Bank financing modes and monitoring: A two-sided matching theory of bank and firm capital*

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Abstract

We consider a model of endogenous matching between heterogeneous banks and firms. Lending relationships are subject to incentive problems in the choice of bank monitoring, and banks can choose between two funding modes – financing through insured deposits and financing through loan securitization. In the equilibrium of the market more capitalized banks fund more capitalized firms following a positive assortative matching pattern. Matches with low aggregate assets choose deposit financing, while those with high aggregate assets opt for loan securitization. Thus, the type of banking institution (traditional or shadow banking) becomes endogenous in the market equilibrium. As a result, both the equilibrium bank monitoring and loan rates are non-monotonic in firm capital. Moreover, we show that a rise in the risk-free rate may have ambiguous implications for the relative size of the banking sector that originates-to-distribute.

JEL codes: C78, D82, G11.
Key words: Bank and firm capital; bank monitoring; bank funding modes.

1 Introduction

One strand of the extant literature on relationship lending postulates that bank (equity) capital enhances a bank's incentives to screen and monitor her borrowers, and hence, loan performance through a lower probability of default (e.g. Hölmstrom and Tirole, 1997; Allen, Carletti, and Marquez, 2011; Mehran and Thakor, 2011; Schwert, 2016). A well capitalized bank has stronger incentives to monitor because insolvency is exorbitantly costly for the shareholders. Thus, in the absence of bailout, one may observe a positive association between bank capital and monitoring.[1] Borrower collateral or capital, on the other

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[1] A competing view (e.g. Diamond and Rajan, 2001) asserts that if a larger portion of the investment is financed through uninsured bank deposits, the threat of bank run may discipline a bank. Consequently, this establishes a negative association between bank equity and monitoring, which is a liability-side story as opposed to the asset-side explanation of a positive association analyzed in the aforementioned papers.

hand, may enhance a lender's incentives to monitor (e.g. Rajan and Winton, 1995; Ono and Uesugi, 2009; Cerqueiro, Ongena, and Roszbach, 2016). When the value of the assets pledged is at risk, in the presence of other claimants, monitoring adds value because it allows the lender to demand additional collateral. One of the main objectives of our paper is to analyze how bank capital and firm capital jointly influence bank monitoring and the terms of the loan contracts.

We argue that the extent to which bank capital and firm capital affect a bank's monitoring incentives also depend on the type of institution a bank belongs to. In particular, we analyze two types of banking institutions – the 'traditional banking' where banks *originate-to-hold* (OTH) and finance loans through fully insured deposits, and the non-traditional 'shadow banking' where banks *originate-to-distribute* (OTD) and fund loans through securitization without having access to any safety nets such as deposit insurance. At this juncture, it is worth clarifying that we focus only on one form of shadow banking – namely, 'loan securitization' or 'unsecured debt' in which the buyers of such securities have claims on a bank's total asset in the event of insolvency, and not on other forms such as money-market mutual funds and repos. Thus, we also shed light on how banks choose the financing mode optimally depending on the levels of bank capital and firm capital.

We develop a simple two-sided *assignment game* (e.g. Shapley and Shubik, 1971) between banks (lenders) and firms (borrowers). In the competitive loan market, both the banks and the firms are risk neutral and heterogeneous with respect to their capital endowments. Each firm owns a project of fixed size which yields a stochastic cash flow (success and failure), and borrows the dollar required from one bank by writing a binding contract. A bank monitors her borrower at a cost, which enhances the probability of success. Non-verifiability of bank monitoring gives rise to a moral hazard problem in the choice of monitoring effort. A loan contract, which specifies a loan rate to be paid to the bank and a financing mode, is subject to limited liability. Therefore, firm capital serves as pledgeable collateral because in the event that a project fails the bank collects the entire firm capital. Banks do not invest their capital in loans, rather raise the project costs from risk averse market investors. Each bank has two modes of financing available. The loan originator can either finance a firm's project with fully insured deposits by paying a fixed insurance premium, or she can fund investment through loan sales. Under deposit financing, the depositors receive the exogenous risk-free rate because the deposits are fully insured, and each bank keeps its capital irrespective of the realized cash flow. Therefore, the optimal monitoring in any arbitrary bank-firm match does not depend on the level of bank capital. Under loan securitization, on the other hand, the contract between a bank and her investors is an unsecured debt contract. When the project of the firm succeeds the investors receive a higher return than the risk-free rate; otherwise, in the event of bank insolvency, the total assets of a bank, i.e., firm capital plus bank capital, are collected by the creditors. Therefore, the more capitalized banks monitor more intensely their borrowers to avoid insolvency. Thus, each bank faces a clear trade-off between the two funding modes. Under deposit financing, a bank keeps its capital but pays an insurance premium. By contrast, under loan securitization, a bank does not incur any such costs, but looses her capital in case she is insolvent.

When heterogeneous banks and firms match with each other endogenously, in the equilibrium of the market more capitalized banks finance more capitalized firms following a positively assortative matching pattern. This result is driven by an endogenous *single-crossing* condition resulting from the optimal contracting problem associated with each arbitrary match, which asserts that if a bank with a given amount of capital is indifferent between two firms, then a bank with greater amount of capital prefers to finance the firm with higher collateral. In other words, bank capital and firm capital are complementary under any mode of bank financing. Moreover, the equilibrium bank financing mode is monotonic in bank and firm capital. It turns out that the marginal gain in surplus by switching from deposit financing to loan securitization is increasing in both bank capital and firm capital. In other words, both higher levels of bank capital and firm capital have comparative advantages under loan sales. Therefore in equilibrium, not only the more capitalized banks lend to the more capitalized firms, but they also opt for financing through loan securitization. By contrast, the equilibrium matches with low aggregate assets choose to fund through insured deposits. As a result, the type of banking institution (traditional or shadow banking) becomes endogenous in the loan market equilibrium.

Because of the monotonicity of the bank funding modes, the equilibrium monitoring intensity and loan rate are non-monotonic in firm capital. When the less capitalized banks and firms choose financing through insured deposits, the incentive compatible level of monitoring is independent of the level of bank capital but is decreasing in the level of firm capital. Therefore, both the equilibrium monitoring and the loan rate, which is the marginal compensation of a bank for an additional unit of monitoring, are monotonically decreasing up to the threshold level of firm capital beyond which all matches switch to loan securitization. When all matches choose loan securitization as the optimal financing mode, the incentive compatible level of monitoring is monotonically increasing in both bank capital and firm capital. Because higher levels of bank capital is associated with higher levels of firm capital via a positive assortative matching, the equilibrium monitoring and loan rate are increasing in firm collateral. Note that, given the exogenous risk-free rate in our model, the behavior of the equilibrium loan rate is equivalent to that of the equilibrium loan spread.

In the present paper we aim at offering a framework where the endogenous matching between heterogeneous banks and firms influence the optimal choice of monitoring and bank funding modes. A plethora of empirical works analyze the roles of heterogeneity and endogenous matching between lenders and borrowers in shaping the optimal incentive contracts (e.g. Sørensen, 2007; Chen and Song, 2013; Schwert, 2016). It is worth noting that, in our model, the nature of incentive problem in a lending relationship is crucial for predicting the equilibrium matching patterns. For example, had deposit financing been the only available financing mode, the optimal monitoring in any match would be independent of the level of bank capital, and hence, the equilibrium matching would have been a random matching (see Proposition 5). On the other hand, when loan securitization is the only funding mode available, incentive compatibility requires that bank monitoring is monotonically increasing in the level of bank capital. As a result, the equilibrium matching between bank and firm capital is positive assortative.

The analysis of Martínez-Miera and Repullo (2017) implies that a higher risk-free rate following an exogenous shift in the supply of savings or in the demand for investment induces a smaller relative size of the shadow banking sector. However, empirical evidence suggests that a higher risk-free rate resulting from a contractionary monetary policy may in fact expand the shadow banking sector (e.g. Nelson, Pinter, and Theodoridis, 2015; Mazelis, 2016). This discord between the theoretical prediction and empirical findings motivates us to analyze the correlation between the risk-free rate and loan securitization. A higher risk-free rate decreases the equilibrium utilities of all types of firms because both modes of bank financing are now costlier. Since under loan securitization a solvent bank has to pay more to her investors, the decrease in utility under loan securitization is higher. As a result, the relative size of the shadow banking sector decreases. This we refer to as the *income effect* of an increase in the risk-free rate. On the other hand, an increase in the risk-free rate induces banks to substitute one mode of funding by the other which depends on the relative costs of the two modes of financing, which we refer to as the *substitution effect*. If, because of the substitution effect, some or all matches prefer to switch to loan securitization, the two effects may pull in opposite directions, and hence, the effect of an increase in the risk-free rate on loan securitization may be ambiguous. We show that the risk-free rate can be either *negatively* or *positively* correlated with the relative size of the shadow banking sector.

1.1 Related literature

Our paper contributes to the recent literature on relationship lending where the level of bank capital influences banks' monitoring incentives. Mehran and Thakor (2011) argue that greater equity capital reduces the probability of future bank closure (by the regulator). This in turn strengthens a bank's incentives to monitor so that the bank can reap the marginal benefit of monitoring in the form of increased value of her loan portfolio. Allen et al. (2011) show that banks tend to hold capital in excess of that is required by the regulatory standards to commit to surplus enhancing monitoring activities. Apart from the value enhancing monitoring activities, ex-ante screening by banks are also important for financial stability. In the context of mortgage-backed securities, Purnanandam (2011) finds empirical evidence that more capital constrained loan originators do not spend sufficient resources on screening, and tend to sell poor quality loans.

The result on the monotonicity of bank funding modes in our paper is related to those obtained by Greenbaum and Thakor (1987) and Martínez-Miera and Repullo (2017). Greenbaum and Thakor (1987) consider a partial equilibrium model of loan contract between a bank and its borrower, and analyze the optimal choice between financing through uninsured deposits and loan securitization. They conclude that under asymmetric information about the loan quality, as defined by the probability of solvency, and in the absence of regulatory intervention, it is optimal to securitize the good quality loans, and the bad quality loans are financed through deposits.[2] Apart from being a general equilibrium framework, the main difference between Greenbaum and Thakor (1987) and our paper is that we analyze how the level of bank capital influences monitoring incentives, and how the relative importance of bank monitoring under different institutional arrangements induces the optimal choice of bank funding modes. Greenbaum and Thakor (1987) do not analyze endogenous monitoring,[3] and they are concerned with the choice between financing through uninsured deposits and loan sales, whereas we analyze the choice between funding through insured deposits and loan securitization. Martínez-Miera and Repullo (2017) analyze the optimal incentive contracting between banks and their creditors, and show that banks that fund the safer firms would choose not to monitor (shadow banking), while riskier firms can obtain financing from the monitoring banks (traditional banking) because firm quality (safe or risky) and bank monitoring are substitutes, i.e., safer borrowers require less monitoring. The main differences between the present paper and that of Martínez-Miera and Repullo (2017) are that we consider banks heterogeneous with respect to capital endowment, and we analyze the optimal loan contracts between lenders and borrowers as opposed to contracts between banks and their creditors. In our model, differences in capital imply differences in bank monitoring. Moreover, the choice between various modes of financing is part of the loan contract design. Martínez-Miera and Repullo (2017) further show that a rise in the risk-free rate decreases the relative size of the shadow banking sector; whereas in our model, the effect of an exogenous increase in the risk-free rate on the relative size of the shadow banking sector is ambiguous.

The endogenous matching game between the lenders and the borrowers in the present paper is con-

[2] Moreover, in analyzing the effect of regulatory intervention, Greenbaum and Thakor (1987) show, under full deposit insurance, that sufficiently low bank capital requirements combined with sufficiently low regulatory costs make deposit financing the dominant choice over securitization, regardless of the borrower quality.

[3] In Greenbaum and Thakor (1987), a single bank always screens her borrower at a fixed cost.

sistent with the recent empirical evidence on the endogenous matching in the venture capital market (e.g. Sørensen, 2007) and the syndicated loan market analyzed by (e.g. Schwert, 2016).[4] In that sense, we extend the assignment game of Shapley and Shubik (1971), which considers the assignment of buyers and sellers, and analyze type-dependent prices in the stable allocations, to relationships permeated by incentive problems. Our approach is thus closer to that of Legros and Newman (2007) who consider the assignment game in an environment with imperfect transferability where the Pareto or bargaining frontier associated with each match is non-linear. In our paper, due to the choice between the two modes of bank financing the bargaining frontier is not only non-linear, but also non-concave. Therefore, our methodology of establishing a positive assortative matching under a non-concave bargaining frontier may be of independent interest.[5]

2 The model

2.1 Matching between bank and firm capital

The economy, which spans three dates $t = 0, 1, 2$, consists of three classes of agents – firms (borrowers), banks (lenders), and market investors. On one side of the credit market, there is a continuum $F = [0, 1]$ of risk neutral firms, and on the other side there is a continuum $B = [0, 1]$ of risk neutral banks. The firms own an ex-ante identical start-up project apiece whose initial outlay is \$1, and yields, at the end of date 2, a stochastic but verifiable cash flow $Q > 1$ in the event of "success", and 0 otherwise. Each firm and each bank are endowed with a fixed amount of capital w and k, respectively. The firms are heterogeneous with respect to w, which we refer to as their "types". Let $G_W(w)$ be the fraction of firms with capital less than w, i.e., $G_W(w)$ represents the cumulative distribution function of w on the support $W = [w_{min}, w_{max}]$ with $w_{min} \geq 0$. We assume that the corresponding density function $g_W(w) > 0$ for all $w \in W$. Similarly, the banks differ in k which has a distribution function $G_K(k)$ on the support $K = [k_{min}, k_{max}]$ with $k_{min} \geq 0$, and the corresponding density function is $g_K(k) > 0$ for all $k \in K$.

Each bank finances a firm's project by raising \$1 from risk averse market investors. At date 0, when a bank of type k agrees to finance a firm of type w, a 'match' or 'partnership' (k, w) is formed. Formally, a partnership is formed via a one-to-one matching function $\mu : W \to K$ which assigns to each $w \in W$ a bank capital level $\mu(w) \in K$. Let $\nu \equiv \mu^{-1}$ denote the inverse matching function. At $t = 1$, each bank exerts a non-verifiable costly *monitoring effort* $m \in [0, 1]$ which determines the probability of success. As in Allen et al. (2011), we assume that the probability of obtaining high cash flow Q is given by m which we refer to as the *monitoring intensity*. The cost of monitoring is given by $c(m)$ which is strictly increasing and convex for $m > 0$ with $c(0) = c'(0) = 0$. In order to obtain closed from solutions to the optimal contracting problems, we assume without loss of generality that $c(m) = m^2/2$. Because monitoring effort is costly and not publicly verifiability, it gives rise to a moral hazard problem in the choice of bank monitoring.

A bank charges contingent loan rates $R(Q)$ and $R(0)$ to a firm for lending a dollar.[6] We assume

[4] Sørensen (2007) finds evidence of a positive assortative matching between firm quality and VC experience.

[5] In different contexts, Alonso-Paulí and Pérez-Castrillo (2012) and Dam and Serfes (2017) analyze assortative matching with non-concave bargaining frontiers.

[6] We could have assumed instead that a type w firm invests w and the bank lends $1 - w$. All our results would hold good (qualitatively) under this modification.

limited liability of the borrowers, and hence $R(0) = w$, i.e., firm capital serves as collateral in the loan contract. We also write $R = R(Q)$.

2.2 Bank financing modes

To each bank there are two modes of funding available. A bank can fund the investment by fully insured deposits, a regime denoted by D. Deposits are supplied perfectly elastically at a fixed deposit rate $r \geq 1$. Because deposits are fully insured, the deposit rate equals the risk-free rate. All banks pay a flat insurance premium $\alpha \geq 0$.[7]

In the second financing mode, a bank can finance a firm through loan securitization, a regime denoted by S.[8] In this regime, the investors collectively write a contingent contract with a type k bank, which finances a type w firm, in which they receive $r_S(Q)$ and $r_S(0)$ with $r_S(Q) > r_S(0)$. In the case when bank is insolvent, i.e., the realized cash flow is 0, the total of firm and bank assets serves as the security for the investors. Bank's limited liability implies that $r_S(0) = w + k$, i.e., the contract is a risky debt contract. Thus, the buyers of securities must receive a rate of return strictly higher than the risk-free rate, and hence, we assume that $r_S(Q) = \beta r$ with $\beta > 1$.

2.3 Timing of events

At date 0, banks and firms form partnerships via a one-to-one matching rule. At $t = 1$, each bank makes a take-it-or-leave-it contract offer to a firm which specifies the loan rate and the funding mode, decides on the monitoring intensity. Finally at $t = 2$, the true cash flow of each firm is realized and the agreed upon payments are made. We solve the model by backward induction.

2.4 Equilibrium

An equilibrium of the loan market consists of a set of matches (k, w) formed through feasible contracts, i.e., bank financing modes and the corresponding loan rate for each match. An allocation for the market (μ, u, v) specifies a one-to-one matching rule $\mu : W \to K$, and payoff functions $u : W \to \mathbb{R}_+$ and $v : K \to \mathbb{R}_+$ for the firms and the banks, respectively.

Definition 1 (Equilibrium) *Given the type distributions $G_W(w)$ and $G_K(k)$, an allocation (μ, u, v) at a given r is an equilibrium allocation of the credit market if the following conditions hold:*

(a) **Feasibility:** *the payoffs to the bank an the firm in each equilibrium match are feasible given the realized cash flow and the risk-free rate r;*

[7] Deposit insurance could have been 'fairly priced', i.e., $\alpha = (1 - m)r$ where $1 - m$ is the probability that a bank is insolvent. Since m is non-verifiable, using fair insurance premium would make the choice of monitoring intensity problem trivial.

[8] Greenbaum and Thakor (1987) show that financing through uninsured deposits and that through loan sales are equivalent when there is perfect information about borrower types. This equivalence also holds good in our model as w is public information.

(b) **Optimization:** *Each bank of a given type chooses optimally a firm, i.e., given $u(w)$ for each $w \in W$,*

$$\mu^{-1}(k) = argmax_w \, \phi(k, w, u(w)), \tag{$\mathscr{M}_k$}$$

for each $k \in K$. The function $\phi(k, w, u(w))$ is the bargaining frontier *or* Pareto frontier *associated with the match (k, w), which is the maximum payoff that accrues to each type k bank given that the firm of type w consumes $u(w)$;*

(c) **Market clearing:** *The equilibrium matching function satisfies the following 'measure consistency' condition. For any subinterval $[i_0, i_1] \subseteq B$, let $i_h = G_K(k_h)$ for $h = 0, 1$, i.e., k_h is the capital of the bank at the i_h-th quantile. Similarly, for any subinterval $[j_0, j_1] \subseteq F$, let $j_h = G_W(w_h)$ for $h = 0, 1$. If $[k_0, k_1] = \mu([w_0, w_1])$, then it must be the case that*

$$i_1 - i_0 = G_K(k_1) - G_K(k_0) = G_W(w_1) - G_W(w_0) = j_1 - j_0. \tag{MC}$$

Definition 1-(b) asserts that each bank chooses her partner optimally. Part (c) of the above definition says that one cannot match say half of the banks to one-third of the firms because the matching is constrained to be one-to-one.

3 Equilibrium sorting, financing modes and loan contracts

We proceed as follows. In Section 3.1, we derive the bargaining frontier under regimes D and S, and determine the optimal financing mode for an arbitrary bank-firm pair as a function of exogenously given payoffs. In Section 3.2 we endogenize the payoffs and derive that the equilibrium matching is positive assortative. In Section 3.3 we establish that the equilibrium funding modes is monotonic, and analyze the behavior of the equilibrium bank monitoring and loan rate with respect to firm capital.

3.1 Optimal financing mode of an arbitrary match

We analyze the optimal loan contract and bank financing mode of an arbitrary match (k, w). We assume that the bank possesses all the bargaining power in the lending relationship and makes a take-it-or-leave-it offer to the firm. We first analyze each financing mode separately.

3.1.1 Insured deposits

Take an arbitrary bank-firm match (k, w). The optimal contract under insured deposit financing will depend on the types of the bank and the firm. To save on notations, we suppress the arguments (k, w) for the time being. An optimal loan contract for an arbitrary match solves the following maximization problem:

$$\max_{\{R,m\}} \quad V(R, m) \equiv m(R + k - r) + (1 - m)(w + k) - \alpha - \frac{1}{2}m^2 - k, \tag{$\mathscr{P}_D$}$$

$$\text{subject to} \quad U(R, m) \equiv m(Q + w - R) = u, \tag{PCF$_D$}$$

$$m = \text{argmax}_{\hat{m}} \{\hat{m}(R + k - r) + (1 - \hat{m})(w + k) - \alpha - \frac{1}{2}\hat{m}^2 - k\} = R - r - w. \tag{ICB$_D$}$$

We assume that $Q-r>1$ so the first-best monitoring is given by $m_D^{fb}=\min\{Q-r,1\}=1$.[9] In the event of success, i.e., when the cash flow is Q the firm pays R to the bank and keeps $Q+w$. In the event of failure, on the other hand, the firm pays w to the bank and earns nothing. The constraint (PCF_D) is the firm's *participation constraint*. We take this constraint to be binding as we will show later that this must be the case in equilibrium, i.e., the firm's expected payoff $U(R,m)$ is equal to his outside option $u\geq 0$. In the objective function ($\mathscr{P}_D$) of the bank we normalize the opportunity cost of capital to 1. Note also that the level of bank capital k does not affect her decision to monitor as k does not appear in the *incentive constraint* (ICB_D). The following lemma characterizes the optimal contract for an arbitrary match (k,w):

Lemma 1 *When in an arbitrary match (k,w), the project is financed through insured deposits, the optimal monitoring and loan rate are respectively given by:*

$$m^D(k,w,u)=\frac{1}{2}\left(Q-r+\sqrt{(Q-r)^2-4u}\right),$$
$$R^D(k,w,u)=\frac{1}{2}\left(Q-r+\sqrt{(Q-r)^2-4u}\right)+r+w.$$

Moreover,

$$\frac{\partial m^D}{\partial k}=0,\quad \frac{\partial m^D}{\partial w}=0 \ \ \textit{and} \ \ \frac{\partial m^D}{\partial u}<0,$$
$$\frac{\partial R^D}{\partial k}=0,\quad \frac{\partial R^D}{\partial w}>0 \ \ \textit{and} \ \ \frac{\partial R^D}{\partial u}<0.$$

Notice that both the optimal monitoring and loan rate are decreasing in the firm's outside option u. When the outside option u of the firm increases, he gains greater bargaining power, and hence, the bank is forced to pay him more. Since the participation constraint is binding, this additional payment cannot be given in the form of additional rent, rather has to be given in the form of a reduction in the loan rate. As a consequence, the optimal R decreases. A decrease in R implies weaker monitoring incentives, and hence, a lower monitoring intensity at the optimum. The optimal monitoring intensity is independent of firm capital w. Because an increase in w leaves the optimal monitoring unaltered, in order to satisfy the incentive constraint (ICB_D), the optimal loan rate R must be increasing in w.

[9]The first-best monitoring is given by

$$\text{argmax}_m V(R,m)+U(R,m)\equiv m(Q-r)+w-\alpha-\frac{1}{2}m^2=Q-r.$$

Since m cannot exceed 1, we have $m_D^{fb}=\min\{Q-r,1\}=1$.

3.1.2 Loan securitization

An optimal loan contract under loan securitization for an arbitrary match solves the following maximization problem:

$$\max_{\{R,m\}} \ V(R,m) \equiv m(R+k-\beta r) - \frac{1}{2}m^2 - k, \qquad (\mathscr{P}_S)$$

$$\text{subject to } \ U(R,m) \equiv m(Q+w-R) = u, \qquad (\text{PCF}_S)$$

$$m = \text{argmax}_{\hat{m}} \{\hat{m}(R+k-\beta r) - \frac{1}{2}\hat{m}^2 - k\} = R - \beta r + k. \qquad (\text{ICB}_S)$$

We assume that $Q - \beta r + w + k > 1$ so the first-best monitoring is given by $m_S^{fb} = \min\{Q+w+k-\beta r, 1\} = 1$. When the realized cash flow is zero in the event of failure, the bank is insolvent and the creditors collect her total asset $w+k$. The participation constraint of the firm remains unaltered as the firm is ex-ante indifferent between the two modes of financing. Interestingly, the bank's incentive constraint changes under loan securitization as now the monitoring intensity depends on her level of capital. The following lemma characterizes the optimal contract for an arbitrary match (k, w):

Lemma 2 *When in an arbitrary match (k, w), the project is financed through loan securitization, the optimal monitoring and loan rate are respectively given by:*

$$m^S(k,w,u) = \frac{1}{2}\left(Q - \beta r + w + k + \sqrt{(Q-\beta r+w+k)^2 - 4u}\right),$$

$$R^S(k,w,u) = \frac{1}{2}\left(Q - \beta r + w + k + \sqrt{(Q-\beta r+w+k)^2 - 4u}\right) + \beta r - k.$$

Moreover,

$$\frac{\partial m^S}{\partial k} > 0, \quad \frac{\partial m^S}{\partial w} > 0 \ \textit{and} \ \frac{\partial m^S}{\partial u} < 0,$$

$$\frac{\partial R^S}{\partial k} > 0, \quad \frac{\partial R^S}{\partial w} > 0 \ \textit{and} \ \frac{\partial R^S}{\partial u} < 0.$$

For similar reasons as explained before, under loan securitization, both the optimal monitoring and loan rate are decreasing in the firm's outside option, and increasing in firm capital. Unlike the case of deposit financing, under loan securitization, both the optimal monitoring and loan rate depend on bank capital. Without surprise both are increasing functions of k. The greater the bank capital, the higher is the amount at stake in the case when the bank is insolvent, and hence, the bank has stronger incentives to monitor to avoid insolvency. Consequently, the bank requires a greater marginal compensation in the form of a higher loan rate. Note that, under both financing modes, the creditors of a bank are able to influence her monitoring incentives. In particular, a lower required rate of return r, i.e., a higher intermediation margin implies a greater monitoring intensity.

3.1.3 Choice of financing mode and the bargaining frontier

Now we derive the bargaining frontier $v \equiv \phi(k, w, u)$ associated with an arbitrary bank-firm match (k, w) when it can choose between the two modes of financing. Let $\phi^D(k, w, u)$ and $\phi^S(k, w, u)$ be the bargaining frontiers associated with financing modes D and S, i.e., the maximum value functions of the programs

($\mathscr{P}_D$) and ($\mathscr{P}_S$), respectively. The (combined) bargaining frontier for (k, w) is given by:

$$\phi(k, w, u) = \max\{\phi^D(k, w, u), \phi^S(k, w, u)\}. \tag{BF}$$

The individual bargaining frontiers $\phi^D(k, w, u)$ and $\phi^S(k, w, u)$ are depicted in Figure 1.[10] We assume that $w + k > \alpha$, i.e., the bank's total asset $w + k$ in the event of zero cash flow is higher than what she is supposed to pay as the deposit insurance premium.

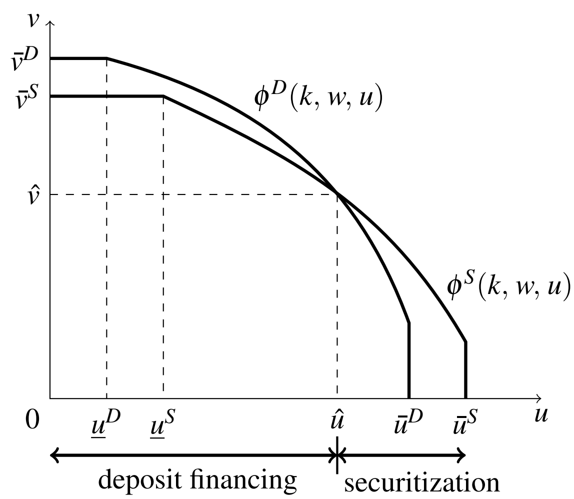

Figure 1: *The bargaining frontier of an arbitrary match when* $w + k > (\beta - 1)r$.

From the expressions of the optimal monitoring levels in Lemmas 1 and 2 it is easy to show that

$$\underline{m}^D \equiv \frac{Q - r}{2} \leq m^D(k, w, u) \leq 1,$$
$$\underline{m}^S \equiv \frac{Q - \beta r + w + k}{2} \leq m^S(k, w, u) \leq 1.$$

Now consider the frontier under deposit financing (loan securitization). The horizontal segment of it corresponds to the utility of the bank at $m^D = 1$ ($m^S = 1$), and its vertical segment corresponds to the utility of the firm at $m^D = \underline{m}^D$ ($m^S = \underline{m}^S$).

For an arbitrary match (k, w), the optimal choice of bank financing mode depends on the aggregate capital $A(k, w) \equiv w + k$. Under the assumption that $A(k, w) > \alpha$, we have $\bar{v}^D > \bar{v}^S$. On the other hand, $\bar{u}^D < \bar{u}^S$ and the non-linear segment of $\phi^D(k, w, u)$ is steeper than that of $\phi^S(k, w, u)$ if and only if $A(k, w) > (\beta - 1)r$. Two cases follow. If $A(k, w) \leq (\beta - 1)r$, then $\phi^D(k, w, u)$ lies above $\phi^S(k, w, u)$ for all values of firm utility, and hence, financing through insured deposits strictly dominates loan securitization for the match (k, w). On the other hand, if $A(k, w) > (\beta - 1)r$, then there is no such clear dominance of one financing mode over the other as depicted in Figure 1. Thus, $\phi^D(k, w, u)$ intersects $\phi^S(k, w, u)$ only once. Here, without loss of generality, we make the assumption that $\phi^D(k, w, \underline{u}^S) > \bar{v}^S$ which implies the intersection always occurs on the non-linear segments of the two frontiers. Otherwise, the horizontal part of $\phi^S(k, w, u)$, i.e., $\bar{v}^S$ would have intersected the non-linear part of $\phi^D(k, w, u)$. The

[10]For the detailed derivations, see Appendix C.

intersection of the two frontiers defines a threshold level $\hat{u}$ of firm utility below which financing through insured deposits is the optimal financing mode for the match (k, w), while financing through loan securitization is the optimal choice if firm utility lies above $\hat{u}$. Note that, as the two frontiers depend on both k and w (each one is increasing in k and w), the indifference point $\hat{u}$ is a function of k and w, i.e., $\hat{u} = \hat{u}(k, w)$. To summarize,

Proposition 1 (Bargaining frontier of an arbitrary match) *For a given match (k, w),*

(a) *If $A(k, w) \leq (\beta - 1)r$, then financing through insured deposits dominates financing through loan securitization. The bargaining frontier is given by $\phi(k, w, u) = \phi^D(k, w, u)$;*

(b) *If $A(k, w) > (\beta - 1)r$, then financing through loan securitization is preferred if and only if $u \geq \hat{u}(k, w)$. The bargaining frontier is given by*

$$\phi(k, w, u) = \begin{cases} \phi^D(k, w, u) & \text{if } u < \hat{u}(k, w), \\ \phi^S(k, w, u) & \text{if } u \geq \hat{u}(k, w). \end{cases}$$

If any bank chooses to finance investments through loan securitization instead of insured deposits, then $(\beta - 1)r$ is the additional amount she has to pay to her investors. When this additional cost exceeds the total asset, i.e., $(\beta - 1)r \geq A$, then the aggregate surplus under D is higher than that under S, and hence, insured deposit becomes the dominant mode of financing. When $(\beta - 1)r < A$, a more interesting case emerges. The firm in any arbitrary match faces a trade-off between provision of incentives and giving up ex ante rent to the bank. In this case, incentive provision is less costly under loan securitization since $m^S \geq m^D$ for all (k, w, u). On the other hand, due to imperfect transferable utility, surplus can not be transferred between the two parties on a one-to-one basis. By taking away a dollar from the firm, more utility can be transferred to the bank under D than S because the bargaining frontier under D is steeper than that under S. When $v \leq \hat{v}$, or equivalently, $u \geq \hat{u}$, the first effect dominates, and hence $\phi^S(k, w, u)$ lies above $\phi^D(k, w, u)$. When $v \geq \hat{v}$, on the other hand, the second effect dominates, and hence, the surplus under D exceeds that under S.

3.2 Equilibrium matching

In a market with endogenous matching, the outside option of each firm is the maximum payoff he could obtain by switching to alternative matches, and hence, it is endogenous. We first argue that in every match (k, w), the participation constraint of the firm must bind in an equilibrium allocation. Suppose that a firm of type w is offered $u(w)$ by a bank of a given type. Because there is a positive measure of each type, one can find an identical bank who would also offer $u(w)$ to the same borrower, and hence, $u(w)$ actually becomes his outside option. Thus, a firm payoff that is strictly above the outside option cannot be an equilibrium payoff. In other words, in an equilibrium allocation, there must be no additional surplus remaining to bargain over, as types are arbitrarily close to each other.

We now analyze the equilibrium matching function $k = \mu(w)$ and show that it is positive assortative (PAM), i.e., $\mu(w)$ is non decreasing in w. At date 0, each type k bank solves ($\mathscr{M}_k$) to choose a firm. We show that the payoff to any type k bank satisfies the following *single-crossing property*:[11]

$$\phi(k', w'', u(w'')) = \phi(k', w', u(w')) \implies \phi(k'', w'', u(w'')) \geq \phi(k'', w', u(w')) \quad \text{(SC)}$$

[11]See Appendix D.

for any $k'' > k'$ and $w'' > w'$, and hence, the equilibrium matching is positive assortative. The single

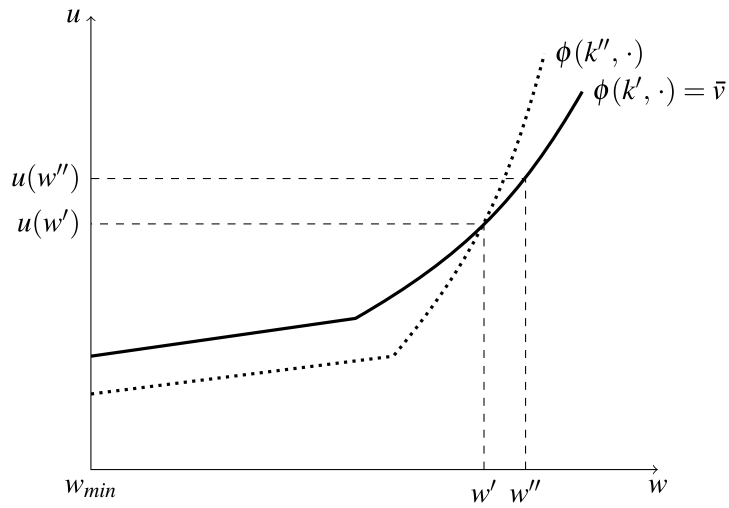

Figure 2: *Single-crossing property.*

crossing property implies that the indifference curves of any two types k' and k'' cross only once. In Figure 2, the solid curve is the indifference curve of a type k' bank and the dotted curve is that of a higher type ($k'' > k'$) bank, both drawn in the 'firm type-firm payoff' space. The flatter segment of each indifference curve (not necessarily linear) corresponds to financing through insured deposits. A type k' bank is indifferent between the type-utility combinations $(w', u(w'))$ and $(w'', u(w''))$ because both yield $\bar{v}$ to her. But a higher type bank (k') strictly prefers to pay more for w''.

Proposition 2 (PAM) *In the equilibrium allocation of the loan market, the matching between k and w is positive assortative, i.e., a more capitalized bank finances a more capitalized firm.*

The result that the single-crossing property implies PAM follows directly from Milgrom and Shannon (1994). Under financing through fully insured deposits, the level of bank capital is irrelevant for the optimal bank monitoring, and hence, assigning a firm to any bank can be part of an equilibrium allocation. Therefore when financing through insured deposits dominates loan securitization, one can select a PAM out of all such equilibrium matchings. By contrast, under loan securitization, bank capital matters for monitoring, and hence, a random matching is not optimal. Condition (SC) implies a complementarity between bank capital and firm capital in this case. Under incentive problems, this complementarity may be two-fold. Bank capital and firm capital are not only complementary in producing surplus, but also in transferring surplus. Legros and Newman's (2007) *generalized increasing differences* (GID) condition is a kind of single-crossing condition, and such two-fold complementarity between firms and banks implies GID, and hence, PAM.

3.3 Equilibrium bank financing modes, monitoring and loan rates

The analysis of Section 3.1 is essentially a partial equilibrium analysis where in each bank-firm match (k, w) the outside option u of the firm is taken as given. In the market equilibrium, the outside option of each bank is endogenized through the matching function $k = \mu(w)$, and hence, the equilibrium contracts are in general different from those previously analyzed.

Consider first a match $(\mu(w), w)$ along the equilibrium path, and let $\hat{w}$ solves $A(\mu(w), w) = (\beta - 1)r$. Clearly, in all matches $(\mu(w), w)$ with $w \leq \hat{w}$ the banks will choose to finance through insured deposits. On the other hand, following Proposition 1-(b), the banks in the matches $(\mu(w), w)$ with $w > \hat{w}$ will choose loan securitization if and only if $u(w) > \hat{u}(w)$ where $\hat{u}(w) \equiv \hat{u}(\mu(w), w)$.

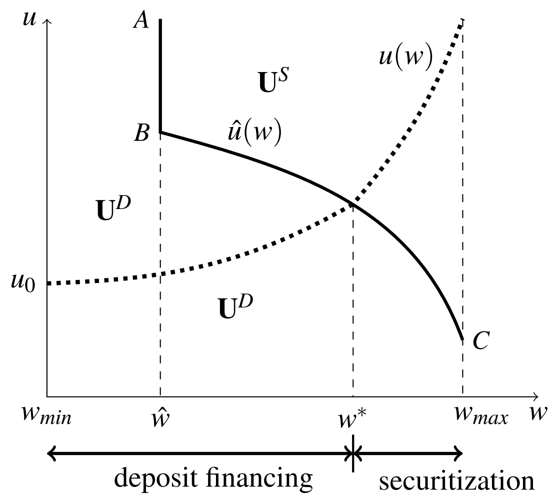

Figure 3: *Monotonic bank financing modes in equilibrium. Low-capital firms, i.e., below w^*, are financed through insured deposits, while high-capital firms, i.e., capital above w^*, are financed through loan securitization.*

In Figure 3 the solid curve *ABC* is the set of (w, u) on which all the equilibrium matches $(\mu(w), w)$ are indifferent between the two modes of bank financing. Let us call *ABC* the *indifference locus* [in equilibrium]. Above the indifference locus and to the right of $\hat{w}$ all matches strictly prefers loan securitization, and in the region below it and to left of $\hat{w}$ all matches prefer financing through insured deposits. We call the above two regions $\mathbf{U}^D$ and $\mathbf{U}^S$, respectively. Which are the matches who will actually choose one mode of financing over the other will also depend on the equilibrium utility function $u(w)$ of the firms.

Now we show how the equilibrium utility function is determined. The first-order condition of the maximization problem ($\mathscr{M}_k$) is given by the following ordinary differential equation (ODE):

$$u'(w) = \begin{cases} -\frac{\phi_2^D(k,w,u(w))}{\phi_3^D(k,w,u(w))} > 0 \quad \text{if } u(w) \in \mathbf{U}^D, \\ \\ -\frac{\phi_2^S(k,w,u(w))}{\phi_3^S(k,w,u(w))} > 0 \quad \text{if } u(w) \in \mathbf{U}^S \end{cases} \tag{ODE}$$

for $k = \mu(w)$. We denote by $u_0 > 0$ the *reservation utility* of each firm, i.e., the utility a firm would consume if he does not enter into any contractual agreement. Clearly, in equilibrium we have $u(w_{min}) = u_0$ because u_0 is the outside option of each type w_{min} firm. The equilibrium utility function (it follows from the (ODE) that it is strictly increasing in w) is depicted by the dotted curve in Figure 3, which has two parts (with different slopes) – the flatter segment is the equilibrium utility of each type w firm under D, and the steeper segment is that under S. Let w^* be the level of firm capital where the equilibrium utility $u(w)$ and the indifference locus intersect each other. Thus, $(\mu(w^*), w^*)$ is the unique match along the equilibrium path which is indifferent between the two modes of bank financing. The following proposition asserts that the equilibrium financing mode is monotonic in w.

Proposition 3 *There is a unique threshold level w^* of firm capital such that each equilibrium match $(\mu(w), w)$ prefers financing through loan securitization over insured deposits if and only if $w \geq w^*$.*

The above proposition along with Proposition 2 says that the matches consisting of more capitalized banks and firms would opt for financing through loan securitization, whereas the partnerships of banks and firms with low aggregate assets would choose deposit financing. This clearly distinguishes between the two types of banking institutions that emerge in the market equilibrium. The less capitalized banks not only finance the projects of the less capitalized borrowers, but also resort to the traditional banking, i.e., the banks that originate-to-hold. On the other hand, more capitalized banks, apart from lending to high-quality borrowers, fund their investment through loan securitization, i.e., they become part of the banking institutions that originate-to-distribute.

Finally, we analyze the behavior of the equilibrium monitoring and loan rate with respect to w. Both under deposit financing and loan securitization the optimal values of these choice variables in each match are determined by the types of bank and firm, and the outside option of the firm. In the market equilibrium, both the matching and the outside option are endogenous. Therefore, we can write the equilibrium functions as the following:

$$m(w) \equiv m(\mu(w), w, u(w)),$$
$$R(w) \equiv R(\mu(w), w, u(w)).$$

We intend to study the signs of $m'(w)$ and $R'(w)$. Differentiating the above expressions with respect to w we obtain

$$m'(w) = \frac{\partial m}{\partial k}\mu'(w) + \frac{\partial m}{\partial w} + \frac{\partial m}{\partial u}u'(w),$$
$$R'(w) = \frac{\partial R}{\partial k}\mu'(w) + \frac{\partial R}{\partial w} + \frac{\partial R}{\partial u}u'(w).$$

Now consider any bank-firm match $(\mu(w), w)$ along the equilibrium path such that $w < w^*$. Following Proposition 3, all these matches will finance their projects through insured deposits. Then, from Lemma 1 and the first-order condition, (ODE) it follows that $m'(w) < 0$ and $R'(w) < 0$. On the other hand, under loan securitization, i.e., $w \geq w^*$, it follows from Lemma 2 and (ODE) that $m'(w) > 0$ and $R'(w) > 0$.[12] Hence,

Proposition 4 *Given the type distributions, $G_W(w)$ and $G_K(k)$, the equilibrium monitoring intensity $m(w)$ and loan rate $R(w)$ are non-monotonic in firm capital. In particular, they are decreasing (increasing) with respect to w if $w < (\geq) w^*$.*

The result of the above proposition is depicted in Figure 4.

[12] See Appendix F for details.

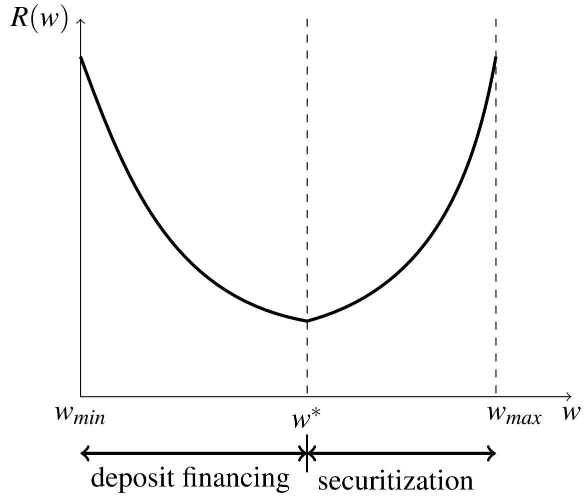

Figure 4: *The U-shaped equilibrium loan rate function. For low (high) values of firm capital, i.e., below (above) w^*, the equilibrium loan rate function $R(w)$ is decreasing (increasing) in w. The equilibrium monitoring intensity function $m(w)$ has a similar shape.*

Under financing through insured deposits, neither bank capital nor firm capital does play any role in determining the optimal monitoring, and hence, the optimal loan rate, and the optimal values are determined only by u. As higher firm capital implies greater firm payoff in the market equilibrium, and both monitoring and loan rate are decreasing with respect to u, we have that the equilibrium monitoring intensity and loan rate are strictly decreasing in w as long as $w < w^*$. On the other hand, when $w \geq w^*$, all matches opt for financing through loan securitization. Under this mode of bank financing, the higher the bank and firm capital, the higher is the total asset at stake in case the bank is insolvent. Therefore, the banks have stronger incentives to monitor in order to avoid insolvency. As a result, the marginal compensation of the banks, i.e., the loan rate must also be increasing in both bank and firm wealth.

Ex post theories of collateral assert that observably riskier borrowers are often required to pledge higher collateral to reduce agency costs, and hence, there is a positive correlation between the loan rate and collateral (e.g., Boot and Thakor, 1994). Ex ante theories of collateral, by contrast, postulate that unobservably safer borrowers often pledge higher collateral to signal their quality, and thus, there exists a negative relation between loan rate and collateral (e.g., Besanko and Thakor, 1987). By combining the above two attributes – moral hazard and private information, Boot, Thakor, and Udell (1991) show that default risk, which is equivalent to the probability of failure, $1-m$ in our model, may be either increasing or decreasing with respect to borrower quality. Berger, Frame, and Ioannidou (2015), on the other hand, show that both loan/default risk and loan rate may be non-monotonic in borrower collateral. These authors argue that collateral may have different desirable economic characteristics such as liquidity, divertability, and outside ownership status, each of which may influence loan risk in a different way; hence, the relation between loan rate and collateral may not be monotonic. Proposition 4 implies that the equilibrium behavior of bank monitoring intensity or default risk, and loan rate with respect to pledgeable collateral crucially depends on the equilibrium choice of bank funding modes because both bank capital and firm capital have differential implications for monitoring incentives under the two different financing arrangements. We therefore provide an alternative explanation for the non-monotonicity of loan rate and default risk with respect to borrower collateral, which is neither based on the interaction between moral hazard and private information as in Boot et al. (1991), nor on the existence of different types of collateral as in Berger et al. (2015).

3.4 Restricted choice of bank funding modes

We now analyze the market equilibrium when the banks face a constraint on the choice of their funding modes – they can finance borrowers either through insured deposits or through loan securitization. In this case, the equilibrium monitoring intensity and loan rate will take the following forms, respectively.

$$\begin{cases} m^D(w) \equiv m^D(\mu^D(w), w, u^D(w)) & \text{under deposit financing,} \\ m^S(w) \equiv m^S(\mu^S(w), w, u^S(w)) & \text{under loan securitization,} \end{cases}$$

and

$$\begin{cases} R^D(w) \equiv R^D(\mu^D(w), w, u^D(w)) & \text{under deposit financing,} \\ R^S(w) \equiv R^S(\mu^S(w), w, u^S(w)) & \text{under loan securitization,} \end{cases}$$

where $\mu^\theta(\cdot)$ and $u^\theta(\cdot)$ are the equilibrium matching and firm utility functions under the financing mode $\theta = D, S$. We state the following result without a formal proof.

Proposition 5 *Suppose the banks are constrained to choose one of the two funding modes. Then,*

(a) *Under financing through insured deposits, the equilibrium matching is random, and the equilibrium monitoring intensity and loan rate are monotonically decreasing with respect to firm capital;*

(b) *Under financing through loan securitization, the equilibrium matching is positive assortative, and the equilibrium monitoring intensity and loan rate are monotonically increasing with respect to firm capital.*

Under deposit financing, bank capital does not matter for the optimal monitoring, and hence, for the determination of the optimal loan rate. Therefore, any bank type can be assigned to any firm type, i.e., the equilibrium matching is a random matching. Under loan sales, on the other hand, greater amounts of bank and firm capital imply more intense monitoring, and the implied marginal cost of monitoring is lower in matches consisting of higher total assets. As a consequence, the equilibrium matching is positive assortative. The non-monotone monitoring intensity and loan rate (in Figure 4) when the banks are free to choose between the two modes of funding are respectively the upper envelopes of the equilibrium monitoring intensity and loan rate functions under each mode of financing, i.e., $m(w) = \max\{m^D(w), m^S(w)\}$ and $R(w) = \max\{R^D(w), R^S(w)\}$.

4 Effect of changes in the risk-free rate on loan securitization

We analyze how an exogenous cross-sectional variation in the risk-free rate r affects the fraction of matches that choose to finance projects through loan securitization in the loan market equilibrium. We will identify two effects – namely, an *income effect* and a *substitution effect*. The income effect refers to a shift in the equilibrium utility $u(w)$ of the firms keeping the indifference locus $\hat{u}(w)$ unaltered. This arises because the utility of each type w firm is endogenized through the equilibrium matching. The substitution effect, by contrast, refers to a shift in the equilibrium indifference locus keeping the $u(w)$ curve constant. Recall that $(\mu(w^*), w^*)$ is the unique match that is indifferent between the two modes of bank financing (see Figure 3). The two aforementioned effects thus refer to how the threshold level of firm capital, w^* responds to a change in the risk-free rate r. Let us write $w^* = w^*(r)$. A complete characterization of this comparative statics exercise requires closed form solutions to our model as all the equilibrium variables depend on w via the matching function $\mu(w)$ and the firm utility function $u(w)$. Unfortunately, under imperfectly transferable utility where the bargaining frontier does not depend linearly on $u(w)$, a closed form solution to the first-order condition, (ODE) does not exist.[13]

The income effect is depicted in Figure 5.[14] Note that both under the old and the new risk-free rates, we have $u(w_{min}) = u_0$. An increase in the risk-free rate increases the costs of bank financing under both financing modes. Therefore, the equilibrium firm utility shifts down from $u(w;r)$ to $u(w;r')$ for all levels of firm capital $w > w_{min}$ following an increase in the risk-free rate from r to r'. Since the indifference locus is kept unaltered, the threshold level of firm capital, w^* increases, i.e., the the fraction of matches who finance their projects with loan securitization decreases.

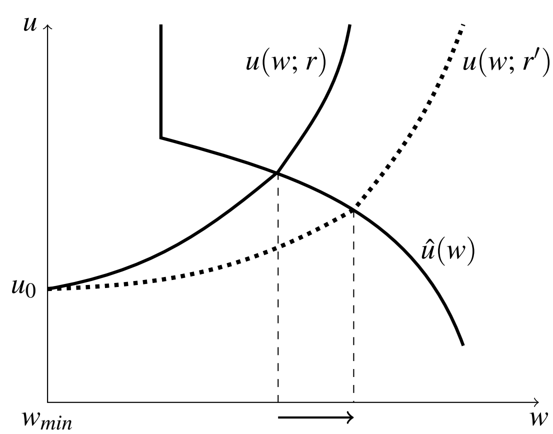

Figure 5: *Income effect: the fraction of matches that finance projects through loan securitization decreases following an increase in the risk-free rate from r to r′.*

When the risk-free rate increases both modes of bank financing become costlier, and hence, some or all matches will substitute one mode of funding by the other. The direction of this substitution will depend on the relative cost of the two financing modes, β. One possible substitution effect is depicted in Figure 6 where the equilibrium utility $u(w)$ of the firms is kept unaltered.

When the risk-free rate increases from r to r' we first observe that $\hat{w}$ goes up from $\hat{w}(r)$ to $\hat{w}(r')$

[13]Following the Picard-Lindelöf Theorem (see Birkhoff and Rota, 1989) a unique solution to (ODE) exists if $u'(w)$ is bounded, Lipschitz continuous in u and continuous in w. It is easy to prove the last two properties. Moreover, $u'(w)$ is bounded as long as the initial condition $u_0 > 0$. The solution to the above ODE is given by:

$$u(w) = u_0 + \int_{w_{min}}^{w} u'(x)dx.$$

[14]See Appendix G for a formal proof.

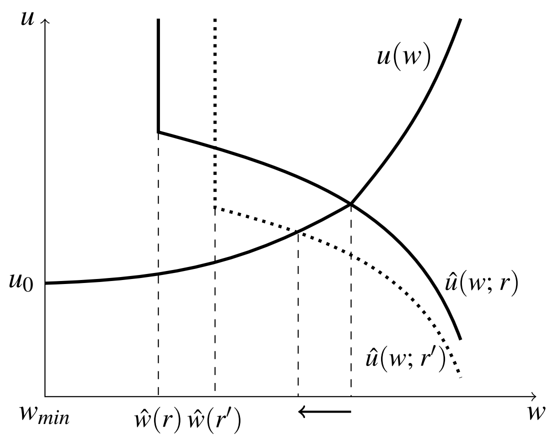

Figure 6: *A possible substitution effect: the fraction of matches that finance projects through loan securitization increases following an increase in the risk-free rate from r to r'.*

because $d\hat{w}/dr > 0$. But how the non-linear segment of the indifference locus respond to an increase in r is difficult to determine. One possible shift is as depicted in Figure 6. The dotted curve is the new equilibrium indifference locus corresponding to the higher risk-free rate r'. In this case, w^* decreases, i.e., the the fraction of matches who finance their projects with loan securitization increases. Therefore, the income and the substitution effects may pull in opposite directions, and the effect of an increase in the risk-free rate on w^*, i.e., the fraction of matches financing their projects through loan securitization is ambiguous. In the following numerical example we show that $w^*(r)$ is non-monotonic in r.

Example 1 Let $Q = 2.13$, $\alpha = 0.02$, $\beta = 1.1$ and $u_0 = 0.2$. Furthermore, assume that both k and w have the same distribution. In particular, they are uniformly distributed on $[0.02, 0.2]$. Using the above parameter values we solve (ODE) numerically using Matlab. The risk-free rate is continuously varied on the interval $[1, 1.12]$. Figure 7 shows that the threshold firm capital $w^*(r)$ is non-monotonic in r. In fact, $w^*(r)$ is monotonically increasing for $r \leq \hat{r} \approx 1.07$. On the other hand, the threshold firm capital is monotonically decreasing beyond $\hat{r}$.

For low values of the risk-free rate (below $\hat{r}$), an increase in the risk-free rate implies that the income effect dominates the substitution effect, and hence, the threshold value of firm capital, w^* increases, i.e., more matches switch from loan securitization to financing through insured deposits. On the other hand, for high values of r, a further increase in the risk-free rate implies that the substitution effect is stronger than the income effect, and hence the fraction of matches that finance their projects through loan securitization increases. Note that although the direction of shift in $u(w)$ following an exogenous increase in the risk-free rate is unambiguous, that in the equilibrium indifference locus is not. Figure 6 depicts only one of the many possible shifts. It may well be the case that the dotted curve lies above that solid curve, and hence, both the income and substitution effects will pull in the same direction. As a result, $w^*(r)$ will be monotonically increasing in the risk-free rate, r, i.e., there will be a negative correlation between the risk-free rate and the relative size of the shadow banking sector.

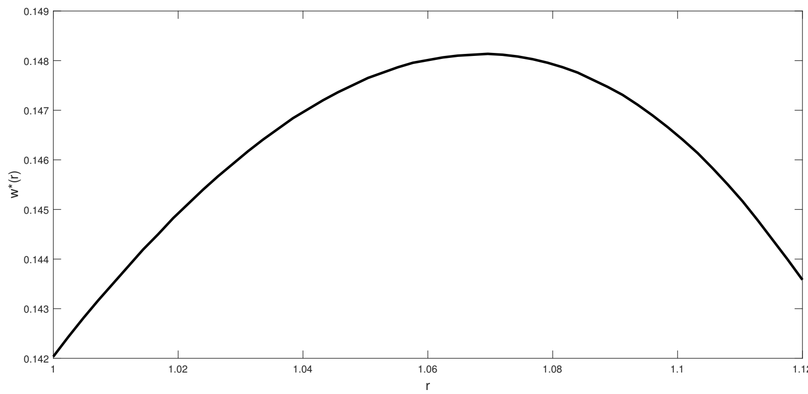

Figure 7: *Non monotonicity of* $w^*(r)$ *with respect to the risk-free rate.*

5 Concluding remarks

In a unified framework where both bank capital and firm capital affect the monitoring incentives of banks, we have shown that not only the endogenous matching between bank capital and firm capital is monotone, but also that the bank funding modes are monotone in the sense that the banks in matches with higher aggregate assets originate-to-distribute, whereas low-asset matches opt for originate-to-hold. As a consequence both the equilibrium bank monitoring and the loan rate functions are non-monotonic in firm capital. Moreover, an increase in the risk-free rate may have an ambiguous effect on the relative size of the banking sector that originates-to-distribute.

In light of the recent financial crisis, shadow banking has been blamed to a large extent for contributing to financial instability. The intermediaries might either have originated and sold lower quality loans, or loan sales might have weakened the banks' incentives to gather soft information regarding their borrowers (e.g. Berndt and Gupta, 2009). Unlike traditional banking, the shadow banking sector has largely been unregulated until recently, which has accounted for its unprecedented growth (see Gorton and Metrick, 2010). In the present paper we do not take a stand on the socially optimal size of the banking system that originates-to-distribute, yet our model suggests that it is not desirable that the low-capitalized banks engage in off-balance-sheet financing. On the other hand, it is not harmful that the high-capitalized banks take part in shadow banking because they are easier to incentivize to monitor their borrowers, and hence, the loans sold in the secondary market are of high-performance. The reasoning is in general consistent with Gorton and Metrick (2010) who argue that setting collateral requirements are essential in any proposal for regulating securitization. In fact, many recent financial regulatory measures directly address this issue. For example, revisions to the Basel II market risk framework now eliminate the ability of the banks to hold less capital against securitization exposures. Also, the Dodd-Frank Act prescribes rules that require financial institutions to retain ownership of some part of the security (see Adrian and Ashcraft, 2012, for details).

Our stylized model is silent about the role that a prudential regulator might have in this framework. In particular, we abstract from analyzing any explicit regulatory policies related to capital requirements. However, our results may shed some light on how financial stability and capital requirements may be tied up depending on a bank's choice of funding modes. In our model, default probability and risk-taking (as defined by the variance of the stochastic cash flow) are inversely related to monitoring intensity. As we have found, bank capital is unrelated to monitoring under deposit financing but it is essential to incentivize the banks to monitor under loan securitization. This in turn implies that bank capital is irrelevant for determining default probability and risk-taking under deposit financing due to full deposit insurance. In contrast, more bank capital implies a lower default probability and risk-taking under loan securitization. Accordingly, the regulator may impose a flat minimum capital requirement for the banks belonging to the traditional OTH sector, given that full deposit insurance is available. On the other hand, setting a non-constant capital requirement under securitization could be a more sensitive policy. In particular, highly leveraged banks involved in securitization would face higher capital requirements to reduce the probability of default and risk-taking. This idea is in line with what the *Squam Lake Report* (French et al., 2010) recommends – capital requirements, measured as a fraction of either total assets or risk-adjusted assets, should be higher for larger banks. The rationale for such proposal, however, is based on the differentiated effects a defaulting bank may have on other financial institutions: presumably, the failure of a large national bank may have a much bigger impact on the banking system than that of small regional banks.

Finally, Martínez-Miera and Repullo (2017) point out that an increase in the risk-free interest rate induces the shadow banking sector to shrink. Thus, any attempt to raise interest rates may be desirable because a large shadow banking sector in their model is socially inefficient. However, empirical evidence (e.g. Nelson et al., 2015) suggests that a contractionary monetary policy leading to a rate increase may imply an expansion of such sector. Our analysis in Section 4 points in the similar direction; it asserts that the risk-free rate may be positively correlated with the size of the shadow banking sector. A research agenda for the future thus would be a full-fledged welfare analysis, as in Martínez-Miera and Repullo (2017), under the current framework in order to study the efficiency properties of our equilibrium allocations.

Appendices

A Proof of Lemma 1

Substituting for R from (ICB_D) into the expected payoffs of the bank and the firm yields the following reduced form maximization problem:

$$\max_{m} \; V(m) \equiv \frac{1}{2}m^2 + w - \alpha, \qquad (\mathscr{P}'_D)$$

$$\text{subject to } \; U(m) \equiv m(Q - r - m) = u. \qquad (\text{PCF}'_D)$$

Solving for m from (PCF'_D) we obtain the expression for $m^D(k, w, u)$, and because $R = m + r + w$ from (ICB_D), we obtain the expression for $R^D(k, w, u)$. The Lagrangean associated with the maximization problem ($\mathscr{P}'_D$) is given by:

$$\mathscr{L}^D = \frac{1}{2}m^2 + w - \alpha + \lambda^D[m(Q - r - m) - u].$$

Then, $\partial \mathscr{L}^D / \partial m = 0$ implies that

$$\lambda^D = \frac{m^D}{2m^D - (Q - r)}.$$

Non-negativity of the Lagrange multiplier implies that

$$m^D \geq \frac{Q-r}{2}.$$

Differentiating (PCF'_D) with respect to k, w and u we get

$$\frac{\partial m^D}{\partial k} = \frac{\partial m^D}{\partial w} = 0, \quad \text{and} \quad \frac{\partial m^D}{\partial u} = -\frac{\lambda^D}{m^D} < 0.$$

Since $R^D = m^D + r + w$, we have

$$\frac{\partial R^D}{\partial k} = 0, \ \frac{\partial R^D}{\partial w} = 1, \quad \text{and} \quad \frac{\partial R^D}{\partial u} = -\frac{\lambda^D}{m^D} < 0.$$

This completes the proof of the Lemma. □

B Proof of Lemma 2

Substituting for R from (ICB_S) into the expected payoffs of the bank and the firm, and writing $A \equiv w + k$ yield the following reduced form maximization problem:

$$\max_m \ V(m) \equiv \frac{1}{2}m^2 - k, \tag{$\mathscr{P}'_S$}$$

$$\text{subject to} \ \ U(m) \equiv m(Q + A - \beta r - m) = u. \tag{PCF'_S}$$

Solving for m from (PCF'_S) we obtain the expression for $m^S(k, w, u)$, and because $R = m + \beta r - k$ from (ICB_D), we obtain the expression for $R^S(k, w, u)$. The Lagrangean associated with the maximization problem ($\mathscr{P}'_D$) is given by:

$$\mathscr{L}^S = \frac{1}{2}m^2 - k + \lambda^S[m(Q + A - \beta r - m) - u].$$

Then, $\partial \mathscr{L}^S / \partial m = 0$ implies that

$$\lambda^S = \frac{m^S}{2m^S - (Q + A - \beta r)}.$$

Non-negativity of the Lagrange multiplier implies that

$$m^S \geq \frac{Q + A - \beta r}{2}.$$

Differentiating (PCF'_S) with respect to k, w and u we get

$$\frac{\partial m^S}{\partial k} = \frac{\partial m^S}{\partial w} = \lambda^S > 0, \quad \text{and} \quad \frac{\partial m^S}{\partial u} = -\frac{\lambda^S}{m^S} < 0.$$

Since $R^S = m^S + r + w$, we have

$$\frac{\partial R^S}{\partial k} = \lambda^S - 1 = \frac{Q + A - \beta r - m^S}{2m^S - (Q + A - \beta r)}, \ \frac{\partial R^D}{\partial w} = \lambda^S, \quad \text{and} \quad \frac{\partial R^S}{\partial u} = -\frac{\lambda^S}{m^S} < 0.$$

Since $Q + A - \beta r > 1$ by assumption, we have $Q + A - \beta r > m$, and hence, $\lambda^S - 1 > 0$. This completes the proof of the Lemma. □

C Proof of Proposition 1

We first derive the individual bargaining frontier under each mode of bank funding. First, consider financing through insured deposits. The bargaining frontier, being the maximum value function of program ($\mathscr{P}_D$), is given by:

$$v(m^D(k,w,u)) = \frac{1}{2}\{m^D(k,w,u)\}^2 + w - \alpha.$$

We know that $\underline{m}^D \leq m^D(k,w,u) \leq 1$. Note that

$$\begin{aligned}\bar{v}^D &\equiv v(1) = \frac{1}{2} + w - \alpha,\\ \underline{v}^D &\equiv v(\underline{m}^D) = \frac{(Q-r)^2}{8} + w - \alpha.\end{aligned}$$

Now let the utility of the firm in the match (k,w) be given by:

$$u(m^D(k,w,u)) = m^D(k,w,u)\{Q - r - m^D(k,w,u)\}.$$

Therefore,

$$\begin{aligned}\underline{u}^D &\equiv u(1) = Q - r - 1,\\ \bar{u}^D &\equiv u(\underline{m}^D) = \frac{(Q-r)^2}{4}.\end{aligned}$$

From the above it follows that the bargaining frontier $\phi^D(k,w,u)$ under D is given by:

$$\phi^D(k,w,u) = \begin{cases} \frac{1}{2} + w - \alpha & \text{for } 0 \leq u \leq \underline{u}^D,\\ \frac{1}{8}\left(Q - r + \sqrt{(Q-r)^2 - 4u}\right)^2 + w - \alpha & \text{for } \underline{u}^D < u < \bar{u}^D,\\ [0, \underline{v}^D] & \text{for } u = \bar{u}^D.\end{cases} \tag{BFD}$$

Now, consider financing through loan securitization. The maximum value function of program ($\mathscr{P}_S$) is given by:

$$v(m^S(k,w,u)) = \frac{1}{2}\{m^S(k,w,u)\}^2 - k.$$

We know that $\underline{m}^S \leq m^S(k,w,u) \leq 1$. Note that

$$\begin{aligned}\bar{v}^S &\equiv v(1) = \frac{1}{2} - k,\\ \underline{v}^S &\equiv v(\underline{m}^S) = \frac{(Q - \beta r + A)^2}{8} - k.\end{aligned}$$

Let the utility of the firm in the match (k,w) be given by:

$$u(m^S(k,w,u)) = m^S(k,w,u)\{Q - \beta r + A - m^S(k,w,u)\}.$$

Therefore,

$$\begin{aligned}\underline{u}^S &\equiv u(1) = Q - \beta r + A - 1,\\ \bar{u}^S &\equiv u(\underline{m}^S) = \frac{(Q - \beta r + A)^2}{4}.\end{aligned}$$

From the above it follows that the bargaining frontier $\phi^S(k, w, u)$ under S is given by:

$$\phi^S(k, w, u) = \begin{cases} \frac{1}{2} - k & \text{for } 0 \le u \le \underline{u}^S, \\ \frac{1}{8}\left(Q - \beta r + A + \sqrt{(Q - \beta r + A)^2 - 4u}\right)^2 - k & \text{for } \underline{u}^S < u < \bar{u}^S, \\ [0, \underline{v}^S] & \text{for } u = \bar{u}^S. \end{cases} \tag{BFS}$$

Now we are ready to derive the combined bargaining frontier which is given by:

$$\phi(k, w, u) = \max\{\phi^D(k, w, u), \phi^S(k, w, u)\}.$$

Notice that $\alpha < w + k$ implies that $\bar{v}^D > \bar{v}^S$. Moreover, $Q - \beta r + A > Q - r$ if and only if $A \equiv w + k > (\beta - 1)r$. So, $\bar{u}^D < \bar{u}^S$ if and only if $A > (\beta - 1)r$. Next we analyze the slopes of the non-linear segments of the two frontiers, which are given by:

$$\phi_3^D(k, w, u) = -\lambda^D = \frac{m^D}{2m^D - (Q - r)},$$
$$\phi_3^S(k, w, u) = -\lambda^S = \frac{m^S}{2m^S - (Q - \beta r + A)}.$$

It is immediate to show that $\lambda^D = \left|\phi_3^D(k, w, u)\right| > \left|\phi_3^S(k, w, u)\right| = \lambda^S$ if and only if $A > (\beta - 1)r$. From the above discussion it follows that

- If $A < (\beta - 1)r$, we have $\bar{u}^D > \bar{u}^S$ and $\lambda^D = \left|\phi_3^D(k, w, u)\right| < \left|\phi_3^S(k, w, u)\right| = \lambda^S$. Therefore, $\phi^D(k, w, u)$ stays always above $\phi^S(k, w, u)$, i.e., funding through insured deposits dominates that through loan securitization for all values of u;
- If $A > (\beta - 1)r$, we have $\bar{u}^D < \bar{u}^S$ and $\lambda^D = \left|\phi_3^D(k, w, u)\right| > \left|\phi_3^S(k, w, u)\right| = \lambda^S$. In this case, $\phi^D(k, w, u)$ intersects $\phi^S(k, w, u)$ only once, say at $(\hat{u}, \hat{v})$ where $\hat{v} = \phi^D(k, w, \hat{u}) = \phi^S(k, w, \hat{u})$. Moreover, if $\bar{v}^S < \phi^D(k, w, \underline{u}^S)$, then the intersection occurs on the non-linear segments of both frontiers.

This completes the proof of the proposition, and explains Figure 1. □

D Proof of Proposition 2

The proof of PAM is nothing but formalizing the logic presented in Figure 2. We first analyze the case when $A > (\beta - 1)r$. Consider the financing mode D. The bargaining frontier is given by:

$$\phi^D(k, w, u) = \frac{1}{2}\left\{m^D(k, w, u)\right\}^2 + w - \alpha.$$

It follows from Lemma 1 that $\phi_2^D = 1$ and $\phi_3^D = -\lambda^D$, and hence,

$$-\frac{\phi_2^D}{\phi_3^D} = \frac{1}{\lambda^D},$$

which is the slope of the indifference curve of a type k bank under D. Clearly,

$$\frac{\partial}{\partial k}\left(-\frac{\phi_2^D}{\phi_3^D}\right) = 0. \tag{1}$$

The above condition is equivalent to having the parallel segments of the two curves depicted in Figure 2. Next consider the financing mode S. The bargaining frontier is given by:

$$\phi^S(k, w, u) = \frac{1}{2}\left\{m^S(k, w, u)\right\}^2 - k.$$

It follows from Lemma 2 that $\phi_2^S = m^S\lambda^S$ and $\phi_3^D = -\lambda^S$, and hence,

$$-\frac{\phi_2^S}{\phi_3^S} = m^S,$$

which is the slope of the indifference curve of a type k bank under S. Therefore,

$$\frac{\partial}{\partial k}\left(-\frac{\phi_2^S}{\phi_3^S}\right) = \frac{\partial m^S}{\partial k} = \lambda^S > 0. \tag{2}$$

The above condition is reflected in the fact that, in Figure 2, the steeper segment of the solid curve is flatter than that of the dotted curve. So, conditions (1) and (2) together imply the single-crossing property of the indifference curves, i.e.,

$$\phi(k', w'', u(w'')) = \phi(k', w', u(w')) \implies \phi(k'', w'', u(w'')) \geq \phi(k'', w', u(w')) \tag{SC}$$

for any $k'' > k'$ and $w'' > w'$. We now prove that (SC) implies PAM. Suppose condition (SC) holds for all (k, w), but given two equilibrium matches $w' = \mu^{-1}(k')$ and $w'' = \mu^{-1}(k'')$ with $k'' > k'$, we have $w'' < w'$, i.e., the matching is not positive assortative. Moreover, without loss of generality, suppose that $\phi(k', w'', u(w'')) = \phi(k', w', u(w'))$. Then by (SC), we have $\phi(k'', w', u(w')) \geq \phi(k'', w'', u(w''))$ since $k'' > k'$ and $w' > w''$ by assumption. The last inequality contradicts the fact that

$$w'' = \mu^{-1}(k'') = \text{argmax}_w\, \phi(k'', w, u(w)).$$

Now consider the case when $A < (\beta - 1)r$. In this case, financing through insured deposits strictly dominates loan securitization, i.e., $\phi(k, w, u) = \phi^D(k, w, u)$. Then, from (1) it follows that the indifference curves of any two $k' \neq k''$ are parallel. Therefore, any matching is an equilibrium matching, and so is a positive assortative matching. This completes the proof of the proposition. □

E Proof of Proposition 3

We first analyze the indifference locus along the equilibrium path. Define by

$$\Delta(k, w, u) \equiv \phi^S(k, w, u) - \phi^D(k, w, u),$$

the marginal gains for an arbitrary match (k, w) by switching from D to S. Then, $\hat{u}(k, w)$ is the level of utility that solves $\Delta(k, w, u) = 0$. Take a typical match $k = \mu(w)$ in an equilibrium allocation, and write

$\hat{u}(w) \equiv \hat{u}(\mu(w), w)$, which is the equilibrium indifference locus. We are interested in finding the sign of $\hat{u}'(w)$. Given $\Delta(\mu(w), w, \hat{u}(w)) = 0$, we have

$$\hat{u}'(w) = -\frac{\partial\Delta/\partial w}{\partial\Delta/\partial u}.$$

Note that by substituting the values of m^S and m^D we obtain

$$\Delta = \frac{1}{8}\left(Q-\beta r+A(w)+\sqrt{(Q-\beta r+A(w))^2-4u}\right)^2 - \frac{1}{8}\left(Q-r+\sqrt{(Q-r)^2-4u}\right)^2 - A(w)+\alpha.$$

Therefore,

$$\begin{aligned}\frac{\partial\Delta}{\partial w} =& \frac{1}{4}[1+\mu'(w)]\underbrace{\left(Q-\beta r+A(w)+\sqrt{(Q-\beta r+A(w))^2-4u}\right)}_{2m^S}\times \\ &\underbrace{\left(1+\frac{Q-\beta r+A(w)}{\sqrt{(Q-\beta r+A(w))^2-4u}}\right)}_{2\lambda^S} - [1+\mu'(w)] \\ =& [1+\mu'(w)](m^S\lambda^S-1)\end{aligned}$$

We now prove that $m^S\lambda^S \geq 1$, which is equivalent to

$$\begin{aligned}&\frac{(m^S)^2}{2m^S-(Q-\beta r+A)} \geq 1 \\ \iff& (m^S)^2-2m^S+(Q-\beta r+A) \geq 0\end{aligned}$$

The left-hand-side of the above inequality is clearly strictly decreasing in m^S, and hence, its minimum value is attained at $m^S = 1$, which is given by $Q-\beta r+A-1$. Since $Q-\beta r+A > 1$ by assumption, we prove that $m^S\lambda^S \geq 1$, and hence, $\partial\Delta/\partial w \geq 0$.

Now,

$$\frac{\partial\Delta}{\partial u} = \underbrace{\frac{1}{2}\cdot\frac{Q-r+\sqrt{(Q-r)^2-4u}}{\sqrt{(Q-r)^2-4u}}}_{\lambda^D} - \underbrace{\frac{1}{2}\cdot\frac{Q-\beta r+A+\sqrt{(Q-\beta r+A)^2-4u}}{\sqrt{(Q-\beta r+A)^2-4u}}}_{\lambda^S} = \lambda^D-\lambda^S > 0. \quad (3)$$

Therefore, we have $\hat{u}'(w) \leq 0$. On the other hand, from (ODE) it follows that $u'(w) > 0$. Therefore, the curve $u(w)$ in Figure 3 intersects the indifference locus $\hat{u}(w)$ only once at w^* which is the threshold level beyond which all equilibrium matches choose to finance through loan securitization. Note that, if $u(w)$ is steep enough, it may cross the indifference locus in its vertical segment at $\hat{w}$, and in this case $w^* = \hat{w}$. This proves the monotonicity of bank funding modes in any equilibrium allocation. □

F Proof of Proposition 4

Note that

$$\begin{aligned}m(w) &\equiv m(\mu(w), w, u(w)) = \max\{m^D(\mu(w), w, u(w)), m^S(\mu(w), w, u(w))\} \equiv \max\{m^D(w), m^S(w)\}, \\ R(w) &\equiv R(\mu(w), w, u(w)) = \max\{R^D(\mu(w), w, u(w)), R^S(\mu(w), w, u(w))\} \max \equiv \{R^D(w), R^S(w)\}.\end{aligned}$$

Therefore, in order to prove the non-monotonicity of $m(w)$ and $R(w)$ with respect to w it suffices to show that

$$\frac{dx^D}{dw} \leq 0 \text{ and } \frac{dx^S}{dw} \geq 0 \text{ for } x = m, R.$$

It follows from Lemmas 1, 2, the first-order condition (ODE), and Proposition 2 that

$$\begin{aligned}
\frac{dm^D}{dw} &= \frac{\partial m^D}{\partial k}\mu'(w) + \frac{\partial m^D}{\partial w} + \frac{\partial m^D}{\partial u}u'(w) = 0 + 0 - \frac{\lambda^D}{m^D}\cdot\frac{1}{\lambda^D} = -\frac{1}{m^D} \leq 0,\\
\frac{dR^D}{dw} &= \frac{\partial R^D}{\partial k}\mu'(w) + \frac{\partial R^D}{\partial w} + \frac{\partial R^D}{\partial u}u'(w) = 0 + 1 - \frac{\lambda^D}{m^D}\cdot\frac{1}{\lambda^D} = -\frac{1-m^D}{m^D} \leq 0,\\
\frac{dm^S}{dw} &= \frac{\partial m^S}{\partial k}\mu'(w) + \frac{\partial m^S}{\partial w} + \frac{\partial m^S}{\partial u}u'(w) = \lambda^S\mu'(w) + \lambda^S - \frac{\lambda^S}{m^S}\cdot m^S = \lambda^S\mu'(w) \geq 0,\\
\frac{dR^S}{dw} &= \frac{\partial R^S}{\partial k}\mu'(w) + \frac{\partial R^S}{\partial w} + \frac{\partial R^S}{\partial u}u'(w) = (\lambda^S - 1)\mu'(w) + \lambda^S - \frac{\lambda^S}{m^S}\cdot m^S = (\lambda^S - 1)\mu'(w) \geq 0.
\end{aligned}$$

The last inequality is true because $m^S\lambda^S \geq 1$ implying that $\lambda^S \geq 1$. This completes the proof of the proposition. □

G The income and substitution effects

First we analyze the income effect. Consider the equilibrium matches $(\mu(w), w)$ that choose deposit financing, i.e., $w < w^*$. Then, as derived in Appendix D,

$$u'(w) = -\frac{\phi_2^D}{\phi_3^D} = \frac{1}{\lambda^D} = \frac{2m^D - (Q-r)}{m^D}.$$

Therefore,

$$\frac{du'(w)}{dr} = \frac{2}{m^D}\cdot\frac{m^D - (Q-r)}{2m^D - (Q-r)} < 0.$$

When the equilibrium matches choose loan securitization, i.e., $w > w^*$, we have

$$u'(w) = -\frac{\phi_2^S}{\phi_3^S} = m^S.$$

It follows from (ICB_S) that

$$\frac{du'(w)}{dr} = \frac{dm^S}{dr} = -\frac{\beta}{\lambda^S} < 0.$$

Therefore, an increase in r shifts the equilibrium $u(w)$ function down for all $w > w_{min}$ as in Figure 5. At $w = w_{min}$ the equilibrium utility does not change since $u(w_{min}) = u_0$ in both equilibria.

The substitution effect refers to a change in the indifference locus $\hat{u}(w)$ keeping the equilibrium utility $u(w)$ constant. First note that

$$A(\hat{w}) = \hat{w} + \mu(\hat{w}) = (\beta - 1)r \implies \frac{d\hat{w}}{dr} = \frac{\beta - 1}{1 + \mu'(\hat{w})} > 0 \text{ since } \beta > 1 \text{ and } \mu'(\cdot) > 0.$$

Therefore, the vertical part of the indifference locus shifts to the right following an increase in r. Next, we analyze the shift in the downward sloping portion of the indifference locus. By the Implicit Function Theorem,

$$\frac{d\hat{u}(w)}{dr} = -\frac{\partial\Delta/\partial r}{\partial\Delta/\partial u}.$$

Because $\partial\Delta/\partial u > 0$ by (3), we have

$$\text{sign}\left[\frac{d\hat{u}(w)}{dr}\right] = -\,\text{sign}\left[\frac{\partial\Delta}{\partial r}\right]. \tag{4}$$

It is easy to derive that

$$\frac{\partial\Delta}{\partial r} = m^D\lambda^D - \beta m^S\lambda^S.$$

The sign of the above expression depends on endogenous variables, and hence, it is difficult to characterize such shifts. But condition (4) is easy to interpret. Note that $\Delta(\mu(w), w, u)$ is the marginal gain for the bank in an equilibrium match $(\mu(w), w)$ by switching to loan securitization. Thus, $\partial\Delta/\partial r < 0$ means that this marginal gain is decreasing in r, and hence, the region over which the equilibrium matches prefer loan securitization (the region labeled $\mathbf{U}^S$ in Figure 3) must shrink. Because β represent the relative cost of loan securitization, the sign of $\partial\Delta/\partial r$ is likely to depend on the values of β. One may expect that for high values of β we should have $\partial\Delta/\partial r < 0$. This is in fact true. We are considering the values of β such that $A \geq (\beta-1)r$, i.e., $\beta \leq 1 + (A/r)$. It is easy to show that

$$\frac{\partial\Delta}{\partial r} = -\frac{A}{r}\cdot\frac{\left(Q-r+\sqrt{(Q-r)^2-4u}\right)^2}{4\sqrt{(Q-r)^2-4u}} < 0 \quad \text{for} \quad \beta = 1+\frac{A}{r}.$$

Therefore, by continuity, we may conclude that $d\hat{u}(w)/dr > 0$, i.e., the indifference locus shifts up following an increase in r for high values of β. It is also easy to show that

$$\frac{\partial\Delta}{\partial r} > 0 \quad \text{for} \quad \beta = 1 \quad \text{and} \quad Q-r, Q-\beta r + A > \frac{4}{3}.$$

Therefore, characterizing the shift in the indifference locus for low values of β is a difficult task. This possible case of substitution effect is depicted in Figure 6. □

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