration

The welfare implications of market integration are illustrated using the two-market example in Section 2. We consider a situation wherein a planner (equivalently, a regulator or antitrust authority) decides whether to integrate the two markets or keep them segmented. In its decision to integrate the two economies, we assume that the planner simply compares welfare under market integration with aggregate welfare when markets are segmented. Importantly, the planner makes no other decision such as

[33] As discussed above, a sufficiently strong bank-customer effect in (22) is equivalent to $\hat{\xi}(m) \geq (-Z_0(m)/Z_1(m))\hat{n}(m)$ whenever $\hat{\xi}(m) > 0$. However, it is difficult to pin down the behavior of $-Z_0(m)/Z_1(m)$ with respect to changes in m without knowing the functional forms of $n(m)$, $a(m)$, and $b(m)$. In this linear example, the functional forms yield that $-Z_0(m)/Z_1(m)$ decreases with m and has a finite upper bound (see the proof of Proposition 5 in Appendix B). Therefore, while the positive bank-customer effect dominates when m is large, the negative bank-competitor effect dominates when m is small.

setting rates or taxes to influence the optimization behavior of market participants.

We begin by deriving the expressions for aggregate welfare in any market j. Following Section 2, we assume $p(\theta) = 1 - \theta/\lambda$ so that the inverse deposit supply and loan demand functions in any segmented market j are given by (2). At any given deposit rate $R_j \equiv R(D_j/a_j)$, loan rate $r_j \equiv r(L_j/b_j)$, and depositor reservation utility v, the expected gross depositor surplus is

$$\begin{aligned} GDS_j(L_j) \equiv & P(L_j/b_j)L_jR(L_j/a_j) + (1 - P(L_j/b_j)) \int_0^{R(L_j/a_j)} vG_j'(v)dv \\ & - \int_0^{R(L_j/a_j)} vG_j'(v)dv \end{aligned} \tag{27}$$

where $P(L_j/b_j) \equiv p(\tilde{\theta}(r(L_j/b_j)))$ and, in any SME, we must have $L_j = D_j$. The second term in (27) is covered by deposit insurance. As a result, deposit surplus net of insurance is given by[34]

$$DS_j(L_j) = P(L_j/b_j)L_jR(L_j/a_j) - \int_0^{R(L_j/b_j)} vG_j'(v)dv. \tag{28}$$

At any given loan rate $r_j \equiv r(L_j/b_j)$ and borrower reservation utility u, the borrower surplus is

$$BS_j(L_j) = L_ju(r(L_j/b_j)) - \int_0^{u(r(L_j/b_j))} uH_j'(u)du. \tag{29}$$

Bank surplus, $\Pi_j(L_j)$, is the sum of profits of individual banks in the symmetric equilibrium

$$\Pi(L_j) = P(L_j/b_j)(r(L_j/b_j) - R(L_j/a_j))L_j. \tag{30}$$

The aggregate welfare in market j is the sum of (28), (29) and (30):

$$\begin{aligned} W_j(L_j) =& P(L_j/b_j)L_jr(L_j/b_j) - \int_0^{R(L_j/a_j)} vG_j'(v)dv + L_ju(r(L_j/b_j)) \\ & - \int_0^{u(r(L_j/b_j))} uH_j'(u)du. \end{aligned} \tag{31}$$

Therefore, all market participants optimize as described in the model above and the planner decides to integrate the two markets if

$$\Delta W \equiv W(L^*) - [W_1(L_1) + W_2(L_2)] > 0,$$

where L^* denotes aggregate loans in the IME and L_j denotes aggregate loans in the SME for market $j = 1, 2$, respectively. We use the distribution functions of the reservation utilities, v and u, to obtain

$$\int_0^{R(L_j/a_j)} vG_j'(v)dv = \frac{L_j^2}{2a_j} \quad \text{and} \quad L_ju(r(L_j/b_j)) - \int_0^{u(r(L_j/b_j))} uH_j'(u)du = \frac{L_j^3}{6\lambda b_j^2}.$$

[34]Following Martinez-Miera and Repullo (2010), we use depositor surplus net of deposit insurance in our calculation of aggregate welfare.

Using $P(L_j/b_j) = L_j/2\lambda b_j$ in (31) gives aggregate welfare in the linear model as

$$W_j(L_j) = \frac{L_j^2}{2\lambda b_j}\left(\lambda - \frac{L_j}{b_j}\right) - \frac{L_j^2}{2a_j} + \frac{L_j^3}{6\lambda b_j^2} = \frac{(a_j - b_j)L_j^2}{2a_j b_j} - \frac{L_j^3}{3\lambda b_j^2}\,. \tag{32}$$

A necessary condition for $W_j(L_j)$ to be positive is that $a_j > b_j$.

Next, we analyze the integration of two economies, $j = 1,\, 2$, that are otherwise identical but vary in the size of their loan markets. In particular, let $a_1 = a_2 = a > 0$, $b_1 = b > 0$, $b_2 = \beta b$ with $\beta > 0$ and $n_1 = n_2 = n \geq 1$. The ratio of borrowers to depositors in the segmented and integrated markets are given by $\xi_1 = b/a$, $\xi_2 = \beta b/a$ and $\xi^* = (1+\beta)b/2a$, respectively. Recall that, with respect to any market $j = 1,\, 2$, $\xi^* - \xi_j > (<)\, 0$ implies that the bank-customer effect is positive (negative) with respect to this market. Note that $\xi^* - \xi_1 = \frac{1}{2}(\beta - 1)\xi_1$ and $\xi^* - \xi_2 = -\frac{1}{2}(\beta - 1)\xi_1$. Therefore, the bank-customer effect with respect to market 1 is positive (negative) if $\beta > (<)\, 1$. In contrast, it is negative (positive) with respect to market 2 if $\beta > (<)\, 1$.

We use a numerical example to illustrate how the change in aggregate welfare from market integration, ΔW, varies with model parameters. Figure 3 plots ΔW as a function of β for a given set of parameters λ, a, b, and n in the linear model. We set $\lambda = 1.5$, $a = 4$, $b = 1.4$, $n = 2$, and vary β on $[0,\, 4]$. In addition to ΔW, Figure 3 also plots $\Delta\theta_j \equiv \theta^* - \theta_j$ where $\Delta\theta_j$ measures change in risk-taking due to market integration. In particular, $\Delta\theta_j$ is the difference in risk-taking between the IME and the SME for individual market $j = 1,\, 2$.

For sufficiently low β, that is $\beta < \beta_2$, the positive bank-customer effect with respect to market 2 is sufficiently strong to outweigh the negative bank-competitor effect, so that loan rate and risk-taking are higher in the IME compared to their SME values in market 2 so that $\Delta\theta_2 > 0$ as shown by the orange line in Figure 3. Also, for $\beta < \beta_2$, the negative bank-customer effect with respect to market 1 reinforces the bank-competitor effect and $\Delta\theta_1 < 0$ as shown by the red line. The situation is reversed at sufficiently high values of β. With $\beta > \beta_1$, the positive bank-customer effect with respect to market 1 is sufficiently strong to outweigh the negative bank-competitor effect, so that loan rate and risk-taking are higher in the IME compared to their SME values in market 1 so that $\Delta\theta_1 > 0$. Again, for $\beta > \beta_1$, the negative bank-customer effect with respect to market 2 reinforces the bank-competitor effect and we get $\Delta\theta_2 < 0$. Lastly, for all intermediate values of β, where $\beta \in (\beta_2,\, \beta_1)$, loan rate and risk-taking in the IME are lower compared to their SME values in both markets 1 and 2.

The change in aggregate welfare, ΔW, is shown by the blue line in Figure 3. For $\beta < \beta^*$, market integration increases aggregate welfare ($\Delta W > 0$) irrespective of whether loan rates and risk-taking are increasing or decreasing. In particular, we find that $\Delta W > 0$ even if risk-taking is increasing for market 1 (when $\beta_1 < \beta < \beta^*$) or for market 2 (when $\beta < \beta_2$). We find that $\Delta W < 0$ only when β is substantially high, that is $\beta > \beta^*$, where declines in aggregate welfare are accompanied by a substantial increase in loan rates and risk-taking with respect to market 1.

The numerical example here yields results that are important in the context of the large empirical literature on competition and risk-taking. First, the effect of competition on risk-taking are not necessarily universal across all integrating markets. Increased risk-taking with respect to one market can be accompanied with reduced risk-taking with respect to another, as shown in Figure 3. Moreover, as the example shows, increased competition under integration can also lower risk-taking in both markets.

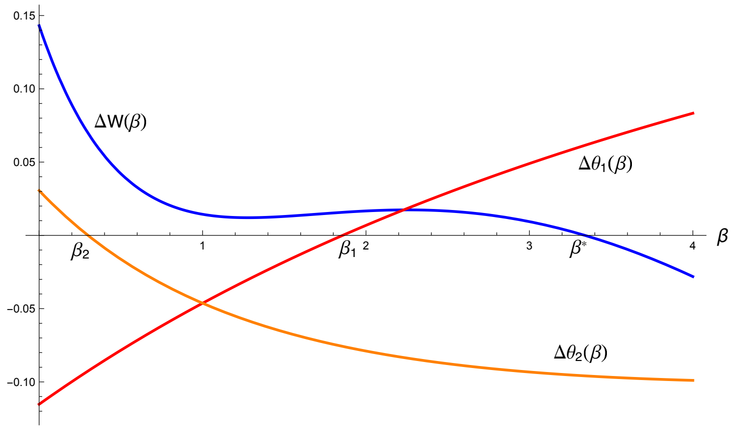

Figure 3: *Welfare comparison between the SME and the IME.*

Second, market integration can increase both risk-taking and aggregate welfare. Restricting attention to only risk-taking implications of competition often ignores the possibility that aggregate welfare can increase with risk-taking. Lastly, market integration can be also decrease aggregate welfare as shown for the case with substantial increases in loan rates and risk-taking for some of the integrating market(s) in the example above.

Before concluding this section, it is important to mention that we have ignored social costs and other negative externalities associated with bank failure and liquidation in our welfare analysis. However, the results are qualitatively similar if we assume a fixed social cost, $C > 0$, so that the expected social loss associated with bank failure in market j is $(1 - P(L_j/b_j))C$.

6. Extensions

We present three extensions of our baseline model. For the IME analyzed above, we have assumed that (1) there is no exit and entry of banks, (2) loan default risks are perfectly correlated, and (3) banks do not transact in the interbank market.[35] In three extensions to the linear model below, we relax each of these assumptions, individually. In addition to the traditional bank-competitor effect, we show that the bank-customer effect of market integration remains an important factor in determining loan rates and risk-shifting under each of the different modeling choices. In so doing, we also point to other factors that affect loan rates and risk-taking when markets integrate.

6.1. Bank mergers

This extension presents a model wherein market integration creates incentives for bank mergers. Empirical evidence on consolidation in the banking industry motivates our choice of bank exit. During 1984-1993, the number of bank mergers and acquisitions in the United States exceeded the number of failures four-fold "even when acquisitions of insolvent banks are counted only as failures" (Wheelock and Wilson, 2000, p. 127). Post-1993, this trend has only grown stronger as described in numerous studies (see, for example Figure 31-3 in DeYoung, 2019, and references therein). We demonstrate the

[35] Fecht, Inderst, and Pfeil (2017) treat interbank lending and bank mergers as alternatives in the process of market integration. However, we model them separately.

importance of the results from the baseline model in a setting where merger activity determines bank exits.

6.1.1. Heterogenous bank operating costs and the SME

We start by introducing market-specific bank operating costs in the two-market linear model in Section 2 so that operating costs of bank i in market j are

$$C_{ij}(D_{ij}) = c_j D_{ij},$$

where $c_j \geq 0$ is the constant marginal operating cost of funds.[36] While all banks operating within market j have identical (marginal) operating cost of funds, we assume that markets are heterogeneous in terms of bank operating costs, so that $c_j \neq c_{j'}$ for any $j \neq j'$. We denote market-j banks with cost of operations c_j as a type-j banks. The assumption of heterogeneity in operating costs (non-interest expenses) captures differences in efficiencies, with more efficient banks having lower operating costs. Differences in efficiency have been attributed to the lack of competition due to restrictions on geographic expansion that provide banks with local market power. Kroszner (2001) argues that, prior to the relaxation of branching restrictions in the United States, geographic variation in banks' cost efficiencies could be linked to variations in the degree of protection across the different banking jurisdictions (cf. footnote 8).

We assume that banks in market 1 are more efficient than those in market 2, so that $c_1 = 0 < c = c_2$. Just as in Section 2, the inverse deposit supply and loan demand functions in any segmented market j are given by (2). Also, $a_1 = a_2 = a > 0$ and $n_1 = n_2 = n \geq 2$, but the loan demand in market 2 is larger than that in market 1, so that $b_1 = b > 0$ and $b_2 = \beta b$ with $\beta > 1$. Therefore, the ratio of borrowers to depositors in the two markets are given by $\xi_1 \equiv b/a$ and $\xi_2(\beta) \equiv \beta b/a$.

Lemma 3 *Risk-shifting in markets 1 and 2 in the SME are given by*

$$\theta_1 = \lambda - \frac{\lambda}{2} \cdot \frac{n+1}{n+2} \cdot \frac{a}{a+b} = \lambda - \frac{\lambda}{2} \cdot \frac{n+1}{n+2} \cdot \frac{1}{1+\xi_1},$$
$$\theta_2 = \lambda - \frac{\lambda - c}{2} \cdot \frac{n+1}{n+2} \cdot \frac{a}{a+\beta b} = \lambda - \frac{\lambda - c}{2} \cdot \frac{n+1}{n+2} \cdot \frac{1}{1+\xi_2(\beta)},$$

respectively. The SME risk-taking is lower in market 1, that is, $\theta_1 < \theta_2$.

All else equal, higher loan demand in market 2 yields a higher loan rate and more risk-taking in market 2, so that the SME loan rate in market 2 increases with β. At the same time, greater inefficiency in market 2 raises marginal operating costs, and consequently, loan rates for banks in market 2, so that the SME loan rate and risk-taking in market 2 increases with c. By construction, both effects point in the same direction and the extent to which loan rates and risk-taking are higher in market 2 depends on both β and c.[37]

[36] Costs are typically a function of both deposits and loans in the Monti-Klein model (Klein, 1971). Given the assumption that all loans are deposit -financed (there is no equity-financing and no interbank market), it is innocuous to assume that costs depend only on the volume of deposits.

[37] This simplification is easily modified if we assume that either (1) banks in market 2 are the low-cost banks or (2) the loan market is smaller in region 2, $\beta \in (0, 1)$. In (2), loan rate and risk-taking in market 2 can be higher or lower than that in market 1 depending on the relative strength of the two opposing forces (i.e., region 1 has a larger loan market, but banks in market 2 are more inefficient). While this alternative complicates the analytical solution, it does not qualitatively alter the

6.1.2. *Heterogenous bank operating costs in the integrated market*

In Section 5.3, we derived the conditions under which market integration increases welfare in the linear model. In this section, we assume that these conditions have been met and that market integration is welfare-enhancing. Given that market integration has been approved, a planner or antitrust authority is now faced with the decision of whether to allow bank mergers. Mergers occur if the planner permits banks to merge and banks find it profitable to merge. On the other hand, if mergers are disallowed or unprofitable, both high- and low-cost banks compete in a Cournot fashion in the integrated market. We assume that there is no entry of banks but that bank exits can occur through mergers and acquisitions.

The timeline is as follows. At $t = 0$, the planner decides whether to allow mergers in the integrated market. To simplify, we assume that the planner allows only pairwise mergers—that is mergers between any two banks are allowed, but mergers of more than two banks are not. With pairwise mergers and $2n$ banks in the integrated market, we can have a maximum of n mergers. At $t = 1$, each pair of banks decide whether to merger or not. We solve the game by backward induction. Note that, in the integrated market, each bank faces the following deposit supply and loan demand schedules

$$R(D/(a_1 + a_2)) = \frac{D}{a_1 + a_2} \equiv \frac{D}{2a} \quad \text{where } D = D_1 + D_2, \tag{33}$$

$$r(L/(b_1 + b_2)) = \lambda - \frac{L}{b_1 + b_2} \equiv \lambda - \frac{L}{(1+\beta)b} \quad \text{where } L = L_1 + L_2. \tag{34}$$

The ratio of borrowers to depositors in the integrated market is given by $\xi^* = (1 + \beta)b/2a \in (\xi_1, \xi_2)$. Given that we have n banks of each type, we may have horizontal mergers between like banks (merger between any two high-cost or any two low-cost banks) or market-extension mergers between unlike banks (merger between a high- and a low-cost bank). We assume that only market-extension mergers yield efficiency gains and the merged entity operates as a low-cost bank. In contrast, there are no efficiency gains from horizontal mergers and the merged entity operates with the same cost as its predecessors.[38]

Using a general model of Cournot competition, Lemma 7 (in Appendix B) shows that pairwise mergers between like banks are never profitable. This general result allows us to focus the analysis below on mergers between unlike banks. We analyze two types of subgame perfect equilibria described below as the "no-merger IME" and the "merger IME".

The no-merger IME. A no-merger IME can prevail under two scenarios—either the planner does not permit mergers or banks do not find mergers profitable even if they are permitted. In both cases, the integrated market has n low-cost and n high-cost banks that compete in a Cournot fashion, each facing deposit supply and loan demand schedules given in (33) and (34), respectively. In equilibrium, we have

results in this extension.

[38] Although not explicitly modeled, the assumption that market-extension mergers yield greater efficiency gains than horizontal mergers is grounded in theories of asymmetric information. Synergies from market-extension mergers of dissimilar banks comes from the local knowledge that each bank possesses about its home market, and therefore, is likely greater in the integrated market than that for horizontal mergers. Moreover, the assumption of zero efficiency gains is a normalization of the synergies from horizontal mergers of similar banks relative to market-extension mergers of dissimilar banks. The assumption also captures the notion that consolidation and competition prior to integration has exhausted all within-market efficiency gains.

$L = D$, and each type-j bank i solves

$$\pi_j^*(n, n) = \max_{L_{ij}} P(L/(b_1 + b_2)) \left(r(L/(b_1 + b_2)) - R(L/(a_1 + a_2)) - c_j\right) L_{ij},$$

where $\pi_j(n_l, n_h)$ denotes the expected profit of each type-j bank when the market has n_l low-cost and n_h high-cost banks, L_{ij} denotes the amount lent by a bank i of type j, and the probability of success is given by $P(L/(b_1 + b_2)) = L/2\lambda(1 + \beta)b$. The result of the maximization problem is summarized in the following lemma.

Lemma 4 *If $c < \bar{c} \equiv \frac{\lambda}{n+2}$, all banks make strictly positive profits by lending. The risk-shifting in the no-merger IME is given by*

$$\theta^* = \lambda - \frac{a\left\{(3n+2)(2\lambda - c) + \sqrt{c^2(3n+2)^2 + 4n^2\lambda(\lambda - c)}\right\}}{8(2a + (1+\beta)b)(n+1)}. \tag{35}$$

If $c \geq \bar{c}$, the difference in costs between the two bank types is sufficiently large so that high-cost banks cannot break even in the no-merger IME. As a result, the no-merger IME comprises only low-cost banks. In what follows, we focus on the interesting case with $c < \bar{c}$.

The expression in (35) reveals that equilibrium loan rates and risk-taking in the no-merger IME depend on three factors. These include the bank-competitor effect, the bank-customer effect, and the effect of efficiency gains which are captured by the effects on the loan rate and risk-shifting of n, β, and c, respectively. Among the three, the effect of efficiency gains is the additional source of heterogeneity imposed on the baseline model. Loan rates and risk-taking in the no-merger IME depend on the relative strength of each of the three factors mentioned above. The following proposition compares the risk-shifting levels under the SME and the IME.

Proposition 6 *Let θ^* be the risk-shifting under the no-merger IME and θ_j be the risk-shifting in the SME in market j, $j = 1, 2$. Assume $\beta > 1$ and $0 \leq c < \bar{c}$. The SME risk-taking in market 2 is always greater than that in the IME, that is, $\theta^* < \theta_2$. On the other hand, there is a unique strictly decreasing function, $\tilde{\beta}(c)$ with $\lim_{c \to \bar{c}} \tilde{\beta}(c) = 1$ so that $\theta^* > (<)\, \theta_1$ according as $\beta > (<)\, \tilde{\beta}(c)$.*

Figure 4 illustrates the result in Proposition 6. First, the negative bank-competitor effect from an increase in the number of banks in the no-merger IME tends to lower rates. Second, with $\beta > 1$, the bank-customer effect is positive (negative) with respect to the low-cost market or market 1 (high-cost market or market 2). Apart from the bank-competitor and bank-customer effects, there is a third effect that stems from differences in the operating costs of banks in the no-merger IME.

For high-cost banks, the bank-competitor and bank-customer effects are both negative, and this tends to lower loan rates and risk-taking in the integrated market. Additionally, competing with low-cost banks also drives high-cost banks to lower loan rates and risk-taking. Therefore, in comparison to the SME in market 2, high-cost banks take lower risk in the no merger IME, that is, $\theta^* < \theta_2$.

For low-cost banks, the bank-competitor effect is negative but the bank-customer effect is positive. As shown in Figure 4, there is a threshold value $\tilde{\beta}(c)$ of β, exceeding which the bank-customer effect is sufficiently strong to outweigh the negative bank-competitor effect. With $c = 0$, the no-merger IME is

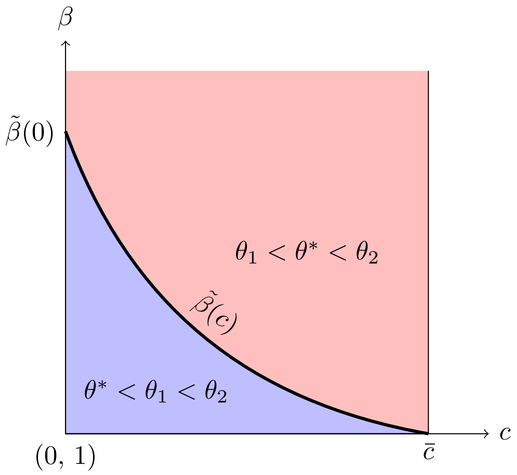

Figure 4: *Comparison of risk levels in the SME, and the no-merger IME.*

the same as the IME with homogenous banks as described in the linear model above. In this case, the differences in risk levels in the IME and SME in market 1 are explained fully by the bank-competitor and bank-customer effects. With $c = 0$ and $\beta > \tilde{\beta}(0)$, it follows that $\theta^* > \theta_1$ as shown in Figure 4.

For $c \in (0, \bar{c})$, the threshold value of β is declining in c according to the downward-sloping function, $\tilde{\beta}(c)$, as seen in Figure 4. Low-cost banks from market 1 can lend at higher rates in the no-merger IME when the cost advantage over their high-cost rivals in the integrated market is large (high c) or when the the bank-customer effect is sufficiently strong (high β). The downward-sloping threshold reflects a tradeoff—equilibrium loan rates in no-merger IME are higher than SME values in market 1 if the bank-customer effect is sufficiently strong at low c, or alternatively, cost differences are sufficiently large at low β. For borrowers in market 1, transitioning to higher loan rates in the no-merger IME compared to the SME implies higher risk-shifting. Moreover, we find that $\lim_{c \to \bar{c}} \tilde{\beta}(c) = 1$, that is, when the differences in efficiency gains are maximum, the tendency for low-cost banks to increase loan rates is exactly offset by the downward pressure on loan rates from the negative bank-competitor effect. As a result, a positive bank-customer effect ($\beta > 1$) is sufficient for low-cost banks to raise loan rates in the no-merger IME.

The merger IME. For an IME with bank mergers, Lemma 7 (in Appendix B) allows us to focus only on mergers between unlike banks. We also assume that the merged entity captures the entire efficiency gains from the merger and, in terms of the linear model, operates with zero costs. Finally, because the integrated market has n banks of each type, symmetry ensures that a merger between any pair of low-cost and high-cost bank allows for the merger between all n pairs. Below, we show that in an IME with bank mergers, n pairwise mergers between each high-cost and each low-cost bank are profitable.

Mergers in the IME will yield n low-cost banks in the post-merger integrated market. Each bank faces deposit supply and loan demand schedules given by (33) and (34) and solves

$$\pi_M(n) = \max_{L_i} P(L/(b_1 + b_2)) \left(r(L/(b_1 + b_2)) - R(L/(a_1 + a_2))\right) L_i.$$

The solution to this maximization problem yields the following result.

Lemma 5 *When the planner allows pairwise bank mergers, we obtain*

$$\pi_M(n) > \pi_1^*(n,\, n) + \pi_2^*(n,\, n) \quad \textit{for all } c < \bar{c}. \tag{36}$$

As a result, all pairwise mergers between low-cost banks and high-cost banks are profitable. The equilibrium risk-shifting is

$$\theta_M = \lambda - \frac{\lambda}{2} \cdot \frac{n+1}{n+2} \cdot \frac{2a}{2a + (1+\beta)b} = \lambda - \frac{\lambda}{2} \cdot \frac{n+1}{n+2} \cdot \frac{1}{1+\xi^*}.$$

Inequality (36) shows that the sum of the profits of a low-cost bank and a high-cost bank in the no-merger IME is less than the profits of the merged entity formed by the two banks in the merger IME. Importantly, mergers are profitable under the same regularity conditions, $c < \bar{c}$, as that for the no-merger IME (Lemma 4). As a result, mergers of unlike banks are always profitable in the integrated market, and such mergers do not occur only if the planner disallows them. Risk-taking in the merger IME is summarized in the following proposition.

Proposition 7 *We obtain $\theta_1 < \theta_M < \theta_2$ for all $c < \bar{c}$, where θ_M and θ_j are the risk-shifting in the merger IME and the SME in market j, $j = 1,\, 2$, respectively.*

We provide the intuition behind equilibrium loan rates in the merger IME, following which, the implications for risk-shifting are straightforward. First, consider the bank-competitor effect. Because the number of banks in the merger IME, n, is the same as the number of banks in each segmented market, the bank-competitor effect with respect to both markets is zero. Below we show that this stylized model can accommodate situations where each segmented market has a different number of banks. Next, the bank-customer effect with respect to market 1 (market 2) is positive (negative) given that $\beta > 1$. Lastly, mergers generate efficiency gains that accrue to the (low-cost) merged entities, which tends to lower loan rates relative to the SME rates charged by their high-cost predecessors in market 2. However, there is no such effect relative to SME loan rates charged in market 1 because mergers do not alter the cost structure of low-cost banks. Given that the bank-competitor effect is zero in our setup (by construction), it allows us to focus on the bank-customer effect and efficiency gains. The negative bank-customer effect together with efficiency gains help lower the merger IME loan rate from their SME values in market 2. Consequently, risk-shifting is also lower so that $\theta_M < \theta_2$. On the other hand, banks were already operating at low cost in market 1 so that the difference between loan rates in the merger IME and SME in market 1 stems entirely from the positive bank-customer effect. It follows that $\theta_1 \leq \theta_M$, that is, risk-taking increases with respect to market 1.

The above analysis emphasizes the importance of the bank-customer effect even after allowing for bank exits through mergers. Proposition 7 shows that loan rates and risk-shifting are lower compared to their SME values in the high-cost market (market 2) even when the number of competitor banks has not increased in the IME. This demonstrates that competition under market integration can yield pro-competitive outcomes without a concomitant increase in the number of banks. Such pro-competitive outcomes can originate from the bank-customer effect that alters price-taking behavior or from efficiency gains from bank mergers or both.

Alternative specifications and the merger IME. While we have assumed that the merged entity enjoys full efficiency gains (i.e., has the same (zero) operating costs of low-cost banks), the model can also accommodate partial efficiency gains where the merged entities' operating costs lie in between the pre-merger operating costs of low-cost and high-cost banks, that is, $c' \in (0, c)$. In this case, efficiency gains tend to lower loan rates relative to SME loan rates in market 2 but also tend to increase loan rates relative to their SME values in market 1. In this way, it reinforces the bank-customer effect discussed above, and the results of this section are robust to partial efficiency gains.

Also, the model results remain qualitative similar if we assume that segmented markets do not have the same number of banks, that is, $n_1 \neq n_2$. With $n_1 > n_2$, there are n_1 low-cost banks in the merger IME, so that borrowers from market 2 have $n_1 - n_2$ more banks to borrow from. The negative bank-competitor effect serves to reinforce the decline in loan rates and risk-taking described in Proposition 7. With $n_1 < n_2$, however, there are n_1 low-cost and $n_2 - n_1$ high-cost banks in the merger IME. As a result, the merger IME resembles the no-merger IME with both high-cost and low-cost banks competing in a Cournot fashion.[39] In particular, if we consider the special case where $n_1 = n$ and $n_2 = 2n$, there will be n high-cost and n low-cost banks in the merger IME and the market structure is equivalent to the no-merger IME analyzed above.

6.1.3. *Welfare analysis and policy implications*

The planner permits mergers if the the expected aggregate welfare in the merger IME exceeds that in the no-merger IME. Let $W^*(c)$ and W_M denote aggregate welfare in the no-merger and merger IME, respectively. Using the expressions for aggregate welfare in the (32), we obtain

$$W^*(c) = \frac{(2a-(1+\beta)b)(L^*)^2}{4a(1+\beta)b} - \frac{(L^*)^3}{3\lambda(1+\beta)^2b^2} - \underbrace{\frac{cL^*L_2^*}{2\lambda(1+\beta)b}}_{P(L^*/(b_1+b_2))cL_2^*},$$

$$W_M = \frac{(2a-(1+\beta)b)(L_M)^2}{4a(1+\beta)b} - \frac{(L_M)^3}{3\lambda(1+\beta)^2b^2},$$

where L_j^* denotes the loans of type-j banks in the no-merger IME with aggregate loans $L^* = L_1^* + L_2^*$, $j = 1, 2$, and L_M denotes aggregate loans in the merger IME. Mergers are permitted if $\Delta W(c) \equiv W^*(c) - W_M < 0$. Using Example 1 below, we show that allowing mergers is the optimal choice for the planner because aggregate welfare in the merger IME is greater than that in the non-merger IME when efficiency gains, c, are large.[40]

Example 1 Set $\lambda = 1.5$, $a = 4$, $b = 1$, $\beta = 1.2$ and $n = 3$ so that $\bar{c} = \lambda/(n+2) = 0.3$. Figure 5 depicts the function $\Delta W(c)$ for $c \in (0, 0.3)$. The function, $\Delta W(c)$, intersects the horizontal axis at $c^* \approx 0.035$. Welfare gains from mergers are low for small differences in cost between high- and low-cost banks, $c \leq c^*$. However, if $c > c^*$, then efficiency gains are sufficiently large so that aggregate welfare in the merger IME exceeds that in the no-merger IME. ■

[39] Any equilibrium where low-cost and high-cost banks compete creates further incentives for pairwise mergers between unlike banks until all possible efficiency gains are exhausted. In this setting, that resembles the merger IME described above.

[40] The sketch of an analytical proof is as follows. It is easy to show that $\Delta W(\bar{c}) = 0$, and $\lim_{c\to\bar{c}} d\Delta W(c)/dc > 0$. Because $\Delta W(c)$ is continuous on $[0, c]$, it must be the case that $\Delta W(c) < 0$ for large values of c (close to $\bar{c}$).

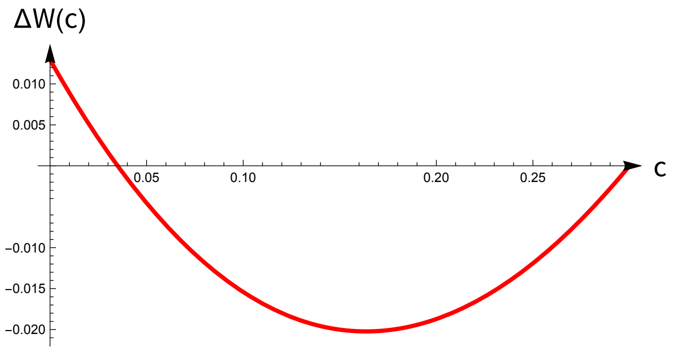

Figure 5: *Welfare effects of bank mergers.*

Example 1 conforms to the welfare analysis of mergers with efficiency gains in oligopoly markets (Williamson, 1968). Mergers increase bank market power and this tends to raise loan rates and lower deposit rates (by lowering loan volumes), thereby reducing expected customer (borrower and depositor) surplus. On the other hand, the opposite is true for efficiency gains from mergers because they tend to lower loan rates and raise deposit rates, thereby increasing expected bank surplus. As a result, net increase in aggregate welfare from mergers depends on the relative magnitudes of these two opposing forces. In sum, mergers are welfare enhancing (i.e., $\Delta W(c) < 0$) if efficiency gains are sufficiently large (i.e., $c > c^*$).

To summarize, at $t = 0$, the planner either allows or disallows bank mergers. This decision is made as described in Example 1. For $c \leq c^*$, the planner disallows bank mergers at $t = 0$ and we obtain the no merger IME described in Lemma 4 and Proposition 4. On the other hand, if $c > c^*$, the planner permits bank mergers at $t = 0$ and we obtain the merger IME described in Lemma 5 and Proposition 7. In our setting, there is no equilibrium wherein the planner permits mergers but banks do not find mergers profitable. Moreover, loan rates and risk-taking are higher in the merger IME because of the negative bank-competitor effect and the fact that there are fewer banks in the merger IME. The result is summarized in terms of the following proposition.

Proposition 8 *In the subgame perfect equilibrium of the merger game, a merger between a pair of high-cost bank and a low-cost bank occurs whenever efficiency gains are large (i.e., $c > c^*$). Moreover, $\theta_M > \theta^*$, that is, risk-taking in the merger IME is greater than that in the no-merger IME.*

This result has important implications. First, even when they are privately optimal (as described in Lemma 5), bank mergers may not be socially optimal. In essence, the model presents a rationale for merger reviews to determine that mergers do not reduce competition substantially. Second, higher aggregate welfare in the merger IME relative to the no-merger IME is also accompanied by higher risk-taking. This has financial stability implications. The planner can lower risk-taking by disallowing mergers. However, this comes at the cost of reducing aggregate welfare. With the inclusion of the *financial stability factor* in merger reviews under the Dodd Frank Act of 2010, welfare considerations are no longer the sole factor in approving bank mergers. Our framework presents scenarios in which the welfare effect and the risk-taking effect can provide conflicting recommendations for merger approvals.

Before concluding this section, it is important to review some of the results for this extension of the model. First, allowing for bank mergers does not materially change the results of the baseline model—

the fact that a positive bank-customer effect tends to increase loan rate and risk-taking in the IME is still robust. Second, the model captures the dominant pattern of market integration, namely, bank mergers: this is because the merger IME dominates and the no-merger IME prevails only if mergers are disallowed by local banking authorities.[41] Third, we find that market integration yields pro-competitive gains even without a concomitant increase in the number of banks (Proposition 7). As discussed above, the negative association between market integration and loan rates can originate from the bank-customer effect that alters price-taking behavior or from efficiency gains from bank mergers or both. While the empirical literature has focused on efficiency gains from mergers, our model shows that this can happen even in the absence of efficiency gains. Fourth, the extension is also motivated by a large empirical literature that has highlighted the efficiency-enhancing role of mergers post integration (Berger et al., 1999; DeYoung et al., 2009). In particular, the model captures how efficient banks merge with relatively inefficient, less profitable banks when heterogenous markets integrate (cf. footnote 10). The model also reflects the dominant pattern following market integration of across-market mergers between local and non-local banks as opposed to within-market mergers between local banks (Berger et al., 1999; Dick, 2006). Fifth, although our specification finds that mergers between like banks are rarely profitable, this is a common feature of Cournot models (Salant, Switzer, and Reynolds, 1983). It is possible that other modeling choices (e.g. merged firms may benefit from merger-related synergies) could account for the prevalence of mergers between similar banks. Lastly, we have focused mainly on the *unilateral effects* of mergers—that is, whether mergers yield welfare gains or losses. On the other hand, the *pro-collusive effects* of mergers can be no less important, especially in any post-merger IME where it is easier to sustain tacit collusion between banks that have similar cost structures and market shares (Bernheim and Whinston, 1990; Boyd and Graham, 1998). Moreover, the pro-collusive effects of mergers can potentially increase loan rates and risk-taking beyond that obtained in the merger IME.

6.2. Market integration and risk diversification

In this extension, we consider the possibility that market integration creates opportunities for diversification.[42] To this end, we return to the two-market linear model with homogenous costs in Section 5.3. We assume that borrower returns are perfectly correlated within each market where the probability of success is $p(\theta) = 1 - \theta/\lambda$. Just as in Section 5.3, inverse deposit supply and loan demand functions in any segmented market j are given by (2). Again we assume, $n_1 = n_2 = n \geq 1$, $a_1 = a_2 = a > 0$, $b_1 = b > 0$ and $b_2 = \beta b$ with $\beta > 1$ so that loan demand in market 2 is larger than that in market 1. Let θ_j denote risk-shifting in the SME of market j. It follows that $\theta_1 < \theta_2$ because $\beta > 1$.

However, project returns are imperfectly correlated across the two markets, where $\sigma \in [0, 1]$ de-

[41] The stylized no-merger IME and merger IME bookend a wide range of outcomes that were observed in the integration process. For example, one may conceive of a "short-run equilibrium" where the banking authority allows only a few mergers initially so that the number of banks in the integrated market still exceeds that in each market prior to integration. As a result, the integrated market comprises both high-cost and low-cost banks and the equilibrium is similar to the no-merger IME discussed above. As the banking authorities allow more mergers, the number banks in the integrated market can even fall below that in any of segmented markets prior to integration. This "long-run equilibrium" resembles more like the merger IME where only low-cost banks prevail. From the perspective of the negative bank-competitor effect, the former tends to lower loan rates and risk-taking while the latter tends to increase loan rates and risk-taking thereby reinforcing the positive bank-customer effect.

[42] Martinez-Miera and Repullo (2010) present a general model wherein projects are imperfectly correlated. In addition to the negative bank-competitor effect (risk-shifting effect in their model), they identify a countervailing margin effect—reduced bank revenues from performing loans due to lower loan rates create incentives for banks to take on more risk.

notes the correlation coefficient between default risks in markets 1 and 2. Moreover, we assume that depositors are repaid by their banks only if projects are successful (and loans are repaid) for both group of borrowers.[43] The probability that projects in both markets succeed is

$$q(\theta) \equiv p(\theta)^2 + \sigma p(\theta)[1 - p(\theta)].$$

In the integrated market with $D = D_1 + D_2$ and $L = L_1 + L_2$, the inverse deposit supply and loan demand functions are given by (33) and (34). Note that $a_1 + a_2 = 2a$ and $b_1 + b_2 = (1 + \beta)b$. The probability of success function of each borrower is given by $p(\tilde{\theta}(r(L/(b_1 + b_2)))) = L/2\lambda(1+\beta)b$, and

$$q(L) \equiv q(\theta(r(L/(b_1 + b_2)))) = \left(\frac{L}{2\lambda(1+\beta)b}\right)^2 + \sigma\left(\frac{L}{2\lambda(1+\beta)b}\right)\left(1 - \frac{L}{2\lambda(1+\beta)b}\right).$$

We find that $q'(L) > 0$. Each bank i in the integrated market solves

$$\max_{L_i} q(L)\left(1 - \frac{L}{(1+\beta)b} - \frac{L}{2a}\right)L_i. \tag{37}$$

The following proposition analyzes the implications of risk-diversification benefits of market integration.

Proposition 9 *Let $\theta^*(\beta, \sigma)$ be the risk-shifting in the IME and θ_j be the risk-shifting in the SME in market j, $j = 1, 2$. Given our assumption that $\beta > 1$ and $0 \leq \sigma < 1$, SME risk-taking in market 2 is always greater than that in the IME, i.e., $\theta^*(\beta, \sigma) < \theta_2$. On the other hand, there is a unique strictly decreasing function, $\tilde{\beta}(\sigma)$ with $\lim_{\sigma\to 1}\tilde{\beta}(\sigma) > 1$ so that $\theta^*(\beta, \sigma) > (<)\, \theta_1$ according as $\beta > (<)\, \tilde{\beta}(\sigma)$.*

Increases in σ indicate higher correlation between returns across the two borrower groups, and this reduces diversification benefits. Reducing diversification benefits reduces bank lending and increases loan rates. Put differently, a greater the diversification benefit (lower σ) tends to lower the loan rate.

The above proposition is similar to Proposition 6. Because $\beta > 1$, the bank-customer effect with respect to market 1 (market 2) is positive (negative). In market 2, the negative bank-customer effect reinforces the negative bank-competitor effect. Moreover, increases in the diversification benefit also tends to lower the loan rate. As a result, we obtain $\theta^* < \theta_2$. In market 1, the positive bank-customer effect opposes the negative bank-competitor effect. Clearly, the threshold at which the positive bank-customer effect outweighs the negative bank-competitor effect depends on the diversification benefit, as measured by σ. The threshold function $\tilde{\beta}(\sigma)$ is decreasing in σ because the greater the diversification benefit (lower σ tends to lower loan rates), the stronger must be the bank-customer effect for the IME loan rate to exceed the SME rate in market 1.

Proposition 9 has important policy ramifications. Consider the welfare analysis in Section 5.3. Note that ΔW, the difference in aggregate welfare between the IME and the SME, and $\Delta\theta_j$, the difference between the risk-shifting in the IME and that in the SME for market j are now functions of the ratio of borrowers to depositors in market 2, β and risk correlation, σ. In terms of Figure 3, Proposition 9

[43] The results obtained here are robust to the assumption that banks' debt obligation are met if at least one group succeeds. Nevertheless, the simplification here does not explicitly model idiosyncratic risk, thereby retaining the feature that entrepreneurial risk-taking is synonymous with risk-taking by banks (cf. footnote 22). As a result, the margin effect modeled in Martinez-Miera and Repullo (2010) is zero in this setting. If we had considered within market correlation, we would also have the same margin effect both in the segmented and the integrated markets.

implies that greater diversification benefits (i.e., lower σ) shift down in the $\Delta\theta_1(\beta)$ curve, and shifts up the $\Delta W(\beta)$ curve. This implies that increases in diversification benefits not only lowers risk-taking in the integrated market relative to market 1 but also increases aggregate welfare from integration. With imperfectly correlated default rates across markets, a planner is therefore more likely to integrate the two markets, and the likelihood of integration increases with the diversification benefit (decrease in σ).

6.3. Interbank lending

Next, we extend the linear model in Section 5 to include an interbank market. In particular, we assume that banks trade funds at the interbank rate, ρ, which is a policy variable chosen by the central bank (Freixas and Rochet, 2008, p. 79). The key assumption here is that all banks take ρ as given irrespective of whether markets are segmented or integrated. Under this assumption, we show below that banks' maximization problems for the SME and the IME are isomorphic. We also show that the bank-customer effect of market integration is significant in determining the loan rate and risk-taking even when banks have this alternative funding source. Moreover, there are additional implications of interbank transactions for the bank-competitor effect of market integration as we describe below.

6.3.1. Interbank lending and the SME

The model setup is similar to that in Section 5.1. Although individual banks operate in only one of m segmented markets, they face the same interbank rate, ρ, chosen by the central bank, where $\rho \in [0, \bar{\rho}]$ where $\bar{\rho} \leq \lambda$. Let the probability of success be given by $p(\theta) = 1 - \theta/\lambda$. Bank i in market j solves

$$\max_{\{L_{ij}, D_{ij}\}} P(L_j/b_j)[(r(L_j/b_j) - \rho)L_{ij} + (\rho - R(D_j/a_j)D_{ij}] \tag{38}$$

where $L_j = \sum_{i=1}^{n_j} L_{ij}$ and $D_j = \sum_{i=1}^{n_j} D_{ij}$. We show below that this problem is isomorphic to that for the IME. It follows that their solutions are similar as described below.

6.3.2. Interbank lending and the IME

As in Section 4.2, we consider the integration of a subset of segmented markets, J_m, where $J_m = \{1, 2, \ldots, m\}$ and $|J_m| = m < |J|$. Banks are allowed to transact with each other at the interbank rate ρ set by the central bank. With (inverse) deposit supply and loan demand given by (23) and (25), respectively, bank i solves

$$\max_{\{L_i, D_i\}} P(L/b(m))[(r(L/b(m)) - \rho)L_i + (\rho - R(D/a(m))D_i], \tag{39}$$

where $L = \sum_{i=1}^{n(m)} L_i$ denotes aggregate loans and $D = \sum_{i=1}^{n(m)} D_i$ denotes aggregate deposits in the integrated market, J_m. The proposition below summarizes the effect of increased competition under the IME on loan rates and risk-shifting in the presence of interbank lending.

Proposition 10 *The following hold for the IME with an interbank market:*

(a) *Suppose that $n(m)$ is invariant to m (i.e., the bank-competitor effect is set to zero). Then, the bank-customer effect is positive (negative) according as $\hat{b}(m) > (<)\, \hat{a}(m)$ or $\hat{\xi}(m) > (<)\, 0$.*

(b) *Suppose that $a(m)$ and $b(m)$ are invariant to m (i.e., the bank-customer effect is set to zero). Then, there is a unique threshold $\rho^* \in [0, \bar{\rho}]$ such that the bank-competitor effect is negative (positive) according as $\rho < (>) \rho^*$. Therefore, r and θ are decreasing (increasing) in m according as the interbank rate ρ is low (high).*

The bank-competitor and the bank-customer effects comprise the aggregate effect of market integration on the equilibrium loan rate and risk-shifting. It follows that the overall effect of market integration on the loan rate, and consequently, risk-taking is indeterminate.

Proposition 10 extends the results in Proposition 3 to the IME with an interbank market. Proposition 10(a) shows that a non-zero bank-customer effect exists even with interbank lending. Moreover, loan rate and risk-taking increase under market integration if this bank-customer effect is positive.

Proposition 10(b) shows that the bank-competitor effect is no longer unambiguously negative in the presence of interbank lending. To demonstrate this, we simplify by setting the bank-customer effect to zero (i.e., $\hat{\xi}(m) = 0$), so that any changes in m yield changes only in the number of banks, n. From the first-order conditions of the maximization problem in (39), we obtain the following relation between loan and deposit rates in the symmetric equilibrium (see Appendix for details):

$$R = \frac{\rho n}{n+1}, \tag{40}$$

$$b(n+2)(\lambda - r)^2 - b(\lambda - \rho)(n+1)(\lambda - r) = aR(\rho - R). \tag{41}$$

Condition (40) helps pin down the deposit rate as an increasing function of the number of banks, n. An increase in the number of banks, n implies a greater volume of aggregate deposits, and therefore, a higher deposit rate in equilibrium. On the other hand, condition (41) reveals that the equilibrium loan rate depends on both the number of banks, n, and additionally, on the deposit rate, R. The dependence of the equilibrium loan rate on the deposit rate stems from the fact that the probability of default is endogenous in our framework (as in Dermine, 1986).[44] Differentiating (41) with respect to n, we obtain

$$\frac{dr^*}{dn} = \frac{\partial r}{\partial n} + \frac{\partial r}{\partial R} \cdot \frac{dR}{dn}. \tag{42}$$

From (41), it follows $\partial r/\partial n < 0$ and $\partial r/\partial R > 0$. Increasing the number of banks decreases individual banks' market power, which tends to raise deposit rates and lower loan rates. Ceteris paribus, increases in deposit costs pass-through to loan rates. The bank-competitor effect on loan rates comprises a negative direct effect—the first term on the right-hand side of (42), and a positive indirect effect that operates through deposit rates—the second term on the right-hand side of (42). Increasing the interbank rate dampens the negative direct effect but strengthens the positive indirect effect.[45] Therefore, at low interbank rates, the negative direct effect dominates and the overall bank-competitor effect is negative. At high interbank rates, on the other hand, the positive indirect effect dominates and the overall bank-competitor effect is positive.

[44] Loan and deposit rates are independent of each other in the presence of strategic competition in both loan and deposit markets, and interbank lending as long as the probability of default is exogenous. Setting $p(\theta) = p^0$ in each bank's maximization problem yields the equilibrium loan rate, $r^* = (\lambda + \rho n)/(n+1)$, and the equilibrium deposit rate, $R^* = \rho n/(n+1)$. Notably, the loan rate is monotonically decreasing and the deposit rate is monotonically increasing in the number of banks, n.

[45] More formally, we obtain that $\partial((\partial r/\partial n))/\partial \rho < 0$ and $\partial((\partial r/\partial R)(dR/dn))/\partial \rho > 0$.

The full effect of market integration comprises the bank-competitor and bank-customer effects. Unlike the results in Sections 4.2 and 5, the bank-competitor effect is no longer unambiguously negative. With interbank lending, both bank-competitor and bank-customer effects can be positive or negative depending on the parameters of the model. It follows that the effect of market integration on loan rates and risk-taking can be positive or negative. Moreover, any existing relationship between competition and risk-taking in the SME can be reversed in the IME with a countervailing and sufficiently strong bank-customer effect. We illustrate this result in terms of the numerical example below. Example 2 confirms the robustness of our baseline findings in Propositions 4 and 5 in the presence of interbank lending.

Example 2 Let $a(m) = m^{\alpha}$, $b(m) = m^{\delta}$ and $n(m) = m^{\nu}$ with $\alpha,\ \delta,\ \nu \in [0,\ 1]$. We allow the number of integrated markets, m, to vary between 2 and 10. We set $\alpha = \nu = 0.3$, $\lambda = 4$ and $\rho = 2.4$. Figure 6 depicts the equilibrium loan rate $r(m)$ as a function of the number of integrated markets. In the left panel, $r(m)$ is drawn for $\delta = 0.3$, thereby setting the bank-customer effect to zero. In this case, the equilibrium loan rate decreases under market integration because of the negative bank-competitor effect. The right panel depicts the reversal because of a positive and sufficiently strong bank-customer effect by setting $\delta = 0.6 > 0.3 = \alpha$. As a result, $r(m)$ is now increasing in m. ■

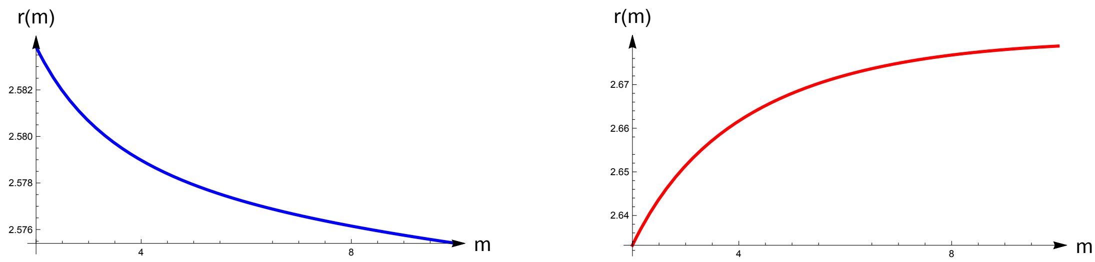

Figure 6: *In the presence of interbank lending, the negative association between loan rates and market integration in the left panel is driven by negative bank-competitor effect and a bank-customer effect set to zero. A strong and countervailing (positive) bank-customer effect reverses this association as shown in the right panel.*

7. Empirical implications

Our analysis offers a framework where bank competition across markets can reverse the within-market relation between competition and risk-taking. The result arises not simply because integrating across markets alters the number of competitor banks, but rather because market integration affects deposit and loan markets asymmetrically—the source of the bank-customer effect. As long as any two integrating markets are heterogenous in terms of their ratios of borrowers to depositors, this bank-customer effect—unmodeled in previous banking theory—can potentially alter the effect on loan rates of any change in the number of competitor banks from integration. The consideration of a direct but asymmetric impact on loan and deposit markets from across-market competition is the key feature of our framework. While no study has revealed its impact on risk-taking, the empirical relevance of this differential impact of integration on deposit and loan markets has been demonstrated in the context of U.S. banking deregulation (see Park and Pennacchi, 2009).[46]

[46] Although Park and Pennacchi (2009) do not directly examine the effects on risk-taking, they find that increased market integration (in their case, with large banks entering local markets populated by small banks) tends to "promote competition in

The asymmetry inherent in integrating markets also generates simple testable implications to help identify the bank-customer effect.

Implication 1 *The bank customer-effect is nonzero as long as the ratio of borrowers to depositors are not identical across any two integrating markets.*

This follows directly from our two-market example in Section 2 for $\xi_1 \neq \xi_2$. The goal would be to compare estimates of loan demand and deposit supply both prior to and following market integration to identify the bank-customer effect. Our results demonstrate that including the bank-customer effect is important for empirical work on competition and risk-taking that focuses on events of deregulation.

Implication 2 *The bank-customer effect is positive and sufficiently strong if observed loan prices are higher in the IME than in the SME.*

The above testable implication follows from (6) and Proposition 3. Market integration has been found to yield an immediate increase in the number of competing banks (see Dick, 2006, Table 10). In terms of our model, therefore, the bank-competitor effect predicts that loan rates decrease following integration. While this result holds on average, Jayaratne and Strahan (1998, Figure 2) finds that loan rates did increase in some states after branching deregulation.[47] In our framework, loan rates can still increase upon integration despite increases in the number of banks because of a strong bank-customer effect (Proposition 3). However, it is unclear if these findings can be reconciled with more traditional models of competition and risk-taking.

Consolidation in the banking industry generates the opposite predictions—increases in market concentration in our framework typically increase loan rates and risk-taking, but a strong negative bank-customer effect can also lower rates (Section 6.1). On average, market integration and ensuing merger activity has been followed by pro-competitive outcomes such as declines in costs of operation, non-performing loans, and loan prices (Jayaratne and Strahan, 1996, 1998), without any significant changes in local market structure (Dick, 2006). While the evidence of efficiency gains and diversification benefits can explain the post-integration declines in operating costs and non-performing loans, respectively, they cannot explain the overall decline in loan rates (Jayaratne and Strahan, 1996, 1998; Dick, 2006). Loan rate declines in the face of growing consolidation upon market integration remains an unresolved question for empirical studies on bank competition. Moreover, a more parsimonious setting that defines increased competition in terms of a within-market increase of competitor banks cannot explain this pattern in the data either. This model offers a mechanism driven by competitive factors wherein integration yields competitive outcomes because of the non-trivial bank-customer effect. Notably, this mechanism is not only independent of efficiency gains and diversification benefits but also of market concentration.

A final contribution in guiding empirical work is methodological. The model suggests that no two cases of market integration are necessarily alike. Recent advances in econometrics have emphasized the role of treatment effect heterogeneity in settings, such as that of banking market integration (for example, intra- and inter-state deregulation in the United States), where policies are introduced "at different times,

retail loan markets but also tends to harm competition in retail deposits."

[47] Although loan rates declined in most states, Jayaratne and Strahan (1998) find that around seven states recorded loan rate increases following deregulation.

to different units" (Callaway and Sant'Anna, 2021; Goodman-Bacon, 2021; Sun and Abraham, 2021). The two-way fixed effects (TWFE) estimator, commonly used in empirical work has been shown to perform "very poorly when (treatment) effects are heterogenous" (Sun and Shapiro, 2022, p. 195). In the context of empirical banking studies examining the process of market integration since the 1970s, our model presents a rationale behind this heterogeneity that is rooted in banking theory. Proposition 4 shows how, depending on the bank-customer effect, any relation between competition and risk-taking in the segmented markets can be reversed upon integration. The bank-customer effect, in turn, depends on the relative sizes of deposit supply and loan demand among the integrating markets. When the set of integrating markets varies with each episode of deregulation, it calls for exploring the possibility that treatment is heterogeneous and deregulation events be treated separately. Studies exploring this possibility and conducting separate assessments for each deregulation event (or group of deregulation events) have shown that the effects to be quite heterogeneous between the various episodes (Jayaratne and Strahan, 1996; Wall, 2004; Huang, 2008). The evidence against the imposition of homogeneity restrictions on different deregulation events has been substantial. Perhaps more importantly, we should not ignore the hypothesis that the lack of consensus about the linkages between bank competition and financial stability described above is not an anomaly but an inherent feature of the problem.

8. Conclusion

The lack of consensus among studies examining the linkages between bank competition and risk-taking poses a significant challenge for policymakers and academics. Our model provides one possible reason behind the mixed empirical evidence on this question. In the context of integrating banking markets, we find that the effect of increased competition extends beyond that explained by a simple increase in the number of rival banks. Importantly, we point to a risk-incentive mechanism, namely the bank-customer effect of market integration, that can potentially reverse *any* observed association between bank competition and risk-taking under within-market competition. Moreover, while the predictions of existing theory are shown to prevail under specific conditions within our generalized framework, they are not the only outcomes of the model.

To the best of our knowledge, the present paper is the first to directly examine the effect of market integration on risk-taking. While future modeling efforts can include richer settings that improve our understanding about the linkages between competition and stability in banking, we have opted for a tractable approach. Even in this parsimonious setting, the richer set of results demonstrate the need for a broader notion of increased competition that captures the evolution of competition. Most theory on bank competition tends to focus on changes in the number of competitor banks within a limited geographic area. Our results suggest the need for reexamining the idea that bank competition is confined to local individual markets.

Appendix A. Proofs of the main results

Proof of Lemma 2. We suppress for the time being the argument m from $a(m)$, $b(m)$ and $n(m)$. Let $P(L/b) \equiv p(\tilde{\theta}(r(L/b)))$. With $L = \sum_{i=1}^{n} D_i = D$, the first-order condition of the maximization problem of bank i in the integrated market J_m is given by:

$$
\begin{aligned}
& P(L/b)[r(L/b) - R(L/a)] + L_i[r(L/b) - R(L/a)]P'(L/b) \cdot \frac{1}{b} \\
& = P(L/b)L_i \left(R'(L/a) \cdot \frac{1}{a} - r'(L/b) \cdot \frac{1}{b} \right) \\
\Longleftrightarrow\ & L_i = \frac{P(L/b)[r(L/b) - R(L/a)]}{P(L/b)\left(R'(L/a) \cdot \frac{1}{a} - r'(L/b) \cdot \frac{1}{b}\right) - [r(L/b) - R(L/a)]P'(L/b) \cdot \frac{1}{b}} \quad \text{for all } i.
\end{aligned}
$$

Because the right-hand side of the above condition depends only on the aggregate loans, L, it immediately follows that $L_i = L_{i'}$ for any $i \neq i'$. Therefore, there are no asymmetric equilibria. In the (symmetric) IME, $L_i = L/n$ for all i. Thus, the above optimality condition boils down to:

$$
\mu(L,\, a,\, b) - F(L,\, a,\, b,\, n) = 0, \tag{43}
$$

where $\mu(L,\, a,\, b) \equiv r(L/b) - R(L/a)$ is the equilibrium intermediation margin, and

$$
F(L,\, a,\, b,\, n) \equiv \frac{[bR'(L/a) - ar'(L/b)]\, P(L/b)L}{a\, [nbP(L/b) + P'(L/b)D]}\,.
$$

The second order necessary condition is given by:

$$
\mu_L(L,\, a,\, b) - F_L(L,\, a,\, b,\, n) < 0. \tag{44}
$$

The equilibrium loan rate is given by $r(L/b)$, and the equilibrium risk-shifting is given by $\tilde{\theta}(r(L/b))$.

Proof of Proposition 3. The first-order condition (43) can be written as

$$
\mu(L,\, a,\, b) = F(L,\, a,\, b,\, n) \equiv \frac{\Phi(L,\, a,\, b)}{\Psi(L,\, b,\, n)}\,,
$$

where $\Phi(L,\, a,\, b) \equiv (b/a)R'(L/a) - r'(L/b) > 0$, and $\Psi(L,\, b,\, n) \equiv \frac{bn}{L} + \epsilon(L/b)$ with $\epsilon(L/b) \equiv P'(L/b)/P(L/b)$. Because the optimal risk-shifting of the entrepreneurs may be increasing or decreasing in the loan rate, the sign of $P'(L/b)$ is indeterminate. If $P'(L/b) > 0$, then $\Psi(L,\, b,\, n)$ is also positive. Therefore, we would assume that $\Psi(L,\, b,\, n) \geq 0$ if $P'(L/b) < 0$. Note that

$$
\mu_L = r'(L/b) \cdot \frac{1}{b} - R'(L/a) \cdot \frac{1}{a} < 0\,, \quad \mu_a = R'(L/a) \cdot \frac{L}{a^2} > 0, \quad \text{and} \quad \mu_b = -r'(L/a) \cdot \frac{L}{b^2} > 0.
$$

On the other hand, from the expression of $F(L,\, a,\, b,\, n)$, we obtain

$$
\begin{aligned}
F_L &= \frac{1}{\Psi}\left[\frac{bR''(L/a)}{a^2} - \frac{r''(L/b)}{b} + F(L,\, a,\, b,\, n)\left(\frac{bn}{L^2} - \frac{\epsilon'(L/b)}{b} \right) \right], \\
F_a &= -\frac{b}{\Psi a^2}\left(R''(L/a)(L/a) + R'(L/a) \right), \\
F_b &= \frac{1}{\Psi}\left[\frac{R'(L/a)}{a} + \frac{Lr''(L/b)}{b^2} + F(L,\, a,\, b,\, n)\left(\frac{L\eta'(L/b)}{b^2} - \frac{n}{L} \right) \right], \\
F_n &= -\frac{\Phi b}{\Psi^2 L} < 0.
\end{aligned}
$$

Note that $F_L - \mu_L > 0$ by (44). Moreover, and $F_a < 0$ because $R''(L/a) \geq 0$, and hence, $\mu_a - F_a > 0$. Lastly, it is immediate to verify that

$$1 - \frac{b(\mu_b - F_b)}{L(F_L - \mu_L)} = \frac{a(\mu_a - F_a)}{L(F_L - \mu_L)} .$$

Totally differentiating the first-order condition (43), we obtain

$$\frac{dL}{L} = -\frac{nF_n}{L(F_L - \mu_L)} \cdot \frac{dn}{n} + \frac{a(\mu_a - F_a)}{L(F_L - \mu_L)} \cdot \frac{da}{a} + \frac{b(\mu_b - F_b)}{L(F_L - \mu_L)} \cdot \frac{db}{b}.$$

Because $r^* = r(L/b)$, we have

$$\begin{aligned} dr^* &= -\, r'(L/b)(L/b) \left(\frac{db}{b} - \frac{dL}{L} \right) \\ &= -\, r'(L/b)(L/b) \left\{ \frac{nF_n}{L(F_L - \mu_L)} \cdot \frac{dn}{n} + \underbrace{\left(1 - \frac{b(\mu_b - F_b)}{L(F_L - \mu_L)} \right)}_{= \frac{a(\mu_a - F_a)}{L(F_L - \mu_L)}} \frac{db}{b} - \frac{a(\mu_a - F_a)}{L(F_L - \mu_L)} \cdot \frac{da}{a} \right\} . \end{aligned} \quad (45)$$

Because $\hat{n}(m) \equiv n'(m)/n(m)$, $\hat{a}(m) \equiv a'(m)/a(m)$, $\hat{b}(m) \equiv b'(m)/b(m)$, and $\hat{\xi}(m) = \hat{b}(m) - \hat{a}(m)$ it follows from (45) that

$$\frac{dr^*}{dm} = -r'(L/b)(L/b) \left[\frac{nF_n}{L(F_L - \mu_L)} \cdot \hat{n}(m) + \frac{a(\mu_a - F_a)}{L(F_L - \mu_L)} \cdot \hat{\xi}(m) \right] .$$

$$\text{Define} \quad Z_0 \equiv -r'(L/b)(L/b) \cdot \frac{nF_n}{L(F_L - \mu_L)} \quad \text{and} \quad Z_1 \equiv -r'(L/b)(L/b) \cdot \frac{a(\mu_a - F_a)}{L(F_L - \mu_L)} .$$

Because $r'(\cdot) < 0$, $F_n < 0$ and $F_L - \mu_L > 0$, we have $Z_0 < 0$. On the other hand, $Z_1 > 0$ as $r'(\cdot) < 0$, $F_L - \mu_L > 0$ and $\mu_a - F_a > 0$. This completes the proof of the Proposition.

Appendix B: Additional results and proofs

Proof of Lemma 1. The proof of this lemma is identical to that of Lemma 2 below. Replace a, b, n and L_i in the proof of Lemma 2 respectively by a_j, b_j, n_j and L_{ij} to obtain the result.

Proof of Proposition 2. This proposition can be obtained as a special case of Proposition 3. Replace $a(m)$, $b(m)$, $n(m)$ and L_i respectively by a_j, b_j, n_j and L_{ij}, and set $\hat{a}(m) = \hat{b}(m) = 0$ in the proof of Proposition 2 to obtain the result.

Proof of Proposition 1. We first characterize the maximization problem of the borrower. We omit the subscript j from the loan rate. Let $\phi(k) \equiv f(k)/k$ be the average product of investment. The first-order condition of (9) is given by

$$\underbrace{\left\{y(\theta) + y'(\theta)\cdot\frac{p(\theta)}{p'(\theta)}\right\}}_{h(\theta)} f(k) - rk = 0 \quad \Longleftrightarrow \quad h(\theta) = \frac{r}{\phi(k)}, \tag{46}$$

which defines $\theta = \theta(k;\, r)$. Note that the objective function of the maximization problem (9) is strictly concave on $\theta \in [0,\, \bar{\theta}]$ because $p(\theta) \geq 0$, $p'(\theta) < 0$, $p''(\theta) < 0$, $y(\theta) \geq 0$, $y'(\theta) > 0$ and $y''(\theta) < 0$, and hence, $\theta(k;\, r)$ is unique. Also, because $y''(\theta) \leq 0 < y'(\theta)$, $p'(\theta) < 0$ and $p''(\theta) \leq 0$, we have

$$h'(\theta) = 2y'(\theta) + \frac{p(\theta)}{p'(\theta)}\left\{y''(\theta) - \frac{y'(\theta)p''(\theta)}{p'(\theta)}\right\} \geq 2y'(\theta) > 0. \tag{47}$$

Differentiating (46) with respect to k and r, respectively, we obtain

$$\theta_k(k;\, r) = \frac{r\varepsilon_\phi(k)}{h'(\theta)k\phi(k)} = \frac{r[1-\varepsilon(k)]}{h'(\theta)k\phi(k)} > 0, \quad \text{and} \quad \theta_r(k;\, r) = \frac{1}{h'(\theta)\phi(k)} > 0, \tag{48}$$

where $\varepsilon_\phi(k) \equiv -k\phi'(k)/\phi(k)$ is elasticity of the average product, $\phi(k)$ with respect to k, and $\varepsilon(k) \equiv kf'(k)/f(k)$ is the output elasticity of investment. It is immediate to show that $\varepsilon_\phi(k) = 1 - \varepsilon(k) > 0$. The last inequality holds because $f(0) \geq 0$ and $f(k)$ is strictly concave.[48]

Now, let $U(k;\, r)$ be the value function of the maximization problem (9). Then, by the Envelope theorem, we have $U_k(k;\, r) = p(\theta)\{y(\theta)f'(k) - r\}$, and hence, the first-order condition of (11) is given by:

$$U_k(k;\, r) = 0 \quad \Longrightarrow \quad y(\theta)f'(k) = r. \tag{49}$$

The second-order necessary condition is given by

$$\begin{aligned} & p(\theta)\{y'(\theta)\theta_k f'(k) + y(\theta)f''(k)\} + p'(\theta)\theta_k\{\underbrace{y(\theta)f'(k) - r}_{=0 \text{ by } (49)}\} \leq 0 \\ \Longrightarrow \quad & \underbrace{y'(\theta)\theta_k f'(k) + y(\theta)f''(k)}_{\Omega(k,r)} \leq 0. \end{aligned} \tag{50}$$

On the other hand, differentiating (49) with respect to r, and using the expression of θ_r from (48), we obtain

$$k'(r) = \frac{1 - (y'(\theta)/h'(\theta))\varepsilon(k)}{\Omega(k,\, r)}. \tag{51}$$

Observe that $h'(\theta) \geq 2y'(\theta)$ implies that $y'(\theta)/h'(\theta) \leq 1/2$. Therefore, the numerator of the last expression is strictly positive because $\varepsilon(k) < 1$. On the other hand, the denominator is negative by the second-order condition. Consequently, $k'(r) \leq 0$.

[48] Note that, for any production function $f(k)$, strictly concave, the average product, $\phi(k)$ is strictly decreasing in k. Moreover, $\phi'(k) < 0$ is equivalent to $\varepsilon(k) < 1 \iff \phi(k) > f'(k)$. We show that $\varepsilon(k) < 1$ for a twice differentiable production function $f(k)$ that is strictly concave with $f(0) \geq 0$. Take any point $(k_0,\, f(k_0))$ on the graph of $f(k)$. Then, there is $\kappa \in (0,\, k_0)$ so that

$$\phi(k_0) \equiv \frac{f(k_0)}{k_0} \geq \frac{f(k_0) - f(0)}{k_0} = f'(\kappa) > f'(k_0).$$

The first (weak) inequality follows from the fact that $f(0) \geq 0$, the second equality follows from the Mean Value theorem, and the last (strict) inequality is implied by $f''(k) < 0$ and $\kappa < k_0$. This proves that $\varepsilon(k) < 1$ as k_0 has been chosen arbitrary.

To show the last part of the proposition, note that

$$\frac{d\tilde{\theta}}{dr} = \theta_r + \theta_k \cdot k'(r) = \frac{1}{h'(\theta)\phi(k)} + \frac{r\varepsilon_\phi(k)}{h'(\theta)k(r)\phi(k)} \cdot k'(r) = \frac{1}{h'(\theta)\phi(k)}\left\{1 + \varepsilon_\phi(k)\cdot\frac{rk'(r)}{k(r)}\right\}.$$

The above condition is an equilibrium condition where $k = k(r)$. Let $\varepsilon_k(r) \equiv -rk'(r)/k(r)$ be the elasticity of the individual loan demand function. Because $h'(\theta),\ \phi(k) > 0$, from the last expression of $d\tilde{\theta}/dr$, it follows that

$$\text{sign}[d\tilde{\theta}/dr] = \text{sign}[1 - \varepsilon_k(r)\varepsilon_\phi(k)].$$

The above condition is intuitive. Recall that the indirect effect works as follows. An increase in r decreases k because $k'(r) < 0$. This decrease in k increases $\phi(k)$ because $\phi'(k) < 0$, which, given (46), decreases $\tilde{\theta}$. Clearly, the strength of the indirect effect depends on both the responsiveness of k with respect to r, i.e., $\varepsilon_k(r)$, and that of $\phi(k)$ with respect to k, i.e., $\varepsilon_\phi(k)$. Thus, if $\varepsilon_k(r)\varepsilon_\phi(k)$ is low (high), i.e., less (greater) than 1, the direct effect dominates (is dominated by) the indirect effect, and hence, risk-taking increases (decreases) with loan rate.

To prove that $d\tilde{\theta}/dr > (<)\, 0$ according as $\varepsilon'(k) < (>)\, 0$, note that

$$\varepsilon'(k) = \frac{d}{dk}\left(\frac{kf'(k)}{f(k)}\right) = \frac{kf''(k) + f'(k)[1-\varepsilon(k)]}{f(k)} \tag{52}$$

Note that

$$\begin{aligned}\frac{d\tilde{\theta}}{dr} &= \underbrace{\frac{r[1-\varepsilon(k)]}{h'(\theta)f(k)}}_{\theta_k}\cdot\underbrace{\frac{1-(y'(\theta)/h'(\theta))\varepsilon(k)}{\Omega(k,\,r)}}_{k'(r)} + \underbrace{\frac{k}{h'(\theta)f(k)}}_{\theta_r} \\ &= \frac{r[1-\varepsilon(k)][h'(\theta)-y'(\theta)\varepsilon(k)] + h'(\theta)k\,\Omega(k,\,r)}{[h'(\theta)]^2 f(k)\Omega(k,\,r)} \\ &\equiv \frac{Q}{[h'(\theta)]^2 f(k)\Omega(k,\,r)}\,. \end{aligned} \tag{53}$$

Using the expressions of θ_k, $\Omega(k,\,r)$, and the fact that $r = y(\theta)f'(k)$ form (49), we obtain

$$Q = h'(\theta)y(\theta)\left\{kf''(k) + f'(k)[1-\varepsilon(k)]\right\} = h'(\theta)y(\theta)f(k)\varepsilon'(k).$$

The last equality follows from (52) Therefore,

$$\frac{d\tilde{\theta}}{dr} = \frac{y(\theta)\varepsilon'(k)}{h'(\theta)\Omega(k,\,r)},$$

which implies that

$$\text{sign}[d\tilde{\theta}/dr] = \text{sign}[1 - \eta(r)\varepsilon_\phi(k)] = -\,\text{sign}[\varepsilon'(k)]$$

because $y(\theta),\ h'(\theta),\ f'(k) > 0$ and $\Omega(k,\,r) \le 0$. This completes the proof of the proposition.

We now provide three examples for which the elasticity of investment, $\varepsilon(k)$ increases, decreases and constant in k, respectively. For all the examples below, we assume that $p(\theta) = 1-\theta$, $y(\theta) = \theta$.

1. Consider $f(k) = k(1-k)$ defined on $[0,\,1/2]$ so that $f'(k) > 0$. For this functional form, $\varepsilon(k)$

decreases with k. In this case, equilibrium risk-shifting is strictly increasing in r_j and is given by:

$$\tilde{\theta}(r_j) = \frac{1}{4}\left(1 + \sqrt{1 + 8r_j}\right).$$

2. Consider $f(k) = \sqrt{k_0 + k}$ with $k_0 > 0$ an $k \geq 0$. In this case, the elasticity of investment is increasing in k, i.e., $\varepsilon'(k) > 0$. The equilibrium risk-shifting is given by:

$$\tilde{\theta}(r_j) = \frac{1}{2} + \frac{\sqrt{2}\left(1 - 24k_0 r_j^2 + \sqrt{1 - 12k_0 r_j^2}\right)}{12\left(1 - 6k_0 r_j^2 + \sqrt{1 - 12k_0 r_j^2}\right)^{1/2}}.$$

   The above expression is decreasing in r_j.

3. Finally, let $f(k) = k^\eta$ with $\eta \in (0,\, 1)$. In this case, $\varepsilon(k) = \eta$ for all k. The optimal risk-taking is given by $\tilde{\theta}(r_j) = (2 - \eta)^{-1}$, which is independent of the loan rate.

Linear loan demand in the general model. We first show that there exist functions $p(\theta)$, $y(\theta)$ and $f(k)$ for which the loan demand function is linear.

Lemma 6 *Let $p(\theta) = 1 - \theta$ and $y(\theta) = \theta$. Then, there exists a unique $f(k)$ on $(0,\, \bar{k}) \subset (0,\, \lambda)$ such that $k(r)$ is a linear function of the form $k(r) = \lambda - r$.*

Proof. If $p(\theta) = 1 - \theta$ and $y(\theta) = \theta$, then $h(\theta) = 2\theta - 1$. The first-order condition (10) of the maximization problem (9) implies that

$$\theta(k;\, r) = \frac{1}{2}\left\{1 + \frac{rk}{f(k)}\right\}.$$

Then, (49) reduces to

$$\frac{1}{2}\left\{1 + \frac{rk}{f(k)}\right\} f'(k) = r \quad \iff \quad r = \frac{f(k)f'(k)}{2f(k) - kf'(k)}.$$

Note that $k = \lambda - r \iff r = \lambda - k$ is equivalent to

$$\underbrace{\frac{f(k)f'(k)}{2f(k) - kf'(k)}}_{r} = \lambda - k \quad \iff \quad f'(k) = \frac{2f(k)(\lambda - k)}{f(k) + k(\lambda - k)} \equiv \psi(k,\, f(k)). \tag{54}$$

Let $f(k_0) = z_0$ such that $(k_0,\, z_0) \in [0,\, \bar{k}] \times \mathbb{R}$. Observe that both $\psi(k,\, z)$ and $\psi_z(k,\, z)$ are continuous functions. The second assertion is true because

$$\psi_z(k,\, z) = \frac{2z(\lambda - k)^2}{\{z + k(\lambda - k)\}^2}$$

whose denominator is different from zero if $z_0 > 0$ which implies that $\psi_z(k,\, z)$ is a bounded function, and hence, $\psi(k,\, z)$ is Lipschitz continuous in z. Therefore, there exists a unique $f(k)$ on $(0,\, \bar{k})$ which satisfies (54) (see Simmons, 2017, Theorem B, p. 634). □

Proof of Proposition 5. With $R = D/a$ and $r = \lambda - (L/b)$, we have $\theta(L/b) = \lambda - (L/2b)$ and $P(L/b) \equiv 1 - \theta(L/b) = L/(2\lambda b)$. Therefore in the integrated market, bank i's objective function, using $L = D$, becomes

$$\frac{L}{2\lambda b}\left\{\lambda - \frac{L}{b} - \frac{L}{a}\right\} L_i.$$

The first-order condition of the above maximization problem with respect to L_i is given by:

$$\lambda L - \left(\frac{1}{a} + \frac{1}{b}\right) L^2 + L_i\left\{\lambda - 2\left(\frac{1}{a} + \frac{1}{b}\right) L\right\} = 0. \tag{55}$$

In the symmetric equilibrium, we have $L_i = L/n$ for all i. Substituting this into (55), and solving for $L > 0$ yields

$$L = \frac{\lambda ab(n+1)}{(a+b)(n+2)}.$$

Writing $n \equiv n(m)$ and $\xi \equiv \xi(m)$, the equilibrium loan rate and risk-shifting of the integrated market are given by:

$$\begin{aligned} r^* &= \lambda - \frac{L}{b} = \lambda\left(1 - \frac{n(m)+1}{n(m)+2}\cdot\frac{1}{1+\xi(m)}\right), \\ \theta^* &= \lambda - \frac{L}{2b} = \lambda\left(1 - \frac{1}{2}\cdot\frac{n(m)+1}{n(m)+2}\cdot\frac{1}{1+\xi(m)}\right). \end{aligned} \tag{56}$$

Differentiating the expression in (56) with respect to m, we obtain

$$\frac{dr^*}{dm} = \underbrace{\left(\frac{-\lambda n(m)}{(n(m)+2)^2(1+\xi(m))}\right)}_{Z_0(m)}\hat{n}(m) + \underbrace{\left(\frac{-\lambda(n(m)+1)}{(n(m)+2)(1+\xi(m))^2}\right)}_{Z_1(m)}\hat{\xi}(m).$$

Let $\gamma \equiv \delta - \alpha$. Because $0 \le \alpha,\ \delta \le 1$, we have $-1 \le \gamma \le 1$. So, $\xi \equiv \xi(m) = m^\gamma$, and hence, $\hat{\xi}(m) = \gamma/m$. So, whenever $\gamma \le 0$, we have $\hat{\xi}(m) < 0$, and hence, the bank-customer effect, $Z_1(m)\hat{\xi}(m)$ is also negative. As a result, $dr/dm < 0$ and $d\theta/dm < 0$. Next, we consider the values of γ on $(0, 1]$ so that the bank-customer effect is positive. We first show that $\tilde{Z}(m) \equiv -Z_0(m)/Z_1(m)$ has a finite upper bound and is strictly decreasing in m for $m \ge 2$. Note that

$$\tilde{Z}(m) \equiv -\frac{Z_0(m)}{Z_1(m)} = \frac{n(m)}{(n(m)+1)(n(m)+2)}\cdot\left(1 + \frac{1}{\xi(m)}\right).$$

The first term of the above expression is strictly decreasing in $n(m)$ for $n(m) \ge 2$ (as there are at least two banks in the integrated market), and the second term is strictly decreasing in $\xi(m)$. Therefore, $\tilde{Z}'(m) < 0$, and hence, $\max \tilde{Z}(m) = \tilde{Z}(2)$. Given that $n(m) = \frac{1}{2}m(m+1)$, $n(2) = 3$ and $\xi(2) = 2^\gamma$. Thus,

$$\tilde{Z}(2) = 0.15\left(1 + \frac{1}{2^\gamma}\right).$$

Because $1 + 1/2^\gamma$ is strictly decreasing in γ, it reaches its maximum at $\gamma = 0$, which is given by 2. Therefore the upper bound on $\tilde{Z}(m)$ is given by $0.15 \times 2 = 0.3$. Now substituting the expressions of

$n(m)$ and $\xi(m)$ into the expression of $d\theta/dm$ above, we obtain

$$\frac{d\theta}{dm} = \frac{\lambda m^{\gamma-1}(2+m+m^2)}{2(1+m^{\gamma})^2(4+m+m^2)} \underbrace{\left(\gamma - \frac{2m(1+2m)}{(2+m+m^2)(4+m+m^2)} \cdot (1+m^{-\gamma})\right)}_{\gamma - h(m;\,\gamma)}.$$

Therefore, sign$[d\theta/dm]$ = sign$[\gamma - h(m;\,\gamma)]$. Note first that $h(m,\,\gamma) > 0$ for any $m > 0$, and hence, for $m \geq 2$. It is easy to show that $\gamma - h(m;\,\gamma)$ is strictly increasing in m for all $m \geq 2$, and strictly increasing in γ, i.e., $\gamma'' > \gamma'$ implies that $\gamma'' - h(m;\,\gamma'') > \gamma' - h(m;\,\gamma')$ for all $m \geq 2$ (as in Figure 7). Moreover,

$$\lim_{m\to\infty} \gamma - h(m;\,\gamma) = \gamma.$$

When the bank-customer effect is positive, i.e., $\gamma \in (0, 1]$, the sign of $d\theta/dm$ is ambiguous. Note that

$$\tilde{h}(\gamma) \equiv \gamma - h(2;\,\gamma) = \gamma - \frac{1}{4}\left(1 + 2^{-\gamma}\right)$$

with $\tilde{h}(0) = -0.5 < 0$ and $\tilde{h}(1) = 0.625 > 0$. Because $\tilde{h}(\gamma)$ is strictly increasing in γ, [by the Intermediate Value theorem] there is a unique $\tilde{\gamma} \in (0,\,1)$ such that $\tilde{h}(\gamma) \equiv \gamma - h(2;\,\gamma) \geq 0$ if and only if $\gamma \geq \tilde{\gamma}$. Thus, $\gamma > \tilde{\gamma}$ implies that $\gamma - h(m;\,\gamma) > 0$, and hence, $d\theta/dm > 0$. On the other hand, whenever $\gamma \in (0,\,\tilde{\gamma})$, there is a unique $\tilde{m}(\gamma) > 0$ with $\tilde{m}'(\gamma) > 0$ such that $\gamma - h(m;\,\gamma) < (>)\, 0$ according as $\gamma < (>)\, \tilde{\gamma}$. Therefore, $\theta(m)$ in U-shaped for all $\gamma \in (0,\,\tilde{\gamma})$.

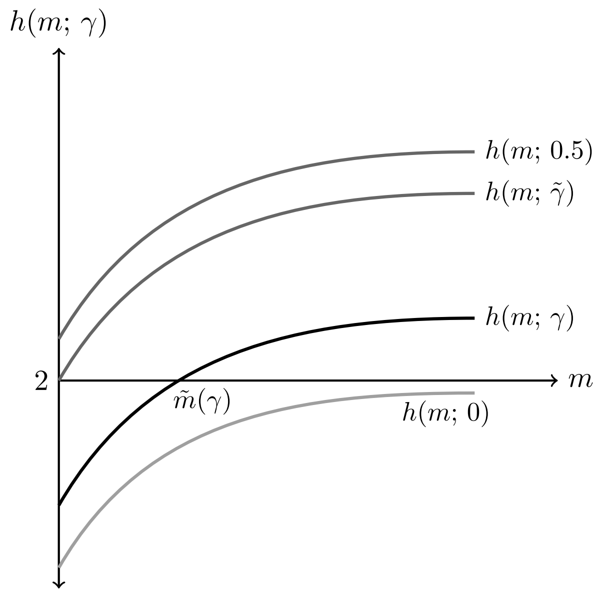

Figure 7: *For $\gamma \leq 0$, $\gamma - h(m;\,\gamma) < 0$, and hence, $d\theta^*/dm < 0$. For any $\gamma \in (0,\,\tilde{\gamma})$, $\gamma - h(m;\,\gamma)$ intersects the horizontal axis at a unique point $\tilde{m}(\gamma)$, that is, $\gamma - h(m;\,\gamma) < (>)\, 0$ if $m < (>)\, \tilde{m}(\gamma)$, and hence, $\theta^*(m)$ is U-shaped with respect to m. For any $\gamma \in (\tilde{\gamma},\,1]$, $\gamma - h(m;\,\gamma) > 0$, and hence, $d\theta^*/dm > 0$.*

Proof of Lemma 3. For market j, we have $R(L_j/a_j) = L_j/a_j$ and $r(L_j/b_j) = \lambda - L_j/b_j$, and hence, $P(L_j/b_j) = L_j/2\lambda b_j$ where $L_j = \sum_{i=1}^{n_j} L_{ij}$ is the aggregate loans in market j. Each bank i in market

j solves

$$\max_{L_{ij}} P(L_j/b_j)[r(L_j/b_j) - R(L_j/a_j) - c_j]L_{ij}.$$

In the symmetric equilibrium, we have $L_{ij} = L_j/n_j$. The first-order condition in the symmetric equilibrium is given by

$$\frac{L_j}{2\lambda b_j}\left(\lambda - \frac{L_j}{b_j} - \frac{L_j}{a_j} - c_j\right) + \frac{L_j}{n_j} \cdot \frac{(\lambda - c_j)a_j b_j - 2(a_j + b_j)L_j}{2\lambda a_j b_j^2} = 0.$$

Solving the above equation, we obtain

$$L_j = \frac{(\lambda - c_j)a_j b_j (n_j + 1)}{(a_j + b_j)(n_j + 2)} . \tag{57}$$

The equilibrium risk-shifting is given by

$$\theta_j = \lambda - \frac{L_j}{2\lambda b_j} = \lambda - \frac{(\lambda - c_j)a_j(n_j + 1)}{2(a_j + b_j)(n_j + 2)} . \tag{58}$$

Substituting $a_1 = a_2 = a$, $b_1 = b$, $b_2 = \beta b$, $n_1 = n_2 = n$, $c_1 = 0$ and $c_2 = c$ in the last expression, we obtain θ_1 and θ_2 in Lemma 3. Note that

$$\theta_2 - \theta_1 = \frac{a(n+1)[(a+b)c + \lambda b(\beta - 1)]}{2(a+b)(a+\beta b)(n+2)} .$$

The above expression is strictly positive for $\beta > 1$, and hence, $\theta_2 > \theta_1$.

Pairwise mergers between like banks. We begin with the following useful result regarding the profitability of mergers between two banks with the same operating costs in any given market.

Lemma 7 *Consider any given market with deposit supply schedule, $R(D/A) = D/A$, and loan demand schedule, $r(L/B) = \lambda - L/B$, where A, $B > 0$ represent the size of the deposit and loan markets, respectively. Suppose further that a merger between two banks with the same operating costs yields no efficiency gains.*

(a) *If the market consists only of banks with the same operating costs, a merger between a pair of banks is profitable only if the pre-merger market structure is a duopoly (i.e., $n = 2$).*

(b) *If there are same number of low-cost and high-cost banks, then any merger between a pair of banks with same operating costs is never profitable.*

The first part of Lemma 7 asserts that mergers in any given market are profitable when there are only two banks. In other words, pairwise mergers must result in monopoly banking in order to be profitable.

Proof. The detailed calculations for the proof are provided in Appendix C (*Mathematica* codes). We provide here only the sketch of the proof. We first prove part (a). Consider any market with parameters $\lambda > 1$, $A > 0$, $B > 0$, $n \geq 2$ and $z \in \{0, c\}$ (each bank's constant marginal cost). Using the expression in (57), the equilibrium aggregate loans are given by

$$L = \frac{(\lambda - z)AB(n+1)}{(A+B)(n+2)}$$

which yields the (common) expected profit for each bank:

$$\pi = \underbrace{\frac{L}{2\lambda B}}_{P(L)} \underbrace{\left(\lambda - \frac{L}{b} - \frac{L}{a} - z\right)}_{r(L)-R(L)-z} \cdot \frac{L}{n}.$$

The above is the common equilibrium profit level of each bank when there are n banks, each with marginal cost z. Let $\pi_j(n_l, n_h)$ be the equilibrium profit of each type $j = 1, 2$ bank when there are n_l low-cost and n_h high-cost banks in the market. So, in a market consisting of only low-cost bank, we have $z = 0$, and the common profit level is given by $\pi_1(n, 0)$. On the other hand, for a high-cost market, we have $z = c$, and the common profit level, $\pi_2(0, n)$. If any pair of banks merge, then the market consists of $n-1$ identical banks. So, for a two-bank merger to be profitable we require that $\pi_1(n-1, 0) > \pi_1(n, n) + \pi_1(n, n) = 2\pi_(n, n)$ (if market 1 is considered), or $\pi_2(0, n-1) > 2\pi_2(0, n)$ (if market 2 is considered) so that each bank in the merged entity obtains at least a profit strictly higher than $\pi_1(n, 0)$ (or $\pi_2(0, n)$), say $\frac{1}{2}[\pi_1(n-1, 0) - 2\pi_1(n, 0)]$ (or $\frac{1}{2}[\pi_2(0, n-1) - 2\pi_2(0, n)]$). We show that the above two strict inequalities does not hold if there are at least 3 banks in a given market, i.e., $n \geq 3$, and it holds only for $n = 2$, i.e., the merger must result in a monopoly banking.[49]

The proof of part (b) is similar. If there are no mergers, then each type-j bank consumes $\pi_j(n, n)$. If two low-cost banks merge, each in the merged entity (as well as each of the $n-2$ same type rivals) obtains $\pi_1(n-1, n)$. On the other hand, if two high-cost banks merger, each in the merged entity (as well as each of the $n-2$ same type rivals) obtains $\pi_2(n, n-1)$. So, for the merger of two low-cost banks to be profitable, we require $\pi_1(n-1, n) > 2\pi_1(n, n)$, and for the merger of two high-cost banks to be profitable, we require $\pi_2(n, n-1) > 2\pi_2(n, n)$. We show that none of the last two inequalities holds if we have $n \geq 2$, i.e., there are at least two banks of each cost type (the requirement to form a merger between two banks of the same type). □

Proof of Lemma 4. Note first that in the integrated market, we have $p(\theta) = 1 - \theta/\lambda$ and $r(L/(b_1 + b_2)) = \lambda - L/(1+\beta)b$, which together imply that the probability of success as a function of the aggregate loan volume is given by $P(L/(b_1+b_2)) = L/2(1+\beta)b\lambda$. Moreover the equilibrium must respect $L = L_1 + L_2 = D$. The IME is characterized by within-group symmetry, i.e., $L_{ij} = L_j/n$ because there are n each type of banks. Therefore, the first-order condition of each type-j bank i yields

$$(L_1+L_2)\left(\lambda - \left(\frac{1}{2a} + \frac{1}{(1+\beta)b}\right)(L_1+L_2)\right) + \frac{L_1}{n}\left(\lambda - \left(\frac{1}{a} + \frac{2}{(1+\beta)b}\right)(L_1+L_2)\right) = 0,$$

$$(L_1+L_2)\left(\lambda - c - \left(\frac{1}{2a} + \frac{1}{(1+\beta)b}\right)(L_1+L_2)\right) + \frac{L_2}{n}\left(\lambda - c - \left(\frac{1}{a} + \frac{2}{(1+\beta)b}\right)(L_1+L_2)\right) = 0.$$

[49] That mergers may reduce the profits of the merged entity is a common feature of Cournot models. In particular, Salant, Switzer, and Reynolds (1983) have shown that for mergers to be profitable under Cournot competition, a critical fraction (about 80%) of the firms must merge.

The above system of non-linear equations have three sets of solutions in (L_1, L_2). The first set is discarded because it has $L_1 = L_2 = 0$. The second set is also discarded because $L_1 < 0$ for $\lambda > 1$, $a, b > 0$, $\beta \geq 1$, $n \geq 2$ and $0 < c \leq \lambda$. The final expressions for L_1 and L_2 in the third set of the solutions are very cumbersome, so we omit them. The third set of solutions (L_1, L_2) yields

$$L^* = \frac{ab(1+\beta)\left\{(2\lambda - c)(3n+2) + \sqrt{c^2(3n+2)^2 + 4\lambda(\lambda - c)n^2}\right\}}{4(2a + (1+\beta)b)(n+1)}.$$

It turns out that $L_2^* > 0$ if $c < \lambda/(n+2) \equiv \bar{c}$. The expression of the equilibrium risk-shifting is obtained from

$$\theta^* = \lambda - \frac{L^*}{2(1+\beta)b}.$$

Proof of Proposition 6. The detailed calculations are in Appendix C. It is easy to show that $\theta_2 > \theta^*$ under $\lambda > 1$, $a, b > 0$, $\beta > 1$, $n \geq 2$ and $0 < c < \bar{c}$. The above holds because $L^* > L_2$. We next compare θ_1 and θ^*. Write θ^* as $\theta^*(\beta, c)$. It is immediate to show that $\theta^*(\beta, c)$ is strictly increasing in both the arguments. On the other hand, θ_1 depends neither on β nor on c. Consider $\theta_1 - \theta^*(\beta, c) = 0$ which defines implicitly β as a function of c, which is denoted by $\tilde{\beta}(c)$. Differentiating $\theta_1 - \theta^*(\tilde{\beta}(c), c) = 0$ with respect to c we obtain

$$\tilde{\beta}'(c) = -\frac{\partial\theta^*/\partial c}{\partial\theta^*/\partial\beta}.$$

Because both the partial derivatives are positive, we have $\tilde{\beta}'(c) < 0$. Next, it follows from the expressions of θ_1 and $\theta^*(\beta, c)$ that

$$\theta_1 - \theta^*(\beta, 0) = 0 \quad \Longrightarrow \quad \beta = 1 + \frac{(a+b)n}{b(n+1)^2} \equiv \tilde{\beta}(0) > 1.$$

On the other hand, given that $\bar{c} = \lambda/(n+2)$,

$$\theta_1 - \theta^*(\beta, \bar{c}) = 0 \quad \Longrightarrow \quad \beta = 1 \equiv \tilde{\beta}(\bar{c}).$$

This completes the proof of the proposition.

Proof of Lemma 5. Calculations for the first part is in Appendix B where we show that $\pi_M > \pi_1^* + \pi_2^*$. Note that, in a merger between a low-cost bank and a high-cost bank, the gain to the merged entity is $\pi_M - (\pi_1^* + \pi_2^*)$. Under the following surplus division rule

$$\tilde{\pi}_1 = \pi_1^* + \frac{1}{2}[\pi_M - (\pi_1^* + \pi_2^*)] \quad \text{and} \quad \tilde{\pi}_2 = \pi_2^* + \frac{1}{2}[\pi_M - (\pi_1^* + \pi_2^*)],$$

both banks gain because $\tilde{\pi}_1 > \pi_1^*$ and $\tilde{\pi}_2 > \pi_2^*$, and hence, such mergers are profitable.

Next, for the merger IME, $P(L/(b_1 + b_2)) = \frac{L}{2\lambda(1+\beta)b}$. So, each bank i chooses L_i to maximize the following expected profit

$$P(L/(b_1+b_2))(r(L/(b_1+b_2)) - R(L/(a_1+a_2)))L_i = \frac{L}{2\lambda(1+\beta)b}\left(\lambda - \frac{L}{(1+\beta)b} - \frac{L}{2a}\right)L_i.$$

The first-order condition evaluated at $L_i = L/n$ (symmetric equilibrium) yields

$$L_M = \frac{2\lambda a(1+\beta)b(n+1)}{(2a+(1+\beta)b)(n+2)}.$$

The expression for θ_M is obtained by substituting the above expression into $\theta_M = \lambda - \frac{L_M}{2\lambda(1+\beta)b}$.

Proof of Proposition 7. Note that

$$\begin{aligned}
\theta_1 - \theta_M &= -\frac{\lambda ab(\beta-1)(n+1)}{2(a+b)(2a+(1+\beta)b)(n+2)} < 0 \quad \text{for } \beta > 1, \\
\theta_2 - \theta_M &= \frac{a[(2a+(1+\beta)b)c + \lambda b(\beta-1)](n+1)}{2(a+\beta b)(2a+(1+\beta)b)(n+2)} > 0 \quad \text{for } \beta > 1,
\end{aligned}$$

which prove the proposition.

Proof of Proposition 8. Note that

$$\theta_M - \theta^* = \frac{a}{8(2a+(1+\beta)b)(n+1)}\left[\sqrt{c^2(3n+2)^2 + 4n^2\lambda(\lambda-c)} - \left(c(3n+2) + \frac{2\lambda n^2}{n+2}\right)\right].$$

So,

$$\begin{aligned}
& \theta_M > \theta^* \\
\iff & \sqrt{c^2(3n+2)^2 + 4n^2\lambda(\lambda-c)} > c(3n+2) + \frac{2\lambda n^2}{n+2} \\
\iff & \lambda - c > \lambda\left(\frac{n}{n+2}\right)^2 + \frac{c(3n+2)}{n+2} \\
\iff & c < \frac{\lambda(n+2)}{4(n+1)}\left[1 - \left(\frac{n}{n+2}\right)^2\right] = \frac{\lambda}{n+2} \equiv \bar{c}.
\end{aligned}$$

Proof of Proposition 9. We provide the sketch of the proof. All the calculations are in Appendix B. The IME aggregate loans, $L^*(\beta, \sigma)$ is determined from the first-order condition of each bank's maximization problem evaluated at $L_i = L/2n$, which is given by

$$q(L)(r(L/(b_1+b_2)) - R(L/(a_1+a_2))) + \frac{L}{2n}\cdot\frac{d}{dL}[q(L)(r(L/(b_1+b_2)) - R(L/(a_1+a_2)))] = 0.$$

The IME risk-shifting is given by $\theta^*(\beta, \sigma) = \lambda - \frac{L^*(\beta,\sigma)}{2(1+\beta)b}$. We next show that $\theta^*(\beta, \sigma)$ are strictly increasing in both arguments. We next compare $\theta_1 - \theta^*(\beta, \sigma)$. To prove the existence of a unique $\tilde{\beta}(\sigma)$ for each σ, we proceed as follows. We first show that $\theta^*(1, \sigma) > 0$ and $\lim_{\beta\to\infty} \theta^*(\beta, \sigma) < 0$. Hence, $\tilde{\beta}(\sigma)$ exists. Because θ_1 does not depend on β, $\theta_1 - \theta^*(\beta, \sigma)$ is strictly decreasing in β as $\theta^*(\beta, \sigma)$ is strictly increasing in β. Therefore, $\tilde{\beta}(\sigma)$ is unique for each σ, and is defined by

$$\theta_1 - \theta^*(\tilde{\beta}(\sigma), \sigma) = 0.$$

Differentiating the above with respect to σ we obtain

$$\tilde{\beta}'(\sigma) = -\frac{\partial\theta^*/\partial\sigma}{\partial\theta^*/\partial\beta} < 0$$

because both the numerator and the denominator are strictly positive. It is also easy to show that $\theta_2 > \theta^*(\beta, \sigma)$ for all (β, σ) (see Appendix C). This completes the proof of the proposition.

Proof of Proposition 10. To save on notations, write $a \equiv a(m)$, $b \equiv b(m)$ and $n \equiv n(m)$. Given that $R(D/a) = D/a$ and $r(L/b) = \lambda - (L/b)$, the first-order conditions associated with the maximization problem (39) with respect to L_i and D_i, under symmetry, i.e., $L_i = L/n$ and $D_i = D/n$ for all i, are given by

$$a(n+2)L^2 - ab(\lambda-\rho)(n+1)L = b(a\rho - D)D, \tag{59}$$

$$a\rho n = (n+1)D, \tag{60}$$

respectively. Solving the above system, we obtain the aggregate loans, L and deposits, D in equilibrium. Using $L = b(\lambda - r)$ and $D = aR$, we get (40) and (41). The equilibrium loan rate is thus given by:

$$r(a,\, b,\, n,\, \rho) = \lambda - \frac{L}{b} = \lambda - \frac{b(\lambda-\rho)(n+1)^2 + \sqrt{b^2(\lambda-\rho)^2(n+1)^4 + 4ab\rho^2 n(n+2)}}{2b(n+1)(n+2)}\,. \tag{61}$$

The equilibrium deposit rate, on the other hand, is given by:

$$R(n,\, \rho) = \frac{D}{a} = \frac{\rho n}{n+1}\,.$$

Note first that our assumption of $m \geq 2$ implies that $\min\{n(m)\} = n(2) \geq 2$. We require that $R(n,\, \rho) \leq \rho \leq r(a,\, b,\, n,\, \rho)$. The first inequality holds for any $n \geq 2$. The second inequality is written as

$$g(\rho) \equiv \frac{b(\lambda-\rho)^2}{a\rho^2} \geq \frac{n}{(n+1)^2}.$$

The last inequality is equivalent to

$$\rho \leq \frac{\lambda b(n+1)}{b(n+1) + \sqrt{an}} \equiv \bar{\rho} \leq \lambda.$$

Note that $\bar{\rho}$ is increasing in n and $\lim_{n\to\infty} \bar{\rho} = \lambda$.

We first prove part (a). Fix $n(m) = \bar{n}$ so that $n'(m) = 0$, i.e., the bank-competitor effect is zero. Note that

$$\frac{\partial r}{\partial a} = -\frac{n\rho^2}{(n+1)\sqrt{b^2(n+1)^4(\lambda-\rho)^2 + 4abn(n+2)\rho^2}} < 0,$$

$$\frac{\partial r}{\partial b} = \frac{an\rho^2}{b(n+1)\sqrt{b^2(n+1)^4(\lambda-\rho)^2 + 4abn(n+2)\rho^2}} = -\frac{a}{b}\cdot\frac{\partial r}{\partial a} > 0.$$

Then, differentiating r with respect to m, we obtain

$$\frac{dr^*}{dm} = \frac{\partial r}{\partial a} \cdot a'(m) + \frac{\partial r}{\partial b} \cdot b'(m) = \left\{ b'(m) - \frac{b(m)}{a(m)} \cdot a'(m) \right\} \frac{\partial r}{\partial b}$$

The above expression is positive (negative) according as $\hat{b}(m) \gtrless \hat{a}(m)$, i.e., the bank-customer effect is positive (negative).

We next prove part (b). By implicitly differentiating (40), we obtain

$$\frac{\partial r}{\partial R} = \frac{a(2R - \rho)}{b\{2(\lambda - r)(n+2) - (\lambda - \rho)(n+1)\}} > 0,$$
$$\frac{\partial r}{\partial n} = -\frac{(\lambda - r)(r - \rho)}{2(\lambda - r)(n+2) - (\lambda - \rho)(n+1)} < 0.$$

Using the expression of r in (61), it is easy to show that the denominator of each of the above two expressions is positive. On the other hand, from (41), it follows that

$$\frac{dR^*}{dn} = \frac{\rho}{(n+1)^2} = \frac{R^*}{n(n+1)} > 0.$$

Note that the sign of the bank-competitor effect is completely determined by that of dr/dn, which is given by:

$$\frac{dr^*}{dn} = \underbrace{\frac{\partial r}{\partial n}}_{\text{direct effect}} + \underbrace{\frac{\partial r}{\partial R} \cdot \frac{dR}{dn}}_{\text{indirect effect}}.$$

We first show that both the terms in the right-hand-side of the above expression is increasing in ρ. Substituting the equilibrium values of r^* and R^* into the expression of $\partial r/\partial n$, and differentiating with respect to ρ we obtain the following:[50]

$$\text{sign}\left[\frac{\partial}{\partial \rho}\left(\frac{\partial r}{\partial n}\right)\right]$$
$$= \text{sign}\left[1 + g(a, b, n, \rho)\right],$$

where

$$g \equiv \frac{b^2(\lambda-\rho)^3(n+1)^7 + 8a^2\rho^3 n^2(n+2)^2 + 2ab\rho(\lambda-\rho)n(n+1)^3(n+2)(\lambda(n+5) + (\lambda-\rho)(n-1))}{b^{\frac{1}{2}}(b(\lambda-\rho)^2(n+1)^4 + 4a\rho^2 n(n+2))^{\frac{3}{2}}},$$

which is strictly positive. Clearly, $\partial(\partial r/\partial n)/\partial \rho < 0$, i.e., the negative direct effect dampens as ρ increases. As far as the indirect effect is concerned, using $R = \rho n/(n+1)$ in $\partial r/\partial R$ and dR/dn, and differentiating the indirect effect with respect to ρ, we obtain

$$\text{sign}\left[\frac{\partial}{\partial \rho}\left(\frac{\partial r}{\partial R} \cdot \frac{dR}{dn}\right)\right] = \text{sign}\left[b(\lambda-\rho)(2\lambda-\rho)(n+1)^4 - 2a\rho^2 n(n+2)(n-3)\right],$$

which is also positive for any $a > 0$, $b > 0$, $\lambda > 0$ and $0 < \rho \leq \lambda$. Therefore, dr/dn is strictly

[50] We omit the unmanageable algebraic expressions. The *Mathematica* codes are available upon request.

increasing in ρ. Let $f(\rho)$ denote dr/dn evaluated at ρ. Note that

$$f(0) = \frac{\lambda(b+1)\{b(n+1) - (n+3)\}}{4b(n+2)^2} \quad \text{and} \quad f(\lambda) = \frac{\lambda^4 a^2 bn\{n(n+1) + b(n^2+n-2)\}}{2(n+1)^2[\lambda^2 abn(n+2)]^{3/2}} > 0.$$

The sign of $f(0)$ ambiguous. There are the following possible cases. First, if $f(0) \geq 0$, i.e., $b \geq \frac{n+3}{n+1}$, then $f(\rho) \geq 0$ for all $\rho \geq 0$. Then, set $\rho^* = 0$. Thus, the equilibrium loan rate is increasing for all possible values of the interbank rate. Next, consider the case when $f(0) < 0$ i.e., $b < \frac{n+3}{n+1}$. Because $f(\lambda) > 0$ and $f'(\rho) > 0$, it follows from the Intermediate Value theorem, there is a unique $\rho^* \in (0, \lambda)$ such that $dr/dn < 0$ if and only if $\rho < \rho^*$. If $\rho^* \geq \bar{\rho}$, then set $\rho^* = \bar{\rho}$, i.e., $dr/dn < 0$ for all $\rho \in [0, \bar{\rho}]$. Otherwise, $\rho^* < \bar{\rho}$, and hence, the bank-competitor effect is positive if and only if $\rho > \rho^*$. This case is more likely for large $n(m)$ because $\bar{\rho} \to \lambda$ and $n \to \infty$ and $f(\lambda) > 0$. This completes the proof of part (b).

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