A Two-Sided Matching Model of Monitored Finance[☆]

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Abstract

We analyze an incentive contracting model of partnership formation between heterogeneous investors and entrepreneurs. Partnerships commonly have double-sided moral hazard in entrepreneurial effort and monitoring by investors. Greater monitoring ability implies stronger incentives to monitor. On the other hand, low-net worth borrowers are more informationally opaque. Hence, the double-sided moral hazard problem is best mitigated by assigning low-net worth entrepreneurs to high-ability investors following a negative assortative matching pattern. Moreover, negative assortative matching implies that the equilibrium loan rate may be non-monotonic in net worth. Finally, our model sheds light on how adverse shocks, which worsen the economy-wide financial conditions of firms, affect the cost of external borrowing and the expected firm value.

JEL classification: D82, J33, J41.
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1. Introduction

Starting with the seminal work of Hölmstrom and Tirole (1997), a plethora of theoretical models have analyzed how underlying informational asymmetry precludes the implementation of an efficient contractual structure in a lending relationship. In particular, the informational failure stems from the

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fact that, in the presence of limited liability, low-net worth borrowers cannot be expected to exert high effort, as they cannot be forced to share losses with lenders in the event of failure. This is the so-called *borrower moral hazard* problem. Therefore, while borrowers with high net worth find it easier to raise funds from uninformed investors, e.g., public debt, firms with weaker balance sheets are obliged to rely on costly intermediated capital. In other words, the wealthier the entrepreneur is, the weaker is his incentive to deviate from the lender's interests due to higher inside equity participation. Monitoring by lenders, which induces the entrepreneurs of borrowing firms to behave diligently, can mitigate the borrower moral hazard problem. However, investors cannot contractually commit to such costly actions; hence, a *lender moral hazard* problem arises. Thus, double-sided moral hazard (in entrepreneurial effort and lender monitoring) often permeates a typical lending relationship.

The main objective of the present paper is to offer a unified general equilibrium framework for a competitive credit market to understand the effects of endogenous sorting between heterogeneous investors (lenders) and entrepreneurs (borrowers) on the optimal loan contracts. In particular, we aim to answer the following questions regarding a typical lender-borrower market.

- When investor-entrepreneur partnerships are formed via endogenous matching and each partnership is subject to double-sided moral hazard, do investors of given types sort themselves into firms with particular characteristics? We therefore provide a theoretical explanation for the recent empirical evidence on endogenous sorting in corporate loan markets (e.g., Fernando, Gatchev, and Spindt, 2005; Sorensen, 2007; Chen, 2013; Chen and Song, 2013; Schwert, 2016).
- Under endogenous sorting, is the optimal loan contract associated with each lender-borrower partnership different from what would have been predicted by the standard agency models in which the lender-borrower relationship is treated in isolation? Our paper thus aims to offer a plausible explanation for the empirical non-monotonic relationship between the loan rate and borrower collateral (e.g., Berger, Frame, and Ioannidou, 2015).

To shed light on the aforementioned issues, we analyze a competitive credit market in which heterogeneous entrepreneurs form *partnerships* with heterogeneous investors. Risk-neutral entrepreneurs that are heterogeneous with respect to net worth lack sufficient funds, and hence, must rely on institutional investors to fund their projects. The lack of verifiability of entrepreneurial effort, which determines the probability distribution of the stochastic cash flow of a project, along with limited liability gives rise to a moral hazard problem in effort choice. Risk-neutral investors that are heterogeneous with respect to monitoring ability monitor their borrowers in order to mitigate the entrepreneurial moral hazard problem. However, an investor cannot credibly commit to a stipulated level of monitoring, and hence, there is an additional incentive problem in monitoring. Thus, partnership formation is subject to a double-sided moral hazard problem that impedes the implementation of efficient outcomes.

In a lending relationship, the ability of the lender is as critical as borrower quality in mitigating incentive problems and enhancing firm value. In the present model, ability is simply the efficiency of monitoring technology an investor owns. For empirical purposes, the efficiency of monitoring technology may sometimes be difficult to quantify, but can be proxied by the types of institutional investors.

For example, Almazan, Hartzell, and Starks (2005) claim that, compared with bank trust departments and insurance companies, investment advisors and investment companies in general entail lower costs of monitoring. Ability may also be proxied by investor attributes such as expertise (e.g., Almazan, 2002), experience (e.g., Sorensen, 2007), size (e.g., Chen, 2013), independence of organizational structure (e.g., Bottazzi, Da Rin, and Hellmann, 2008), or size of the human capital (e.g., Dimov and Shepherd, 2005), which may limit the intensity of investor monitoring. In this sense, the monitoring cost function we propose should be viewed as a reduced-form function in which ability either represents one of the aforementioned investor attributes or has a strong positive association with one of them. Because ability in our model influences an investor's incentive to monitor, as Kaplan and Schoar (2005) argue, differences in the ability of investors imply differences in the performance of borrowing firms.

The role of lender capital in determining investor's monitoring incentives has been paid much attention in the extant literature. There are two competing sets of theories. Diamond and Rajan (2001) assert that if a larger portion of the investment is financed through bank deposits, the depositors may discipline a bank by threatening to withdraw their deposits whenever they think that their claim is at risk. Consequently, the bank would have incentives to invest in specific monitoring skills such as loan collection effort. This establishes a negative association between equity capital and monitoring efforts. By contrast, Mehran and Thakor (2011) argue that greater equity capital reduces the probability of future bank closure (by the regulator). This in turn strengthens a bank's incentive to monitor its borrowers so that the bank can reap the marginal benefit of monitoring in the form of increased value of its loan portfolio. A similar positive association between bank capital and incentives to monitor has been established in Hölmstrom and Tirole (1997), and Allen, Carletti, and Marquez (2011). Given this theoretical disagreement, it may often be misleading to assume a positive association between ability and equity capital. In this sense, our model is fundamentally different from that of Hölmstrom and Tirole (1997) as we think, apart from equity capital, investor ability is an important factor that determines a lender's monitoring incentives. In fact, Almazan (2002) considers a model of bank competition to analyze the roles of expertise and capital. He claims "the banking industry will be more efficient the more important expertise is relative to capital".

The competition for 'good quality' borrowers and lenders naturally emerges in such markets because both net worth and monitoring ability significantly influence the performance of a firm. Limited liability implies that low-net worth entrepreneurs require more intense due diligence, as they tend to be more *informationally opaque*. On the other hand, more able investors who entail lower marginal costs of monitoring have stronger incentives to monitor. Therefore, investors with greater monitoring ability have *comparative advantages* in low-net worth firms. Thus, to maximize efficiency in each partnership, it is optimal to assign low-net worth firms to high-ability investors following a *negative assortative matching* (NAM) pattern. In other words, monitoring ability and net worth are substitutes in mitigating the associated double-sided moral hazard problem.

At this juncture, it is worth discussing how endogenous sorting and incentives interact in each entrepreneur-investor partnership. In a standard agency model where a lending relationship is treated in isolation, one individual, such as the investor, is assumed to posses all the bargaining power, and makes a 'take-it-or-leave-it' contract offer to the borrower taking his outside option as exogenously given. In an investor-entrepreneur market such as ours, the outside option of any borrower is the payoff he can obtain by switching to an alternative partnership. Thus, endogenous matching implies an endogenous type-dependent outside option, and hence, an endogenous bargaining power. This is the general equilibrium effect of a market with endogenous sorting.

Our model also predicts a non-monotonic relationship between the equilibrium loan rate and net worth. The intuition is simple. The optimal loan rate associated with an isolated lender-borrower partnership (with an exogenous outside option), which serves as the instrument that balances double-sided incentive problems, is generally a function of monitoring ability (lender type), net worth (borrower type), and the exogenous outside option of the borrower. An investor with lower monitoring ability has a weaker incentive to monitor, and hence, requires higher marginal compensation (in the form of loan rates) to exert an additional unit of monitoring effort. By contrast, a borrower with high net worth is less informationally opaque and requires less intense monitoring, and hence, the monitor must retain a smaller portion of the realized cash flow. Therefore, the greater the net worth is, the lower the loan rate is. Finally, if the exogenous outside option of an entrepreneur increases, then he must pay a lower loan rate due to the increased bargaining power. To understand why the loan rate in a Walrasian equilibrium, where the outside option is endogenous, may be non-monotonic in net worth, consider two investor-entrepreneur partnerships with two distinct levels of net worth. In equilibrium, the borrower with high net worth is matched with a less able monitor following NAM and has a higher endogenous outside option. Thus, this entrepreneur must pay a lower loan rate than the other borrower. We call this the *outside option effect*. However, a *matching effect* implies that the matched partner of this high-net worth entrepreneur must receive a higher loan rate because she has a weaker incentive to monitor. Because of these two countervailing effects, the equilibrium loan rate may be non-monotonic in net worth.

As is typical with assignment models, the nature of equilibrium matching between investors and entrepreneurs is independent of the distribution of types on each side of the market. However, the shapes of the equilibrium matching and payoff functions may change following a change in the type distributions. Terviö (2008) shows how changes in type distributions change the shape of the equilibrium matching functions and have positive spillover effects on the upper tail of the type distributions. Thus, in our model, changes in type distributions have important comparative statics implications for the equilibrium loan rate as a function of net worth. If, at a given level of net worth, the number of borrowers increases relative to the number of lenders with the corresponding monitoring ability, then these lenders gain higher bargaining power because the borrowers are now relatively abundant. As a result, the same type of borrowers must pay a higher loan rate. By exploiting this simple intuition, we employ a set of numerical exercises to show that both a variance-preserving change in skewness and a mean-preserving spread of the distribution of net worth imply asymmetric changes in the relative bargaining power of borrowers. Consequently, some borrowers pay higher loan rates, whereas others pay lower loan rates.

The contribution of the present paper to the literature on incentive contracting and partnership formation is two fold. First, when the individuals may seek for alternative partners, the model helps endogenize the outside option of each borrower as opposed to the models (e.g., Besanko and Kanatas, 1993; Hölmstrom and Tirole, 1997; Repullo and Suárez, 2000) in which the outside options of the individuals on one side of the market are exogenously given. The aforementioned models are amenable to determine the optimal incentive structure in an organization in the sense that they determine the way fixed surplus must be divided between the lender and borrower in a given lending relationship. A fixed outside option of a borrower also pins down the payoff achievable by his matched partner. In a general equilibrium model such as ours, the endogenous outside option not only determines the structure of incentive pay, but also its level in each partnership, and hence, the bargaining power of each individual. Later in Section 6, we show how changes in type distributions may alter the endogenous bargaining power of investors and entrepreneurs, and analyze the implications for the equilibrium loan rate and firm value (which is the expected retained earning of an entrepreneur). Second, we contribute to the literature on partnership formation (e.g., Farrell and Sctochmer, 1988) which argues that economic agents who differ in abilities will form partnerships by equally sharing the surplus if abilities are complementary. In the context of corporate lending, formation of partnerships is often subject to several market imperfections, among which the informational constraint plays an important role. When partnerships are subject to moral hazard, incentive contract for a particular match gives rise to a non-linear Pareto frontier implying that match surplus cannot be transferred between principal and agent on a one-to-one basis and an equal sharing of surplus cannot be implemented. Thus, substitutability rather than complementarity explains why heterogeneous individuals may form partnerships and agree to share the match output according to endogenously determined sharing rules.

The remainder of the paper is organized as follows. In Section 2, we briefly review the related theoretical literature. In the following section, we describe the fundamentals of our model. In Section 4, we analyze the optimal loan contracts associated with an arbitrary investor-entrepreneur partnership. We analyze the partnerships and contracts in a Walrasian equilibrium in Section 5. In the next section, we present the comparative static results of changes in the distribution of net worth. In Section 7, we discuss some testable empirical implications of our model. We conclude the paper in Section 8. All proofs are relegated to the appendix.

2. Related literature

Our paper contributes to the recent theoretical literature on incentive contracting and endogenous matching between heterogeneous principals and agents. Most of the works in this context extend Sattinger's (1979) 'differential rents' model, which analyzes the assignment of heterogeneous workers to heterogeneous firms. At the heart of this model is the fact that more productive workers are assigned to firms in which they have greater marginal impact (comparative advantage). In other words, the optimal firm-worker matching is assortative if the match surplus function exhibits the *single crossing property* (SCP) or the *difference condition*. When the match surplus is fixed, it induces a linear Pareto frontier, i.e., in each match, utility is fully transferable between the worker and the firm. Legros and Newman (2007) generalize the aforementioned model and propose a *generalized differences* condition, a sufficient condition for assortative matching when utility is not fully transferable. Optimal assignment models are similar in nature to the optimal screening problems (e.g. Maskin and Riley, 1984) in which consumption allocation is monotonic in buyer type if the utility function exhibits SCP. In other words, the optimal mechanism perfectly *separates* the types.[1]

As we have discussed earlier, one novelty of our model is to incorporate double-sided moral hazard into a classical assignment model under imperfect transferability. Because partnerships are formed by writing binding contracts, the sufficient condition for assortative matching arises endogenously. The current model resembles that of Chakraborty and Citanna (2005), which also analyzes partnerships under double-sided moral hazard. Because wealthier individuals are less wealth constrained, they accept occupations with more severe incentive problems. Partnerships are assortative, and an increase in median wealth improves the welfare of poorer agents.

In the context of corporate finance, two classes of papers analyze the effect of endogenous matching on incentive contracts. The first type, of which the present paper is an example, studies the effects of endogenous investor-entrepreneur matching on the optimal financial contracts.[2] Dam and Pérez-Castrillo (2006) analyze the effect of endogenous principal-agent matching on interlinked agrarian contracts in a market with borrower heterogeneity. Because the principals are homogeneous, the agents assume all the (endogenous) bargaining power in each match, and hence, the equilibrium effort and investment are more efficient than that implied by the standard agency models in which a principal makes a take-it-or-leave-it offer to the hired agent. In a similar model, von Lilienfeld-Toal and Mookherjee (2016) show that a less stringent bankruptcy law allows high-net worth borrowers greater leverage. Moreover, a weakened bankruptcy law increases the bargaining power of wealthier borrowers. Besley, Buchardi, and Ghatak (2012) analyze the effect of competition between lenders that are heterogeneous with respect to the cost of capital and consider variations in property rights on optimal loan contracts. If competition is sufficiently intense (more similar lenders), the borrower receives his outside option. Improved property rights, which allows the borrower to pledge a larger proportion of wealth as collateral, relaxes the borrower incentive problem and reduces the loan rate. In contrast to the present paper, the aforementioned works consider matching markets with one-sided heterogeneity, and hence, any changes in market fundamentals affect equilibrium contracts only through the endogenous outside option. However, these models do not take into account the effect of assortative matching. As we show in Section 6, changes in type distributions may have asymmetric effects on equilibrium allocations because of the countervailing matching and outside option effects. One paper that considers the effects of lender-borrower sorting is that of Cabolis, Dai, and Serfes (2015), who analyze a venture capital (VC) market. They show positive assortative matching between VC rank (given by their stage-specific expertise) and firm quality (proxied by the probability of successful exit) at each stage. Moreover, those authors establish a non-monotonic relationship between specialization and competition.

[1] Terviö (2008) provides an excellent discussion of the similarity between the optimal assignment and screening models.

[2] The second class of papers analyzes the effect of assigning managerial talent to firm characteristics on the optimal managerial compensation. There is assortative matching, i.e., more talented managers run larger firms (e.g., Terviö, 2008; Edmans, Gabaix, and Landier, 2009), safer firms (e.g., Li and Ueda, 2012), or firms with greater market power (e.g., Dam, 2015). Many of these papers show that assortative matching significantly explains the observed variations in the level and incentive structure of CEO pay.

3. The model

3.1. Agents

The economy, which spans three dates $t = 0, 1, 2$, consists of two classes of agents: a continuum $I = [0, 1]$ of heterogeneous risk-neutral investors and a continuum $J = [0, 1]$ of risk-neutral entrepreneurs. Each entrepreneur with initial wealth or net worth $w \in W = [0, 1]$ owns a startup project whose initial outlay is 1. A dollar invested in each project yields a stochastic but verifiable cash flow. If a project succeeds, it yields $Q > 1$ with probability e and 0 in the case of failure. Projects are ex ante identical across firms, but the entrepreneurs are assumed to be heterogeneous with respect to net worth. Let $F(w)$ be the fraction of borrowers with net worth lower than or equal to w. In other words, $F(w)$ is the cumulative distribution function of net worth, and let $f(w)$ be the corresponding density function with $f(w) > 0$ for all $w \in W$. In what follows, we refer to w as the 'type' of entrepreneur, and this information is publicly observable. We assume that the probability of success, e, is the same as the non-verifiable effort level chosen by an entrepreneur at the total cost

$$C(e) = \frac{e^2}{2c},$$

with $c > 0$. The effort cost function is identical across entrepreneurs. No borrower can commit to a pre-specified level of effort, and hence, lack of verifiability gives rise to the *borrower moral hazard* problem.[3]

An entrepreneur with net worth w is thus required to raise $1 - w$ through external borrowing.[4] The institutional investors have unlimited funds, but funding a project requires mitigating the aforementioned moral hazard problem by monitoring the borrower.[5] Following the agency model of Besanko and Kanatas (1993), we assume that the intensity of monitoring is positively correlated with the difference between the *monitoring* and *non-monitoring* efforts of a borrower. If the financier of a project monitors her borrower, she can oblige the entrepreneur to exert a stipulated effort e, which is referred to as the monitoring effort. On the other hand, let e_0 denote the effort exerted by a borrower if he is not monitored, which we call the non-monitoring effort. The difference $e - e_0$ thus represents the level or intensity of monitoring by a lender. If more resources are spent on monitoring, greater effort would be exerted relative to the

[3] We assume $cQ \leq 1$ such that the first best effort level $e^{FB} = \text{argmax}_{e\in[0,1]} \{eQ - C(e) - 1\} = \min\{cQ, 1\} = cQ$. We also assume that the first best surplus $e^{FB}Q - C(e^{FB}) - 1 = (1/2)cQ^2 - 1 > 0$, i.e., each project is viable at least at the first best level. Note that $Q > 2$ is a necessary condition for both $cQ \leq 1$ and $cQ^2 > 2$ to hold simultaneously.

[4] An entrepreneur can potentially borrow more than $1 - w$ and consume part of it, but such financing arrangement can be shown to be suboptimal.

[5] Monitoring may often mean something broader than what we analyze here. There are mainly three types of investor actions classified as monitoring. First, one type includes the "screening" activities that investors undertake to gather information about their borrowers, and the financing decision depends on the acquired information. The second type is "ex ante monitoring", a costly action that an investor chooses before the true (verifiable) cash flow is realized. Finally, the third genre is "ex post monitoring" or "costly state verification", an action that an investor may choose to undertake at a cost after the true non-verifiable cash flow is yielded. We model the second class of investor action, which is more prevalent in the corporate loan markets (e.g., Hölmstrom and Tirole, 1997; Sufi, 2007).

non-monitoring effort. We assume that the cost incurred by an investor in monitoring is given by the following:

$$D(e-e_0;m)=\begin{cases}\frac{(e-e_0)^2}{2m} & \text{if } e>e_0,\\ 0 & \text{if } e\le e_0,\end{cases}$$

where $m \in M = [0, m_{max}]$ with $m_{max} < c$.[6] The parameter m in the monitoring cost function represents the 'ability' or 'efficiency' of a lender or the investor 'type'. The higher the m is, the greater the ability is, as an investor with higher m has a lower cost for each additional unit of monitoring. No lender can pre-commit to such actions, and hence, costly monitoring gives rise to an additional *lender moral hazard* problem. The particular form of the above monitoring cost function implies that at the optimum, there is no excessive monitoring in the sense that a lender would not exert monitoring effort if $e \le e_0$. The assumption that $m_{max} < c$ places a lower bound on the marginal cost of monitoring and guarantees the second-order conditions. We assume that investors are heterogeneous with respect to monitoring ability. Let $G(m)$ be the cumulative distribution function of net worth. Furthermore, let $g(m)$ be the corresponding density function with $g(m) > 0$ for all $m \in M$. Lender types are also publicly observable, and the type distributions are taken as the primitives of our economy under consideration. Let $\xi = (G, F)$ denote a generic investor-entrepreneur economy or market with two-sided heterogeneity.

3.2. Partnerships and loan contracts

On date 1, if an investor agrees to finance an entrepreneur's project, then an investor-entrepreneur partnership or match forms. As types, not individual names, matter, a typical partnership will be denoted by (m, w). We treat partnership formation as an endogenous matching problem in which an investor with a given ability m is assigned to an entrepreneur with a given level of net worth w. To this end, we extend Sattinger's (1979) "differential rents" model to an environment in which utility is not perfectly transferable. Formally, each partnership (m, w) forms via a one-to-one matching rule $\lambda : W \to M$, which assigns to each net worth level $w \in W$ an investor ability $\lambda(w) \in M$. One of our main objectives is to determine the equilibrium matching pattern, i.e., which types of lenders and borrowers will form partnerships. The following definition describes a negative assortative matching (NAM) pattern.

Definition 1 (Negative assortative matching) *Investor-entrepreneur matching is negatively assortative if $\lambda'(w) \le 0$, i.e., a lender with high (low) ability forms a partnership with an entrepreneur with low (high) net worth.*

[6] The following is an alternative formulation, as in Hölmstrom and Tirole (1997); Repullo and Suárez (2000), under which all our results remain unchanged. Assume instead that each project, apart from the verifiable cash flow, yields non-verifiable private benefits $B(e)$ that are strictly decreasing and concave in e. Assume, without loss of generality, that

$$B(e)=\frac{1-e^2}{2c}.$$

Monitoring by a lender with intensity $e-e_0$ implies a loss of private benefits of amount $B(e_0)-B(e)$ for the borrower, i.e., monitoring mitigates but cannot eliminate completely borrower moral hazard.

Each partnership (m, w) writes a binding *loan contract* that specifies state-contingent transfers $r(0)$ and $r(Q)$ to the investor at $t = 2$. We assume *limited liability* such that in the event of failure, no agent is paid, i.e., $r(0) = 0$. Let $R = r(Q)$ denote the 'loan rate'.

4. An arbitrary investor-entrepreneur partnership

We first analyze the optimal loan contract for an arbitrary partnership (m, w). The Nash incentive compatibility constraints that determine the optimal monitoring and non-monitoring efforts as functions of the loan rate R are given by the following:[7]

$$e_0 = \text{argmax}_{\hat{e}_0} \{U(R, e, \hat{e}_0 \mid m, w) \equiv \hat{e}_0(Q - R) - C(\hat{e}_0) - w\} = c(Q - R) \equiv e_0(R), \tag{ICE}$$

$$e = \text{argmax}_{\hat{e}} \{V(R, \hat{e}, e_0 \mid m, w) \equiv \hat{e}R - D(\hat{e} - e_0) - (1 - w)\} = e_0(R) + mR \equiv e(R), \tag{ICI}$$

If the above is substituted into $V(R, e - e_0, e \mid m, w)$ and $U(R, e - e_0, e \mid m, w)$, the expected payoffs of the investor and the entrepreneur in an arbitrary match (m, w) are reduced to the following:

$$V(R \mid m, w) = e(R)R - D(e(R) - e_0(R); m) - (1 - w),$$
$$U(R \mid m, w) = e(R)(Q - R) - C(e(R)) - w.$$

Limited liability requires that $R \in [0, Q]$. Thus, the type m investor solves the following maximization problem:

$$\phi(m, w, \bar{u}(w)) = \max_{R \in [0, Q]} \{V(R \mid m, w) \text{ such that } U(R \mid m, w) = \bar{u}(w)\}, \tag{$\mathscr{P}$}$$

where

$$U(R \mid m, w) = \bar{u}(w) \tag{PC}$$

is the *participation constraint* of the borrower, with $\bar{u}(w) \geq 0$ being his type-dependent outside option.[8] The maximum value function $\phi(m, w, \bar{u}(w))$ is the Pareto frontier associated with an arbitrary partnership (m, w). The following lemma characterizes the optimal loan contract associated with an arbitrary match.

[7]Monitoring adds value to a firm by relaxing an entrepreneur's incentive problem. The first-order necessary condition (ICE) can be written as follows (allowing for both the interior and corner solutions):

$$Q - R \leq C'(e).$$

If the non-monitoring effort level is chosen, then the above condition is given by $Q - R \leq e_0/c$. Now suppose that monitoring is able to oblige the borrower an effort level $e > e_0$. Thus, e must satisfy $Q - R \leq e/c$, and monitoring is meant to relax the incentive constraint of a borrower by $(e - e_0)/c$. In the present contracting model we assume that the extent of relaxation of the borrower incentive constraint is directly proportional to the monitoring effort exerted by a lender, and hence, there is no loss of generality in assuming the particular form of monitoring cost function.

[8]It is well known that, under limited liability, the participation constraint of the borrower may not bind for low values of his outside option, i.e., he earns an *efficiency wage*. In Section 5, we show that in the equilibrium of the investor-entrepreneur economy, the participation constraint of each borrower must be binding, and hence, we ignore the optimal contracts under a slack participation constraint.

Lemma 1 *In an arbitrary partnership* (m, w),

(a) *The optimal loan rate* $R(m, w, \bar{u}(w))$ *is determined by (PC) with* $0 < R(m, w, \bar{u}(w)) < Q$. *The optimal entrepreneurial effort* $e(m, w, \bar{u}(w))$ *and monitoring level* $a(m, w, \bar{u}(w)) \equiv e(m, w, \bar{u}(w)) - e_0(m, w, \bar{u}(w))$ *are simultaneously determined by (ICE) and (ICI);*

(b) *The optimal loan rate is monotonically decreasing in investor ability m, net worth w and the entrepreneur's outside option* $\bar{u}(w)$*; and*

(c) *The optimal monitoring is monotonically increasing in investor ability m, and monotonically decreasing in net worth w and the entrepreneur's outside option* $\bar{u}(w)$.

Consider first the effect of an increase in m. Note that the incentive constraint (ICI) of the investor dictates that his marginal income R must be equal to her marginal cost of monitoring. Because the marginal cost is monotonically decreasing in m, other things being equal, she requires a lower marginal compensation R, and hence, the optimal loan rate decreases with m. By contrast, an increase in ability for a given level of R implies that it is now less costly for the investor to marginally increase monitoring effort, and hence, the optimal monitoring increases. Next, an increase in w implies greater inside equity participation by the entrepreneur. Because the participation constraint of the entrepreneur binds at the optimum and his expected utility (gross of w) is strictly decreasing in R, the only way to induce the borrower to accept the contract is to lower the loan rate. However, this undermines the incentive for the investor to increase monitoring, and hence, the optimal monitoring intensity decreases. Finally, consider the effect of an increase in the entrepreneur's outside option. In optimal contracting problems under moral hazard and limited liability, it is typical for the investor to face a trade-off between incentive provision and giving up ex ante rent to the borrower. When $\bar{u}(w)$ increases, the entrepreneur gains greater bargaining power, and hence, the investor is forced to pay him more. Because the participation constraint is binding, this extra payment cannot be given in the form of additional rent; rather, it must be given in the form of stronger incentives. As a consequence, $Q - R$ increases, i.e., R decreases. A decrease in R implies lower monitoring at the optimum.

It is worth noting that when the loan contracts in any partnership (m, w) are subject to incentive problems, the associated Pareto frontier is not separable in the entrepreneur's outside option $\bar{u}(w)$, i.e., $\phi(m, w, \bar{u}(w))$ cannot be expressed as $\Phi(m, w) - \bar{u}(w)$, where $\Phi(m, w)$ is the exogenously given match surplus. In other words, the match surplus is not "fully transferable" between the investor and the entrepreneur because, under double-sided moral hazard, the size of the pie crucially depends on how the pie is divided. The imperfect transferability of surplus will be the crux of our analysis.

5. The Walrasian equilibrium

We now analyze the set of equilibrium allocations. An allocation for the economy ξ is a matching rule λ, and the corresponding vectors of payoffs $\boldsymbol{v}$ and $\boldsymbol{u}$, where $v(m) \in \boldsymbol{v}$ represents the payoff of each type m investor and $u(w) \in \boldsymbol{u}$ is the payoff of each type w entrepreneur. In a Walrasian equilibrium allocation, each type m investor chooses an entrepreneur type w to maximize her payoff $\phi(m, w, u(w))$.[9] Formally,

Definition 2 (Walrasian allocation) An allocation $(\lambda, \boldsymbol{v}, \boldsymbol{u})$ is a Walrasian equilibrium allocation for the investor-entrepreneur economy ξ if the following conditions are satisfied:

(a) **Maximization:** Given $u(w) \in \boldsymbol{u}$ for each $w \in W$,

$$v(m) = \max_w \phi(m, w, u(w)), \tag{$\mathscr{M}$}$$

for each $m \in M$.

(b) **Market clearing:** For any subinterval $[i_0, i_1] \subseteq I$, let $i_k = G(m_k)$ for $k = 0, 1$, i.e., m_k is the ability of the investor at the i_k-th quantile. Similarly, for any subinterval $[j_0, j_1] \subseteq J$, let $j_h = F(w_h)$ for $h = 0, 1$. If $[m_0, m_1] = \lambda([w_0, w_1])$, then it must be the case that $G(m_1) - G(m_0) = F(w_1) - F(w_0)$.

We treat $\boldsymbol{u}$ as the Walrasian price vector posted by entrepreneurs. Part (a) of the above definition asserts that each type m investor chooses her partner's type optimally by taking the price vector as given. Part (b) is a measure-consistency requirement, the standard 'demand-supply equality' condition of a continuum economy. Note that the Lebesgue measure of a subinterval $[i_0, i_1]$ of investors is $i_1 - i_0 = G(m_1) - G(m_0)$, and that of a subinterval $[j_0, j_1]$ of entrepreneurs is $j_1 - j_0 = F(w_1) - F(w_0)$. Thus, measure-consistency requires that if $[j_0, j_1]$ is matched to $[i_0, i_1]$, then these two subintervals cannot have different measures.[10]

5.1. *Equilibrium payoffs*

In a market with endogenous matching, the outside option of each entrepreneur is the maximum payoff he could obtain by switching to alternative matches, and hence it is endogenous. We first argue that in every type (m, w) partnership, the participation constraint of the entrepreneur must bind in a Walrasian equilibrium. Suppose that an entrepreneur of type w is offered $u(w)$ in an equilibrium allocation. Because there is a continuum of types, one can find an identical investor who would also offer $u(w)$ to the same borrower, and hence, $u(w)$ actually becomes his outside option. Thus, any borrower payoff that is strictly above the outside option cannot be an equilibrium payoff. In other words, in a Walrasian allocation, there must not be any additional surplus remaining to bargain over, as types are arbitrarily close to one another.

[9] The payoff that each type m investor maximizes is the maximum payoff accrued from the optimal loan contract analyzed in the previous section. That is, the utility allocation would lie on the associated Pareto frontier, i.e., the loan contract is Pareto optimal. If the utility allocation lies strictly below the frontier, then both the investor and the entrepreneur would be better off by proposing a different contract, which would contradict the definition of a Walrasian allocation. Because $u(w)$ is endogenous for each type w borrower, a take-it-or-leave-it contract offer is equivalent to any other bargaining protocol (see Chakraborty and Citanna, 2005).

[10] The notion of Walrasian equilibrium is equivalent to that of *stability* in a matching model that asserts that an allocation is stable if (a) there is no individual who can propose a contract different from what he/she receives in the current allocation and can be made strictly better off, and (b) there does not exist any investor-entrepreneur pair which could sign a feasible contract, different than that have been agreed upon in the current allocation, which would make both the investor and entrepreneur strictly better off.

The first-order condition of the maximization problem ($\mathscr{M}$) associated with each type m investor implies that

$$u'(w) = -\frac{\phi_2(m, w, u(w))}{\phi_3(m, w, u(w))} \equiv \psi(m, w, u(w)) \qquad \text{for } m = \lambda(w). \tag{1}$$

It is easy to show that $\phi_1(m, w, u(w)) > 0$, $\phi_2(m, w, u(w)) > 0$ and $\phi_3(m, w, u(w)) < 0$ (see Lemma 2 in Appendix B), and hence, $u'(w) = \psi(m, w, u(w)) > 0$ for all $w \in W$. The above differential equation is the "demand function for type w", which is nothing but the equality between the marginal earnings of a type w entrepreneur and his marginal contribution to the match surplus. Moreover, the above first-order condition is similar to the *local downward incentive constraint* of an optimal screening problem. Applying the Envelope theorem to the maximization problem ($\mathscr{M}$), we obtain

$$v'(m) = \phi_1(m, w, u(w)) \qquad \text{for } m = \lambda(w). \tag{2}$$

Given that $\phi_1(m, w, u(w)) > 0$, we have $v'(m) > 0$ for all $m \in M$.

Let $u_0 \geq 0$ be the reservation utility of all entrepreneurs irrespective of the level of net worth, which represents the utility obtained by any entrepreneur if his project is not financed by any investor, and hence, is not undertaken. Note that the notion of outside option differs from that of reservation utility, which is an exogenous object. In a Walrasian allocation, we must have $u(w) \geq u_0$ for all $w \in W$. This is the "individual rationality" condition that has been omitted in the maximization problem ($\mathscr{M}$). Because $u(w)$ is strictly increasing in w, u_0 must also be the outside option of the entrepreneur with the minimum net worth $w = 0$, i.e., $u(0) = u_0$. Therefore, the equilibrium payoff function $u(w)$, which is the solution to the differential equation (1), is given by the following:

$$u(w) = u_0 + \int_0^w \psi(\lambda(x), x, u(x))dx. \tag{3}$$

The above condition implies that $u(w) - u_0$ is the area below the curve $\psi(\lambda(w), w, u(w))$ between 0 and w, and hence, $u(w) > u_0$ for all $w > 0$, i.e., all borrower types except the least wealthy type earn type-specific rents. This is similar to the *informational rent* of an agent in an optimal screening problem, which is monotonically increasing in w.

5.2. *Equilibrium matching*

Next, we analyze the matching pattern in a Walrasian allocation. The sign of $\lambda'(w)$ is determined by the second-order condition associated with the maximization problem ($\mathscr{M}$) of each type m investor, which is given by the following:

$$\Psi(m, w, u(w))\lambda'(w) \geq 0, \quad \text{for } m = \lambda(w), \tag{4}$$

where

$$\Psi(m, w, u(w)) \equiv \phi_2(m, w, u(w))\phi_{31}(m, w, u(w)) - \phi_3(m, w, u(w))\phi_{21}(m, w, u(w)).$$

We show that $\Psi(m, w, u) < 0$ for all (m, w, u), and hence the inequality (4) is satisfied only if $\lambda'(w) \leq 0$. In summary,

Proposition 1 *In any Walrasian allocation* $(\lambda, \boldsymbol{v}, \boldsymbol{u})$*, we have* $\lambda'(w) \leq 0$*, i.e., more able lenders invest in firms with lower net worth following a negative assortative matching pattern.*

The intuition behind the above proposition, although not extremely surprising, calls for a careful explanation. From condition (2), it follows that

$$\frac{\partial}{\partial w} v'(m) = \frac{\partial}{\partial w} \phi_1(m, w, u(w)) = -\frac{\Psi(m, w, u(w))}{\phi_3(m, w, u(w))} < 0.$$

The above inequality implies that the marginal payoff of any type m lender is lower, corresponding to higher values of w. In other words, the investor payoff function corresponding to any two w and w' with $w < w'$ can cross only once. This is the same as the 'single crossing property' (SCP) in an optimal screening problem (e.g., Maskin and Riley, 1984), or the 'decreasing difference' property in an optimal assignment problem (e.g., Sattinger, 1979; Legros and Newman, 2007). [11] The reason the equilibrium matching is negatively assortative is best understood with only two types of firms w and w' with $w < w'$. Recall that $\Psi(m, w, u(w)) < 0$ implies that $v(m)$ corresponding to w is steeper than that corresponding to w'. In Figure 1, the steeper curve is labeled $v(m; w)$, and the flatter one is labeled $v(m; w')$, and they cross one another only once. Because each type m investor must maximize payoff by choosing w optimally [Definition 2-(a)], the equilibrium investor payoff function is given by $\max\{v(m; w), v(m; w')\}$, which is the upper envelope of the two payoff functions in Figure 1.

[Insert Figure 1 about here]

This clearly separates the investor type space into two disjoint subintervals: $[0, m^*]$ is matched with the higher net worth level w', and $(m^*, c]$ is assigned to the lower net worth level w. This is nothing but NAM in this special case with only two entrepreneurial types, which is analogous to an optimal separating mechanism in the screening models.

The single crossing property, and hence, NAM are driven by the fact that $\phi_{21} < 0$ and $\phi_{31} < 0$, which together imply that m and w are substitutes. Under imperfect transferability, this substitutability has two aspects. First, investor and entrepreneur types are not only substitutes in creating match surplus, i.e., $\phi_{21} < 0$, but also substitutes in transferring surplus, i.e., $\phi_{31} < 0$, in the sense that it is less costly (at the margin) for low-ability monitors to transfer surplus to high-net worth entrepreneurs. This two-fold substitutability of m and w implies that more able monitors have comparative advantage in firms with lower net worth, and hence, it is optimal to assign high monitoring ability to low-net worth firms in a Walrasian allocation.

For investors with ability $m \to 0$, monitoring is prohibitively costly, and hence, they will be unable to exert any monitoring effort in equilibrium. Such investors can be thought of as uninformed intermediaries, e.g., market investors. In principle, all firms have the option to form partnerships with market

[11] The SCP of the Pareto frontier is equivalent to that of the match surplus function $\Phi(m, w, u(w)) = \phi(m, w, u(w)) + u(w)$. Legros and Newman's (2007) generalized difference condition is a condition on the Pareto frontier, whereas Mailath, Postlewaite, and Samuelson (2014) use the property of the match surplus function to characterize the matching equilibrium.

investors. But an NAM in the Walrasian equilibrium implies that only high-net worth entrepreneurs, i.e., those with $w \to 1$, will be able to exercise this option. In other words, the financing of high-net worth firms are akin to public debt.

It is also worth noting that the Walrasian allocation is efficient. Note first that the loan contract for each partnership formed in a Walrasian equilibrium is Pareto optimal because the contract selects a point on the Pareto frontier for each match (m, w). Moreover, the equilibrium investor-entrepreneur matching $m = \lambda(w)$ is *Pareto efficient* in the sense that this one maximizes the aggregate match surplus of the market among all possible assignment rules. To visualize this, suppose that the equilibrium matching is NAM, but is not Pareto efficient. Then, there must be at least two matches (m_1, w_1) and (m_2, w_2) with $m_1 < m_2$ and $w_1 < w_2$ such that

$$\phi(m_1, w_1, u(w_1)) + \phi(m_2, w_2, u(w_2)) > \phi(m_1, w_2, u(w_2)) + \phi(m_2, w_1, u(w_1)). \tag{5}$$

The above inequality is strict because at least one agent must be strictly better off. From the above, it follows that

$$\int_{w_1}^{w_2} [\phi(m_2, w, u(w)) - \phi(m_1, w, u(w))]dw > 0,$$

which holds only if

$$\Psi(m, w, u(w)) > 0,$$

which is a contradiction. This property of a Walrasian allocation is a type of first welfare theorem, but the notion of Pareto efficiency is stronger. In an equilibrium allocation, both the matching rule and the contract are optimal, and hence, the utility allocations for each match are optimal. Therefore, the inequality in (5) is true only if there exists a loan contract satisfying incentive compatibility and limited liability constraints that would make a deviating lender-borrower pair strictly better off.

5.3. Equilibrium loan contracts

The recent empirical literature on principal-agent matching (e.g., Ackerberg and Botticini, 2002) claims that optimal incentive contracts under endogenous matching can be very different from those predicted by the standard agency theory that treats a principal-agent partnership in isolation. To understand this point, suppose that there are two types of entrepreneurs (those with high and low net worth) and two types of investors (those with high and low monitoring ability). Standard agency models would predict that a higher loan rate must be associated with low net worth. On the other hand, low-ability investors must receive higher loan payments because incentive problems are more stringent with such lenders. We have already shown that in the Walrasian equilibrium of this economy, borrowers with low net worth are matched with more able lenders following a negative assortative matching pattern. Because optimal loan rate is decreasing in monitoring efficiency, through endogenous matching, a lower repayment obligation will be associated with low net worth. Therefore, the outcome of an assignment model will offer predictions about the equilibrium loan rate with respect to net worth that are exactly the opposite of what would have been predicted by standard agency theory. Thus, a natural question is to ask whether, under double moral hazard, the equilibrium loan rate and monitoring intensity are monotone functions of net worth.

Note that the Walrasian equilibrium variables are functions of net worth via endogenous matching and that $u(w) = \bar{u}(w)$ for each $w \in W$. Therefore, the equilibrium loan rate and monitoring can be expressed as functions of w alone, which are given by the following:

$$R(w) \equiv R(\lambda(w), w, u(w)), \tag{6}$$

$$a(w) \equiv a(\lambda(w), w, u(w)). \tag{7}$$

First, we analyze the behavior of the equilibrium loan rate with respect to net worth. Differentiating (6) with respect to w, we obtain the following:[12]

$$R'(w) = \underbrace{R_1\lambda'(w)}_{\text{matching effect}} + \underbrace{R_3\,[1+u'(w)]}_{\text{outside option effect}}. \tag{8}$$

Thus, the behavior of the equilibrium loan rate with respect to net worth can be decomposed into two countervailing effects: a *matching effect* and an *outside option effect*. Because $R_1 < 0$ (by Lemma 1) and $\lambda'(w) < 0$, the first effect is positive. The second effect, however, is negative because $R_3 < 0$ (by Lemma 1) and $u'(w) > 0$. Thus, the relationship between the equilibrium loan rate and net worth is generally non-monotonic, and the monotonicity of the equilibrium loan rate with respect to w thus depends on which of the two countervailing effects dominates.

Proposition 2 *The equilibrium loan rate $R(w)$ is generally non-monotonic with respect to net worth.*

Rearranging the terms in (8), we have $R'(w) \gtrless 0$ if and only if

$$|\lambda'(w)| \gtrless \chi(m, w, u(w)), \tag{9}$$

where

$$\chi(m, w, u(w)) \equiv \frac{c^2(Q - R(m, w, u(w))) + m^2 R(m, w, u(w))}{m(cQ - (2c - m)R(m, w, u(w)))[R(m, w, u(w))]^2}.$$

The above is a condition on the endogenous variables, and hence, it is difficult to provide an analytically tractable sufficient condition under which the equilibrium loan rate is monotonic in net worth. One may find many such sufficient conditions, one of which is when the monitors are identical. In this case, the matching function is horizontal, i.e., $\lambda'(w) = 0$, implying a null matching effect, and hence, $R'(w) < 0$. This would be the prediction of a matching model with one-sided heterogeneity. The monotonicity of the equilibrium loan rate may also depend on the characteristics of the distribution functions $F(w)$ and $G(m)$, and hence, one may find alternative sufficient conditions for monotonic loan rates. The upper panel of Figure 2 depicts two equilibrium loan rate functions under the assumption that monitoring ability is uniformly distributed on $[0, 0.2]$, whereas net worth follows a beta distribution with parameters $\alpha, \beta > 0$.

[12] It follows from the proof of Lemma 1 that $R_2 = R_3$, and hence,

$$R'(w) = R_1\,\lambda'(w) + R_2 + R_3\,u'(w) = R_1\,\lambda'(w) + R_3[1 + u'(w)].$$

[Insert Figure 2 about here]

The non-monotonic function corresponds to the parameter values $Q = 4$, $c = 0.25$, $u_0 = 0.1$, $\alpha = 1$ and $\beta = 20$. This implies that approximately 88% of entrepreneurs have net worth less than 0.1. In this case, for low values of w, $|\lambda'(w)|$ is very high, i.e., the matching effect is dominant, and hence, $R'(w) > 0$. However, corresponding to high values of net worth, $|\lambda'(w)|$ is very low, and hence, $R'(w) < 0$. Intuitively, in this economy with a large proportion of low-net worth firms, the average level of the borrower incentive problem is high, and hence, the role of monitoring becomes important. Therefore, for low values of net worth, the matching effect dominates. For high values of net worth, borrower moral hazard decreases, and the role of the outside option in ameliorating the incentive problem is dominant. Next, the downward-sloping function corresponds to $Q = 4$, $c = 0.25$, $u_0 = 0.1$, $\alpha = 20$ and $\beta = 1$. In this case, approximately 88% of entrepreneurs have net worth more than 0.9. Because the average incentive problem is low in this economy, the role of monitoring becomes less important. Therefore, the outside option effect dominates the matching effect for all values of w and $R'(w) < 0$.

One can interpret w as the pledgeable collateral of entrepreneurs instead of inside equity. In this case, entrepreneurs with higher collateral will be easier to incentivize because differences in collateral imply differences in liability limits.[13] However, the loan rate associated with each investor-entrepreneur partnership is equivalent to the loan risk premium (the loan rate minus the risk-free rate) because the risk-free rate has been taken as exogenous. Therefore, Proposition 2 can be rewritten as describing the nature of the equilibrium relationship between the loan risk premium and collateral. Ex post theories of collateral assert that observably riskier borrowers are often required to pledge higher collateral to reduce agency costs, and hence, there is a positive correlation between the loan risk premium and collateral (e.g., Boot and Thakor, 1994). Ex ante theories of collateral, by contrast, postulate that unobservably safer borrowers often pledge higher collateral to signal their quality, and thus, there exists a negative relation between loan risk premium and collateral (e.g., Besanko and Thakor, 1987). Empirical findings endorse both strands of the literature.[14] Boot, Thakor, and Udell (1991) analyze a model of secured lending by combining the above two attributes – private information and moral hazard – and show that default risk may be either increasing or decreasing with respect to borrower quality. In a recent study, Berger et al. (2015) find empirical evidence of a non-monotonic relation between the loan risk premium and collateral. These authors argue that collateral may have different desirable economic characteristics, such as liquidity, divertibility, and outside ownership status, each of which may influence loan risk in a different way; hence, the empirical relation between the loan risk premium and collateral may not be monotonic. We provide an alternative explanation for the non-monotonicity of the loan risk premium and default risk with respect to net worth,[15] which complements the claim of Berger et al. (2015). En-

[13]One requires a slight modification of the contracting model in Section 4. If w is to be treated as collateral, then each investor finances the entire project, i.e., lends 1. In this case, an entrepreneur would have a gross income of w in the event of failure, of which he would make a transfer $r(0) = r \in [0, w]$. Under binding limited liability, we would then have $r = w$. All our results would remain qualitatively unchanged under this modification.

[14]See Berger and Udell (1990); Brick and Palia (2007) for a positive relationship and Degryse and Van Cayseele (2000); Agarwal and Hauswald (2010) for a negative relationship.

[15]Note that in our model, the equilibrium default risk is given by $1 - e(w)$, where $e(w) \equiv e(\lambda(w), w, u(w))$, which is

trepreneurial net worth is a homogeneous good, as measured in terms of money; however, in the market equilibrium, this becomes differentiated as different levels of net worth must be combined with different levels of monitoring ability through endogenous matching.

The behavior of equilibrium monitoring with respect to net worth can also be decomposed into a matching effect and an outside option effect. Differentiating (7) with respect to w, we obtain the following:

$$a'(w) = \underbrace{a_1 \lambda'(w)}_{\text{matching effect}} + \underbrace{a_3 [1 + u'(w)]}_{\text{outside option effect}} .$$

However, in this case, the two effects point in the same direction, as it follows from Lemma 1 that $a_1 > 0$ and $a_3 < 0$, and hence, $a'(w) < 0$. Therefore,

Proposition 3 *Equilibrium monitoring $a(w)$ is monotonically decreasing in net worth.*

The main implication of Proposition 3 is that lenders with greater incentives to monitor, i.e., higher m, actually monitor more in equilibrium. By contrast, borrowers who require more intense due diligence, i.e., borrowers with lower w, are in fact monitored more intensely in equilibrium. The extant empirical literature has used the leverage ratio and credit rating (e.g., Sufi, 2007; Nini, Smith, and Sufi, 2009) as broad measures of the information opacity of borrowers. However, measuring monitoring has been a somewhat challenging task. Ono and Uesugi (2009) measure monitoring intensity by the frequency of document submissions to banks in order to analyze the use of collateral in the Japanese SME loan market. Cerqueiro, Ongena, and Roszbach (2014) study the effect of a decline in collateral value on bank monitoring. They analyze data from a major Swedish bank, for which monitoring is measured as the time in months elapsed between two consecutive collateral revaluations to the same borrower. Sufi (2007) finds that more informationally opaque firms are monitored more intensively. Cerqueiro et al. (2014) find that a lender less frequently monitors the assets pledged by firms as collateral. Therefore, Proposition 3 conforms to the findings of the two aforementioned papers, among others. One must interpret Proposition 3 carefully while mapping our results to the empirical evidence regarding monitoring intensity in loan contracts. The established equilibrium relationship between the incentive to monitor and net worth, i.e., the equilibrium matching function, is not necessarily equivalent to the equilibrium relationship between the amount of monitoring and net worth. Given the particular specifications of our agency model, an NAM implies a monotonically decreasing monitoring effort function (with respect to net worth), as both the matching and outside option effects point in the same direction.

6. Effects of changes in the type distribution

The characteristics of a Walrasian allocation of our investor-entrepreneur economy, particularly equilibrium assortative matching, have been derived under very general assumptions and for the given arbitrary type distributions $G(m)$ and $F(w)$. Although the assortative matching property of equilibrium

perfectly positively correlated with the equilibrium loan rate.

allocations is distribution-free, the shape of the equilibrium matching function $\lambda(w)$ depends crucially on the shapes of the distribution functions of types. Such dependence allows us to conduct meaningful comparative statics exercises, namely, the effects of changes in the type distributions on Walrasian allocations.

First, let us establish how the shape of the equilibrium matching depends on the distributions of ability and net worth. Suppose that a type w entrepreneur is matched with a type m investor. An immediate consequence of Definition 2-(b) and NAM is that

$$G(m) = 1 - F(w) \implies m = G^{-1}(1 - F(w)) \equiv \lambda(w). \tag{10}$$

The two type distribution functions also help determine the slope of the matching function. Differentiating the above equation, we obtain

$$\lambda'(w) = -\frac{f(w)}{g(m)} \quad \text{for} \;\; m = \lambda(w).$$

Because, by assumption, $g(m) > 0$ for all $m \in M$ and $f(w) > 0$ for all $w \in W$, we must have a matching function $\lambda(w)$ that is strictly decreasing. In other words, in an equilibrium allocation, each net worth level must be matched with a unique monitoring efficiency level, i.e., $\lambda(w)$ is not a 'correspondence'. Note that the density functions $g(\cdot)$ and $f(\cdot)$ are local measures of the dispersions of the corresponding distributions. Therefore around any equilibrium matching $m = \lambda(w)$, $g(m) > (<) f(w)$ implies that more (less) mass is concentrated around $m = \lambda(w)$ than around w, i.e., the relative dispersion of the type distributions determines the slope of the equilibrium matching function.

6.1. Effects on equilibrium matching and borrower payoff

Our first objective is to analyze the effects of changes in type distributions on the equilibrium matching function $\lambda(w)$ and the borrower payoff function $u(w)$. Because only the relative density $f(w)/g(m)$ is relevant to the shape of the matching function, henceforth, we assume without loss of generality that m is uniformly distributed on M. In this case, $g(m)$ is constant for all m, and the equilibrium matching function is a (scaled) mirror image of $F(w)$. Therefore, a change in $F(w)$ would affect $u(w)$ via shifts in the matching function.

Consider two investor-entrepreneur economies $\xi^1 = (F_1, G)$ and $\xi^2 = (F_2, G)$, which differ only in the distributions of net worth. Recall that

Definition 3 (Stochastic dominance) *Let F_1 and F_2 be two cumulative distribution functions of net worth defined on the same support $W = [0, 1]$.*

(a) *F_1 is said to first-order stochastically dominate F_2, or $F_1 \succeq_{FOSD} F_2$ if*

$$F_1(w) \le F_2(w) \;\; \textit{for all} \;\; w \in W;$$

(b) *F_1 is said to second-order stochastically dominate F_2, or $F_1 \succeq_{SOSD} F_2$ if there is a unique $w^* \in (0, 1)$ such that*

$$F_1(w) \le (\ge) F_2(w) \text{ for all } w < (\ge) w^*,$$
$$\text{and } \int_0^1 [F_1(x) - F_2(x)]dx = 0.$$

Definition 3-(b) is a restrictive definition of second-order stochastic dominance, but it is the most popular example of such a stochastic order. It asserts that the distribution functions $F_1(w)$ and $F_2(w)$ cross one another only once, and they have the same expectation. We can immediately show that Definition 3-(b) implies that $F_1 \succeq_{SOSD} F_2$, or F_2 is a *mean-preserving spread* (MPS) of F_1 (see Diamond and Stiglitz, 1974).[16] The definition also implies that the distribution of net worth in ξ^2 is more unequal than that in ξ^1. With the above two types of stochastic orders in mind, we are ready to state the following results.

Proposition 4 *Consider two investor-entrepreneur economies ξ^1 and ξ^2 that differ only in the distributions F_1 and F_2 of net worth. Furthermore, let $\lambda_k(w)$ be the equilibrium matching function and $u_k(w)$ be the equilibrium payoff of each type w entrepreneur in economy ξ^k for $k = 1, 2$.*

(a) *If $F_1 \succeq_{FOSD} F_2$, then $\lambda_1(w) \ge \lambda_2(w)$, and $u_1(w) \le u_2(w)$ for all $w \in W$;*
(b) *If $F_1 \succeq_{SOSD} F_2$, then $\lambda_1(w) \ge (\le) \lambda_2(w)$ for $w \le (\ge) w^*$, and either* (i) *$u_1(w) \le u_2(w)$ for all $w \in W$ or* (ii) *there is a unique $\hat{w} \in (w^*, 1)$ such that $u_1(w) \le (\ge) u_2(w)$ for $w \le (\ge) \hat{w}$.*

The nature of the shift in the equilibrium matching function resulting from a change in the distribution of net worth is trivial. Because the equilibrium matching function in any investor-entrepreneur economy is a mirror image of the cumulative distribution function $F(w)$, it must be the case that $\lambda_h(w) \ge \lambda_k(w)$ whenever $F_h(w) \le F_k(w)$ for $h, k = 1, 2$ and $h \ne k$. Therefore, if for any given $w \in W$, we have $F_1(w) \le (\ge) F_2(w)$; then, any entrepreneur with this net worth level must be matched with an investor with lower (higher) monitoring ability in the equilibrium of ξ^2.

The nature of the shift in the equilibrium borrower payoff function $u(w)$ following a change in the type distribution function $F(w)$ is less trivial. First, let us discuss the channel through which a change in the distribution of net worth affects the equilibrium payoff of an entrepreneur. Note that

$$\frac{\partial}{\partial m} \psi(m, w, u(w)) = \frac{\Psi(m, w, u(w))}{[\phi_3(m, w, u(w))]^2}.$$

In an economy with NAM, the above expression is negative because $\Psi(m, w, u(w)) < 0$. Therefore, $\lambda_1(w^0) \ge \lambda_2(w^0)$ for a given $w^0 \in W$ implies that $\psi(\lambda_1(w^0), w^0, u(w^0)) \le \psi(\lambda_2(w^0), w^0, u(w^0))$. Because $u(w) - u_0$ is the area under the curve $\psi(\lambda(w), w, u(w))$, we have $u_2(w^0) \ge u_1(w^0)$. In other words, a change in the distribution of net worth shifts bargaining power from entrepreneurs to investors.

When F_1 first-order stochastically dominates F_2 and, consequently, $\lambda_1(w) \ge \lambda_2(w)$ for all $w \in W$, each investor in market ξ^1 gains greater bargaining power relative to ξ^2, as each entrepreneur is matched

[16] Second-order stochastic dominance may imply multiple crossings that we do not consider as the results of the analysis would not be very tractable.

with a more efficient monitor (except the entrepreneurs with 0 and 1) in ξ^1. Therefore, the shift in bargaining power from entrepreneurs to investors is uniform in the sense that all investors gain and all entrepreneurs lose in the equilibrium of economy ξ^1. Thus, although the equilibrium surplus of each match is higher in ξ^1, each investor consumes a larger proportion of the match surplus, i.e., $u(w)$ is lower for each w.

With a change in the distribution of net worth in the sense of second-order stochastic dominance, the shift in bargaining power from entrepreneurs to investors is asymmetric because under such a change, the $\psi(\lambda(w), w, u(w))$ curve rotates down around some w. Therefore, for low levels of net worth, the bargaining power of associated investors increases, whereas for high values of net worth, entrepreneurs gain increased bargaining power. The results of Proposition 4-(b) are obtained depending on which of the two countervailing forces is dominant.

6.2. *Effects on the equilibrium loan rate: numerical results*

As the equilibrium loan rate is expressed as a function of net worth, any change in $F(w)$, the cumulative distribution of net worth, would also change the equilibrium loan rate. In a static model such as ours, such exercises should be viewed as the effects of cross-sectional variations in the investor-entrepreneur market. To analyze the effect of a change in the distribution of net worth on $R(w)$, the equilibrium loan rate, one must first solve the differential equation in (1). When we substitute for $\phi_2(m, w, u(w))$ and $\phi_3(m, w, u(w))$, the differential equation is reduced to the following:

$$u'(w) = \psi(\lambda(w), w, u(w)) = \frac{[c^2 - c\lambda(w) + (\lambda(w))^2]R(\lambda(w), w, u(w))}{c[cQ - (2c - \lambda(w))R(\lambda(w), w, u(w))]}.$$

Under imperfectly transferable surplus for which the Pareto frontier does not depend linearly on $u(w)$, the above ordinary differential equation does not have an analytical solution. Moreover, it is difficult to determine the nature of shifts in the equilibrium loan rate. We therefore resort to numerical simulation of the model to examine some particular comparative statics results. For this purpose, we set $Q = 4$, $c = 0.25$, and $u_0 = 0.1$. We assume that monitoring ability is uniformly distributed on $M = [0, 0.2]$ and that net worth follows a beta distribution with parameters α and β, i.e.,

$$F(w) = \frac{\int_0^w x^{\alpha-1}(1-x)^{\beta-1}dx}{\int_0^1 w^{\alpha-1}(1-w)^{\beta-1}dw}, \quad \alpha, \beta > 0.$$

There are two principal reasons for choosing a beta distribution. First, one must define bounded support for net worth, w, which is consistent with the model's assumptions. Second, as we have seen from the theoretical analysis, heterogeneities in the distributions of m and w are crucial to determine the relative importance of the matching and outside option effects on the equilibrium variables; thus, a beta distribution is sufficiently flexible to consider alternative specifications for the relative heterogeneity between investors and entrepreneurs. For example, if $\alpha = \beta = 1$, the beta distribution reduces to uniform distribution. As we have discussed earlier, because two comparable markets differ only in the corresponding distributions of net worth, there is no loss of generality in assuming a uniform distribution of monitoring ability.

We conduct two comparative statics exercises. First, we assume that F_1 is a beta distribution function with parameters $\alpha = 1$ and $\beta = 20$, whereas F_2 is a beta distribution function with parameters $\alpha = 20$ and $\beta = 1$. Therefore, the distribution of net worth in economy ξ^1 is positively skewed, whereas that in ξ^2 is negatively skewed, both with the same level of inequality, i.e., the same variance.[17] This is a special case of $F_2 \succeq_{FOSD} F_1$. The following result summarizes the effect of such a change on the equilibrium loan rate.

Result 1 (Change in skewness) *Consider two economies, $\xi^1 = (F_1, G)$ and $\xi^2 = (F_2, G)$ where the distribution of net worth in ξ^1 is positively skewed, whereas that in ξ^2 is negatively skewed, both with the same variance. Then, there is a unique $\hat{w}$ such that an entrepreneur with $w < (>)\, \hat{w}$ pays a lower (higher) loan rate in the equilibrium of ξ^2.*

Let $R_k(w) \equiv R(\lambda_k(w), w, u_k(w))$ be the equilibrium loan rate associated with a common net worth level w in market ξ^k for $k = 1, 2$. Then,

$$\Delta R(w) \equiv R_2(w) - R_1(w) \approx \frac{\partial R}{\partial m}\left[\lambda_2(w) - \lambda_1(w)\right] + \frac{\partial R}{\partial u}\left[u_2(w) - u_1(w)\right]. \tag{11}$$

Proposition 4-(a) implies that if $F_2 \succeq_{FOSD} F_1$, then $\lambda_1(w) < \lambda_2(w)$ and $u_1(w) > u_2(w)$ for all $w \in (0, 1)$. Thus, the first term of the above expression is negative, whereas the second term is positive because both $\partial R/\partial m < 0$ and $\partial R/\partial u < 0$. Therefore, depending on which of the two countervailing effects dominates, $R_1(w)$ can be higher or lower than $R_2(w)$. The intuition of the above result is best understood by analyzing how a change in the skewness of the distribution of net worth affects the bargaining power of entrepreneurs. Because in ξ^2, high-net worth borrowers are relatively abundant, they have lower bargaining power and thus pay higher loan rates. By contrast, low-net worth entrepreneurs are scarce in economy ξ^2, and hence, they have greater bargaining power relative to ξ^1 and pay lower loan rates. The relative strength of these two countervailing effects thus determines the sign of the difference between the equilibrium loan rates $R_1(w)$ and $R_2(w)$, which are depicted in the upper panel of Figure 2.

For the second comparative statics exercise, we assume that F_1 and F_2 are both symmetric beta distribution functions with parameters $\alpha = \beta = 3$ and $\alpha = \beta = 1/3$, respectively. Clearly, F_2 is a mean-preserving spread of F_1, both distributions crossing at $w^* = 0.5$.[18] The effect of a mean-preserving spread

[17] Recall that the variance and skewness of a beta random variate w with parameters α and β are respectively given by the following:

$$\mathrm{Var}[w \mid \alpha, \beta] = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)},$$
$$\gamma_1[w \mid \alpha, \beta] = \frac{2(\beta-\alpha)}{\alpha+\beta+2}\sqrt{\frac{\alpha+\beta+1}{\alpha\beta}}.$$

Therefore, $\mathrm{Var}[w \mid \alpha, \beta] = \mathrm{Var}[w \mid \beta, \alpha]$, and $\gamma_1[w \mid \alpha, \beta] = -\gamma_1[w \mid \beta, \alpha]$.

[18] Recall that the mean of a beta random variate w with parameters α and β is given by the following:

$$E[w \mid \alpha, \beta] = \frac{\alpha}{\alpha+\beta}$$

Therefore, $E[w \mid 3, 3] = E[w \mid 1/3, 1/3] = 0.5$, and $\mathrm{Var}[w \mid 3, 3] = 0.036 < 0.15 = \mathrm{Var}[w \mid 1/3, 1/3]$.

of the distribution of net worth on the equilibrium loan rate is described in the following result.

Result 2 (Change in the inequality of net worth) *Consider two economies, $\xi^1 = (F_1, G)$ and $\xi^2 = (F_2, G)$, where F_2 is a mean-preserving spread of F_1, i.e., the distribution of net worth in ξ^2 is more unequal than that in ξ^1. Then, there is a unique $\bar{w} \in [0, 0.5)$ such that an entrepreneur with $w \leq (\geq)\, \bar{w}$ pays a higher (lower) loan rate in the equilibrium of ξ^2.*

The upper panel of Figure 3 describes the above result.

[Insert Figure 3 about here]

The above case corresponds to part (i) of Proposition 4-(b). When $F_1 \succeq_{SOSD} F_2$, $u_1(w) < u_2(w)$ for all $w \in (0, 1)$, and $\lambda_1(w) > (\leq)\, \lambda_2(w)$ for $w < (\geq)\, 0.5$. Therefore, both expressions of (11) are negative for $w \geq 0.5$, and hence, $R_2(w) < R_1(w)$ for all $w \geq 0.5$. However, for $w < 0.5$, the first term of (11) is positive, but the second term is negative. Therefore, the effect of a mean-preserving spread on the equilibrium loan rate is ambiguous for values of $w < 0.5$. Thus, the overall effect of such change in the distribution is determined by the resulting change in the bargaining power of entrepreneurs. In a more unequal economy, ξ^2, entrepreneurs with high net worth have greater bargaining power because the competition for better entrepreneurs is more fierce compared with the competition in ξ^1. Conversely, low-net worth borrowers have lower bargaining power in economy ξ^2. Therefore, high-net worth entrepreneurs pay lower loan rates in economy ξ^2, whereas low-net worth borrowers face an increased cost of external financing in ξ^2.

The theory of *financial accelerator* (e.g., Bernanke and Gertler, 1989; Bernanke, Gertler, and Gilchrist, 1999) asserts that an economy-wide decline in the average net worth or collateral can amplify the business cycle through contractions in the availability of credit and investment. In a fixed investment model, the equivalent consequence of a negative collateral shock is a uniform rise in the lending rate. The above two results provide additional insights in understanding the effects of cross-sectional changes in the distribution of net worth. Both a decline in the average net worth caused by a variance-preserving change in skewness and an increase in net worth inequality have similar implications for equilibrium loan rates – the cost of external financing increases for low-net worth borrowers but declines for high-net worth entrepreneurs. Thus, in a model with two-sided heterogeneity such as ours, the predictions of financial accelerator theory apply to only a segment of the loan market, namely, low-net worth firms. Interestingly, the prediction that an economic contraction at the aggregate level may have an asymmetric effect across firms is consistent with the broad empirical findings for the U.S. manufacturing sector (e.g., Gertler and Gilchrist, 1994; Bernanke et al., 1996). Result 1 shows that the nature of variation in the distribution of net worth that induces a decline in the average net worth plays an important role in explaining how an aggregate shock could have substantial heterogeneous effects on firms' balance sheet conditions through the credit channels of the financial market.

7. Empirical implications

In this section, we discuss a set of empirical implications of our model and relate them to existing empirical evidence. We further provide some potentially testable predictions of our theory.

NAM between monitoring incentives and informational opacity. Our model predicts a negative assortative matching between monitoring ability and entrepreneurial net worth (Proposition 1). Note that an investor with higher m has a stronger incentive to monitor because she entails lower marginal costs of monitoring. By contrast, an entrepreneur with lower w is more informationally opaque, as the underlying effort incentive problem is more severe. Therefore, Proposition 1 has the following general implication.

Implication 1 *A lender with a stronger incentive to monitor forms a partnership with a more informationally opaque borrower.*

Chen (2013) analyzes data on the U.S. corporate loan market between 1996 and 2003, and find evidence of positive assortative matching of sizes – larger banks tend to invest in larger companies. He argues that in larger banks, loan officers have weaker incentives to collect soft information about borrowers. By contrast, smaller firms tend to be more informationally opaque because they lack thorough records. Therefore, one implication of the positive assortative matching of sizes in Chen (2013) is that banks with stronger monitoring incentives (smaller banks) tend to be matched with more informationally opaque borrowers (smaller firms), which supports our theoretical prediction regarding NAM.

It is well known that the ability to impose more restrictive covenants improve the lender's incentive to monitor (e.g., Rajan and Winton, 1995). Nini et al. (2009) study a sample of private credit agreements between public firms and banks to evaluate the effect of the use of capital expenditure restrictions (covenants) on firm investment. They show that when contracts can be renegotiated, creditors are more likely to use capital expenditure restrictions on borrowers with greater credit risk, as measured by firms' debt-to-cash-flow ratio and credit rating. In this regard Nini et al.'s (2009) paper documents indirect evidence of NAM, i.e., a stronger incentive to monitor is associated with borrowers with higher debt-to-cash-flow ratios, i.e., lower net worth.

Analyzing the syndicated loan market in the U.S., Sufi (2007) shows that a syndicate that lends to more informationally opaque borrowers, i.e., borrowers without SEC filings and third-party credit ratings, is more concentrated, and the lead arranger in such a syndicate tends to hold larger stakes in the loan.[19] By contrast, lending to more creditworthy borrowers is akin to public debt, as the syndicates are

[19] A loan syndicate typically consists of one lead bank that negotiates loan terms and monitors the borrower, and a number of participants who help diversify loan risk. We do not pretend to model the structure of syndicated lending because in our framework, each investor finances one firm and no firm can resort to multiple lenders. However, our model is not an outrageous simplification of reality; in general, a borrower directly negotiates the loan contract with the lead bank, not with the participants. Thus, we can easily extend our contracting model to a situation in which each partnership consists of one borrower and many lenders – one investor (the lead bank) negotiates the loan terms with the borrower and the rest of the lenders (the participants) act as residual claimants. Moreover, there are motives other than risk diversification under which a syndicate forms, but this is beyond the scope of the present paper. Brander, Amit, and Antweiler (2002) provide an excellent analysis of the motives of syndicate formation.

more dispersed with lead banks having smaller cash-flow rights. Because in a larger syndicate the lead arranger tends to hold a smaller proportion of the syndicated loan and the loan risk is well diversified, the lead bank has a weaker incentive to monitor the borrowing firm. Thus, the result of Sufi (2007) provides indirect evidence of NAM in the sense that lenders with stronger monitoring incentives (lead arrangers in concentrated syndicates) lend to more informationally opaque (less creditworthy) borrowers.

If the 'equity monitoring hypothesis' i.e., greater equity capital of lenders implies stronger incentives to monitor, is true (e.g., Hölmstrom and Tirole, 1997; Mehran and Thakor, 2011), then our model should predict a negative association between lender capital and borrower quality. Schwert (2016) provides evidence of NAM in a syndicated corporate loan market – bank-dependent (more informationally opaque) firms tend to obtain funds from well-capitalized banks, whereas rated firms with access to the public debt market borrow from banks with less equity capital. More interestingly, Schwert (2016) shows that there is endogenous sorting between borrower quality and the capital of lead banks, whereas the matching between firm quality and participant capital is random. This finding confirms the 'equity monitoring hypothesis' because in a typical syndicate, the task of providing due diligence is conferred on the lead arranger and the participant lenders do not have any monitoring role.

Non-monotonicity of firm value. Our model provides novel testable implications for the expected firm value, or simply *firm value*, which is defined as

$$p = e(Q-R).$$

Note that the above expression is the expected retained earnings of a firm, which is the analogue of the stock price of a publicly held firm. From the maximization problem ($\mathscr{P}$) associated with an arbitrary partnership (m, w), it is easy to show that firm value p is monotonically increasing in monitoring ability, net worth and the outside option of the entrepreneur. Because in any Walrasian allocation, firm value can be expressed as a function of net worth alone, i.e., $p(w) \equiv p(\lambda(w), w, u(w))$, it follows that

$$p'(w) = \underbrace{p_1\lambda'(w)}_{\text{matching effect}} + \underbrace{p_3\left[1+u'(w)\right]}_{\text{outside option effect}}.$$

The above two effects are countervailing, and hence,

Implication 2 *The equilibrium firm value may be non-monotonic with respect to net worth.*

The lower panel of Figure 2 depicts a non-monotonic $p(w)$ corresponding to the parameter values $Q = 4$, $c = 0.25$, $u_0 = 0.1$, $\alpha = 1$ and $\beta = 20$. As before, we assume that monitoring ability is uniformly distributed on $[0, 0.2]$. For low values of net worth, firm value is decreasing in w, but it is increasing when corresponding to high values of w.

Firm value in the cross-section. A central feature of an optimal assignment model is the equilibrium payoff determination, the condition (3), which is implied by condition (1), the analogue of the local downward incentive constraint of an optimal screening model. An entrepreneur with a given net worth is affected by changes in the types of others below him in the rankings, and hence, the optimality condition bears the qualifier 'downward'. In other words, if the equilibrium payoff of an entrepreneur with a given net worth w improves, it has positive spillover effects on all the borrowers above him in the rankings. Therefore, the relative ranking of an entrepreneur with respect to net worth plays an important role when one considers cross-sectional variations in the type distributions, as in Section 6.

The lower panels of Figures 2 and 3 present the changes in equilibrium firm value resulting from cross-sectional variations in the distribution of net worth. A decline in the mean net worth induced by a variance-preserving change in skewness and an increase in asset inequality due to a mean-preserving spread have similar effects on the equilibrium firm value. Both a change in skewness from $\gamma_1[w \mid 20, 1]$ to $\gamma_1[w \mid 1, 20]$ and an increase in the net worth variance from $\mathrm{Var}[w \mid 3, 3]$ to $\mathrm{Var}[w \mid 1/3, 1/3]$ imply that low-net worth firms realize lower values, whereas high-net worth firms have higher firm values in the new economy. These asymmetric effects are due to the asymmetric changes in the relative rankings of the same firms. Consider, for example, the case of a mean-preserving spread that increases the inequality of net worth, and take the net worth levels 0.1 and 0.7. Notice that at the distribution with $\alpha = \beta = 1/3$ (greater inequality) relative to the one with $\alpha = \beta = 3$, the firm value declines from 1.94 to 1.61 for the net worth level 0.1, whereas the firm value increases from 2.54 to 2.78 for the net worth level 0.7. This occurs because at the more equal distribution, 0.9% of firms are below 0.1, and 83.7% are below 0.7. By contrast, when inequality rises, 26.7% of firms are below 0.1, and 59.9% are below 0.7. Therefore, in the new economy, the competition for the 'worse types' is relaxed for the entrepreneur with net worth 0.1 but is tighter for high-net worth borrowers.

Implication 3 *Following a cross-sectional variation in the distribution of net worth, caused either by a worsening of mean net worth or by a mean-preserving spread, low-net worth firms realize lower firm value, whereas high-net worth firms enjoy higher firm value.*

Changes in the average firm value. Both types of changes in the distribution of net worth imply changes in the average firm value, which is given by the following:

$$AFV(\alpha, \beta) = \int_0^1 e(w)(Q - R(w))f(w)dw.$$

If the firms under consideration were public firms, then we could have interpreted the above expression as some stock market index, a measure of the performance of the financial market.[20] Thus, any change in the average firm value would indicate a change in market performance. To see this, we set the parameter values as follows: $Q = 4$, $c = 0.25$, $u_0 = 0.1$ and $m \sim U[0, 0.2]$. We then conduct exercises similar to those in Section 6. First, consider a decline in the average net worth following a variance-preserving change in skewness from $\gamma_1[w \mid 5, 1]$ to $\gamma_1[w \mid 1, 5]$. Consequently, $AFV(5, 1) = 2.982$ and $AFV(1, 5) = 1.626$. Such a change in skewness implies a decrease in the average net worth by 80%, which explains an approximately 45% loss in the average firm value. However, a mean-preserving spread of the distribution of net worth, which tightens credit constraints for low-net worth borrowers but relaxes constraints for wealthier entrepreneurs, increases the average firm value. For example, $\mathrm{Var}[w \mid 4, 4] = 0.028$ and $\mathrm{Var}[w \mid 1, 1] = 0.083$ imply $AFV(4, 4) = 2.334$ and $AFV(1, 1) = 2.373$, respectively. An almost three-

[20] Stock market indices are generally weighted averages of the stock prices of firms included in the indices.

fold increase in variance explains an increase of approximately 1.67% in the average firm value. Therefore,

Implication 4 *A decrease in mean net worth following a variance-preserving change in skewness, which implies a economy-wide worsening of the balance sheet position of firms, implies a decline in the average firm value. By contrast, a mean-preserving spread of the distribution of net worth implies an increase in the average firm value.*

8. Concluding remarks

Compared with contracts for an isolated investor-entrepreneur pair, incentive contracts may be quite different in a market with many heterogeneous investors and entrepreneurs. In the equilibrium of a market, individual contracts are influenced by the two-sided heterogeneity via endogenous investor-entrepreneur matching. In this paper, we have developed a simple two-sided matching model of incentive contracting between lenders and borrowers. Entrepreneurs who differ in net worth and investors who differ in monitoring ability are matched into pairs in order to accomplish projects of fixed size. In the equilibrium of the market, both the sorting and payoffs that accrue to each individual are determined endogenously.

Although our stylized model is built on a number of simplifying assumptions, the analyses in Section 7 show that our conclusions are actually very general, and they can be extended to credit relationships other than those analyzed in the present paper. Under double-sided moral hazard, in each investor-entrepreneur partnership, what is crucial is the identification of lenders and borrowers who are "easier to incentivize". When investors perform the monitoring function, in equilibrium, investors with greater incentive to monitor form partnerships with more informationally opaque borrowers following a negative assortative matching pattern. Thus, any empirical analysis that finds direct or indirect evidence of negative sorting between the incentive to monitor and the informational opacity of borrowers by using a set of appropriate measures is consistent with our theoretical finding.

We have discussed above that our model is an extension of Sattinger (1979) to an environment where partnerships are pervaded by underlying incentive problems, which give rise to a concave Pareto frontier. In a principal-agent relationship, fundamentals different from the agency problem may also generate a concave frontier. For example, if at least one of the two contracting parties is risk averse, then the associated frontier is concave. Therefore, in such markets, as argued by Legros and Newman (2007), partnerships are formed because of pure risk-sharing motives, whereas in our model, each partnership (which is subject to limited liability) implies an optimal trade-off between the provision of incentives and rent extraction. Therefore, an extension of the current paper to an environment with risk-averse individuals would shed light on the implications of risk-averse investors in the corporate loan markets.

A more ambitious model would consider many-to-many matching among investors and entrepreneurs. When a lender is allowed to invest in more than one firm, additional complications arise because the monitoring cost function is generally not additively separable. Thus, non-zero interaction terms induce externalities across matches. However, allowing an entrepreneur to borrow from multiple sources may imply the inability of lenders to write binding exclusive contracts. Non-exclusivity may also lead to an externality across matches. Second, an important assumption in the paper is that the relationship between an investor and an entrepreneur lasts only for one period. Such relationships could also involve dynamic considerations, which in turn imply some degree of relaxation of the limited liability constraints, and the conclusions of the current paper could thus change. In a dynamic model, when there are possibilities of wealth accumulation, the income distributions of an economy are generally endogenous. The literature on two-sided matching (e.g., Shapley and Shubik, 1971) has largely been silent on the context of dynamic bilateral relationships. At this juncture, the paper by Mookherjee and Ray (2002) is worth mentioning, as it considers a dynamic model of lending relationships in which lenders and borrowers are randomly matched into pairs. These authors analyze a model of equilibrium short-period credit contracts and assume that bargaining power is exogenously distributed between lenders and borrowers. When lenders have all the bargaining power, less wealthy borrowers have no incentive to save, and poverty traps emerge. Conversely, if borrowers have all the bargaining power, income inequality is reduced as a result of the strong incentives for savings. One significant difference between our model and that of Mookherjee and Ray (2002) is that bargaining power in the current model is distributed endogenously among the principals and agents because the outside option of each individual is endogenous. The above-mentioned extensions of the current model would be an interesting research agenda for the future.

Appendix

A. Proof of Lemma 1

We analyze the optimal loan contract for an arbitrary partnership (m, w), which solves the program $(\mathscr{P})$. Write $u \equiv \bar{u}(w)$. The binding participation constraint (PC) defines the optimal loan rate $R(m, w, u)$ as a function of m, w and u, which is given by:

$$R(m, w, u) = \frac{c^2 Q - \sqrt{c^2 m^2 Q^2 + 2c(c^2 - m^2)(w+u)}}{c^2 - m^2}.$$

The larger root is ignored since it is strictly larger than Q, and hence, does not satisfy the limited liability constraint. It is also easy to check that $0 < R(m, w, u) < Q$. Differentiating implicitly the participation constraint with respect to m, w and u, respectively, we obtain the following:

$$R_1(m, w, u) = -\frac{mR^2}{c^2(Q-R) + m^2 R} < 0,$$
$$R_2(m, w, u) = -\frac{c}{c^2(Q-R) + m^2 R} < 0,$$
$$R_3(m, w, u) = -\frac{c}{c^2(Q-R) + m^2 R} < 0.$$

From the incentive compatibility constraints, the optimal monitoring is given by $a(m, w, u) = mR(m, w, u)$. Therefore,

$$\begin{aligned} a_1(m, w, u) &= mR_1(m, w, u) + R(m, w, u) = \frac{c^2R(Q-R)}{c^2(Q-R)+m^2R} > 0, \\ a_2(m, w, u) &= mR_2(m, w, u) < 0, \\ a_3(m, w, u) &= mR_3(m, w, u) < 0. \end{aligned}$$

This completes the proof of the lemma.

B. Proof of Proposition 1

First, we prove the following result prior to showing that the equilibrium matching is NAM.

Lemma 2 *Let $\phi(m, w, u)$ be defined as the maximum value function of the maximization problem ($\mathscr{P}$). Then,*

(a) $\phi_1(m, w, u) > 0$, $\phi_2(m, w, u) > 0$ *and* $\phi_3(m, w, u) \le 0$*;*
(b) $\phi_{21}(m, w, u) < 0$ *and* $\phi_{31}(m, w, u) < 0$.

Proof The Lagrangean associated with the maximization problem ($\mathscr{P}$) is given by:

$$\mathscr{L} = cR(Q-R) + \frac{1}{2}mR^2 - (1-w) + \mu\left[\frac{1}{2c}\{c^2(Q-R)^2 - m^2R^2\} - w - u\right],$$

where μ is the associated Lagrange multiplier. Note that the first-order condition $\partial\mathscr{L}/\partial R = 0$ implies that

$$\mu = \frac{c(cQ-(2c-m)R)}{c^2(Q-R)+m^2R} = \frac{c^2Q-(2c^2-cm)R}{c^2Q-(c^2-m^2)R}.$$

Since $2c^2 - cm > c^2 - m^2 \iff (c-m)^2 + cm > 0$, we have that $\mu < 1$. Moreover, note that $\mu > 0$ because (PC) is binding at the optimum, which implies that

$$R < \frac{cQ}{2c-m} \equiv R^0. \tag{12}$$

By the Envelope theorem,

$$\begin{aligned} \phi_1(m, w, u) &= \frac{\partial\mathscr{L}}{\partial m} = \left(\frac{1}{2} - \frac{m\mu}{c}\right) > 0; \\ \phi_2(m, w, u) &= \frac{\partial\mathscr{L}}{\partial w} = 1 - \mu > 0; \\ \phi_3(m, w, u) &= \frac{\partial\mathscr{L}}{\partial u} = -\mu < 0. \end{aligned}$$

The first inequality is true because

$$\frac{1}{2} - \frac{m\mu}{c} > 0 \iff R < \frac{cQ(c-2m)}{c^2+m^2-4cm}, \text{ and } R^0 = \frac{cQ}{2c-m} < \frac{cQ(c-2m)}{c^2+m^2-4cm}. \tag{13}$$

This completes the proof of Part (a). Note also that

$$\phi_{21}(m, w, u) = \phi_{31}(m, w, u) = -\frac{\partial \mu}{\partial m}.$$

Therefore, to prove Part (b), we must show that $\partial \mu / \partial m > 0$. Differentiating μ with respect to m and substituting for the expression of $\partial R / \partial m$, we obtain

$$\frac{\partial \mu}{\partial m} = \frac{[\{c(2c-m) - (c^2 - m^2)\mu\}mR + (c - 2m\mu)\{c^2(Q-R) + m^2 R\}]R}{c^2(Q-R) + m^2 R} \tag{14}$$

Because $Q - R > 0$ and $R > 0$, to prove that the above expression is strictly positive, it suffice to show that (1) $c(2c-m) - (c^2 - m^2)\mu > 0$ and (2) $c - 2m\mu > 0$. Earlier, we have shown that $c(2c-m) > c^2 - m^2$ and $\mu < 1$, and hence, (1) holds. The second inequality is also true because of the inequality in (13). This completes the proof of the lemma. ■

Now, recall that the first-order condition of the maximization problem ($\mathscr{M}$) of each type m investor is given by (1), and the second-order condition is given by:

$$[\phi_{22} + \phi_{23} u'(w)] + [\phi_{32} + \phi_{33} u'(w)]u'(w) + \phi_3 u''(w) \leq 0 \quad \text{for } m = \lambda(w). \tag{15}$$

Differentiating (1) at $m = \lambda(w)$, we obtain the following:

$$u''(w) = -\frac{1}{\phi_3^2}[\phi_3(\phi_{21}\lambda'(w) + \phi_{22} + \phi_{23}u'(w)) - \phi_2(\phi_{31}\lambda'(w) + \phi_{32} + \phi_{33}u'(w))] \tag{16}$$

By substituting the expressions for $u'(w)$ and $u''(w)$ into (15), the above inequality is reduced to the following:

$$\begin{aligned} &\left[\phi_{21}(m, w, u(w)) - \phi_{31}(m, w, u(w)) \frac{\phi_2(m, w, u(w))}{\phi_3(m, w, u(w))}\right] \lambda'(w) \geq 0 \\ \implies &\Psi(m, w, u(w))\lambda'(w) \geq 0. \end{aligned} \tag{17}$$

Because, by Lemma 2, $\phi_2 > 0$, $\phi_3 < 0$, and $\phi_{21} = \phi_{31} < 0$, we have that $\Psi(m, w, u(w)) < 0$. Therefore, form (17), it follows that $\lambda'(w) \leq 0$. This completes the proof of the proposition.

C. Proof of Proposition 4

First, we prove part (b), which can be easily adapted from (Määttänen and Terviö, 2014, Proposition 4) who analyze a similar result for one-sided matching market. Note that

$$F_1(w) < (>) F_2(w) \text{ for } w < (>) w^* \implies \lambda_1(w) > (<) \lambda_2(w) \text{ for } w < (>) w^*.$$

Moreover, in the equilibria of both markets, $u_1(w_{min}) = u_2(w_{min}) = u_0$. Therefore,

$$\Delta u(w) \equiv u_1(w) - u_2(w) = \int_{w_{min}}^{w} [\psi(\lambda_1(w), w, u(w)) - \psi(\lambda_2(w), w, u(w))]dw.$$

As $\psi(m, w, u(w))$ is strictly decreasing in m, the integrand of the above expression, which is the slope of $\Delta u(w)$, is strictly negative (positive) for $w < (>) w^*$. At $w = w^*$, the above definite integral is strictly negative because $\Delta u(w_{min}) = 0$, and it is strictly decreasing on $[w_{min}, w^*)$. Because $\Delta u(w)$ is strictly increasing on $[w^*, w_{max}]$, there are two possibilities: (a) $\Delta u(w)$ does not intersect the horizontal axis, and hence, it is strictly negative for all $w \in [w_{min}, w_{max}]$. Otherwise, (b) $\Delta u(w)$ intersects the horizontal axis at some point $\hat{w} \in (w^*, w_{max}]$, which is unique because $\Delta u(w)$ is strictly increasing on $(w^*, w_{max}]$. To prove Proposition 4-(a), take $w^* = w_{max}$, which completes the proof of the proposition.

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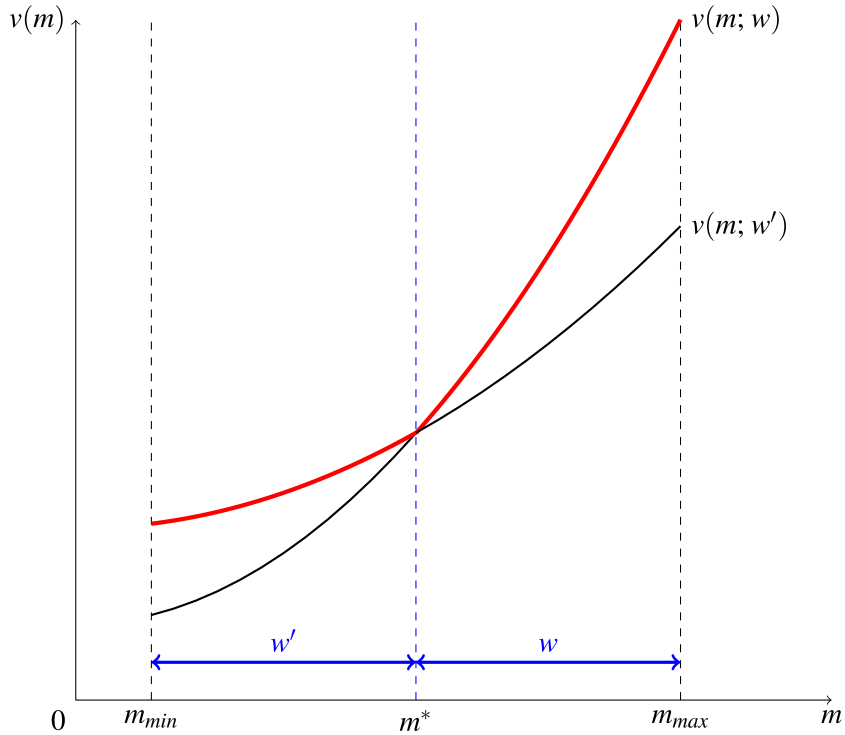

Figure 1: The single crossing property of the investor payoff function with two entrepreneurial types w and w', where $w' > w$. The equilibrium induces an NAM with low-ability (high-ability) investors being matched with high-net worth (low-net worth) entrepreneur.

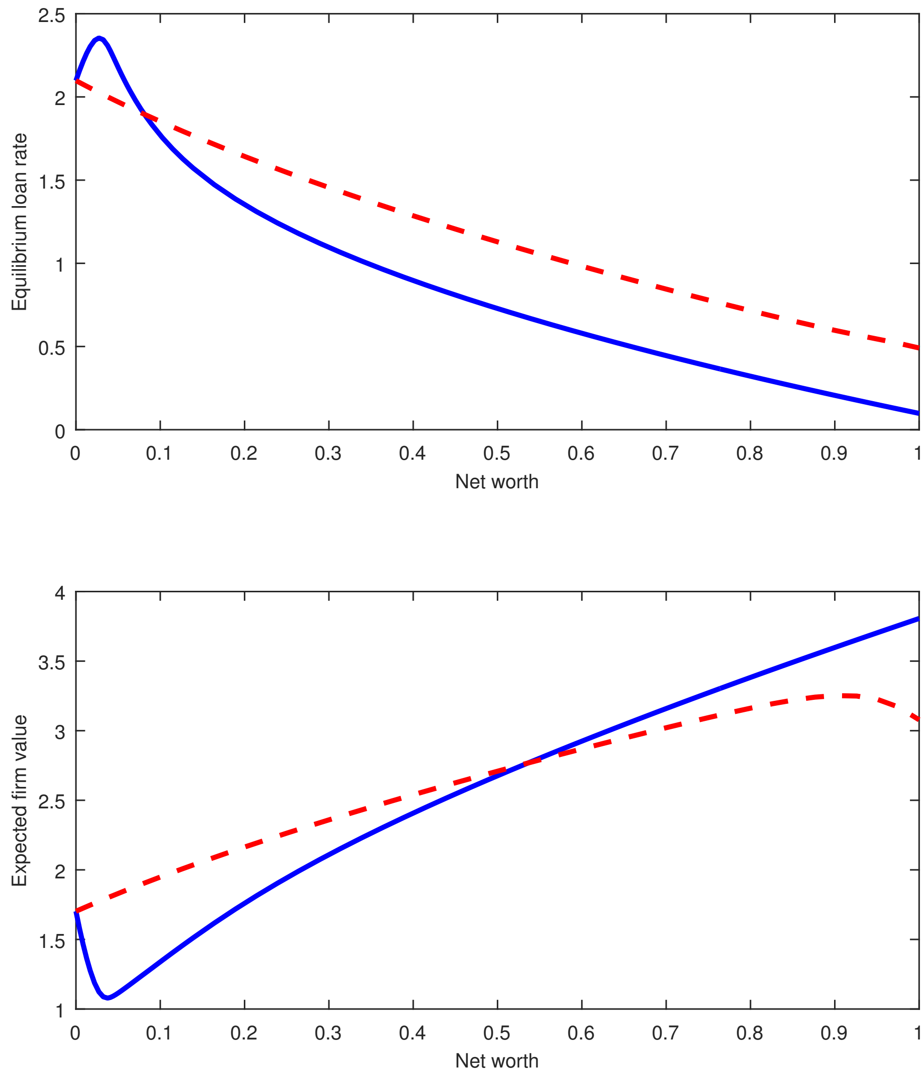

Figure 2: Effect of a change in skewness of the distribution of net worth from $\gamma_1[w \mid 1, 20]$ (the solid curve) to $\gamma_1[w \mid 20, 1]$ (the dotted curve). Both distributions have the same variance. The upper panel describes the effect on the equilibrium loan rates and the lower panel describes the effect on the expected firm value.

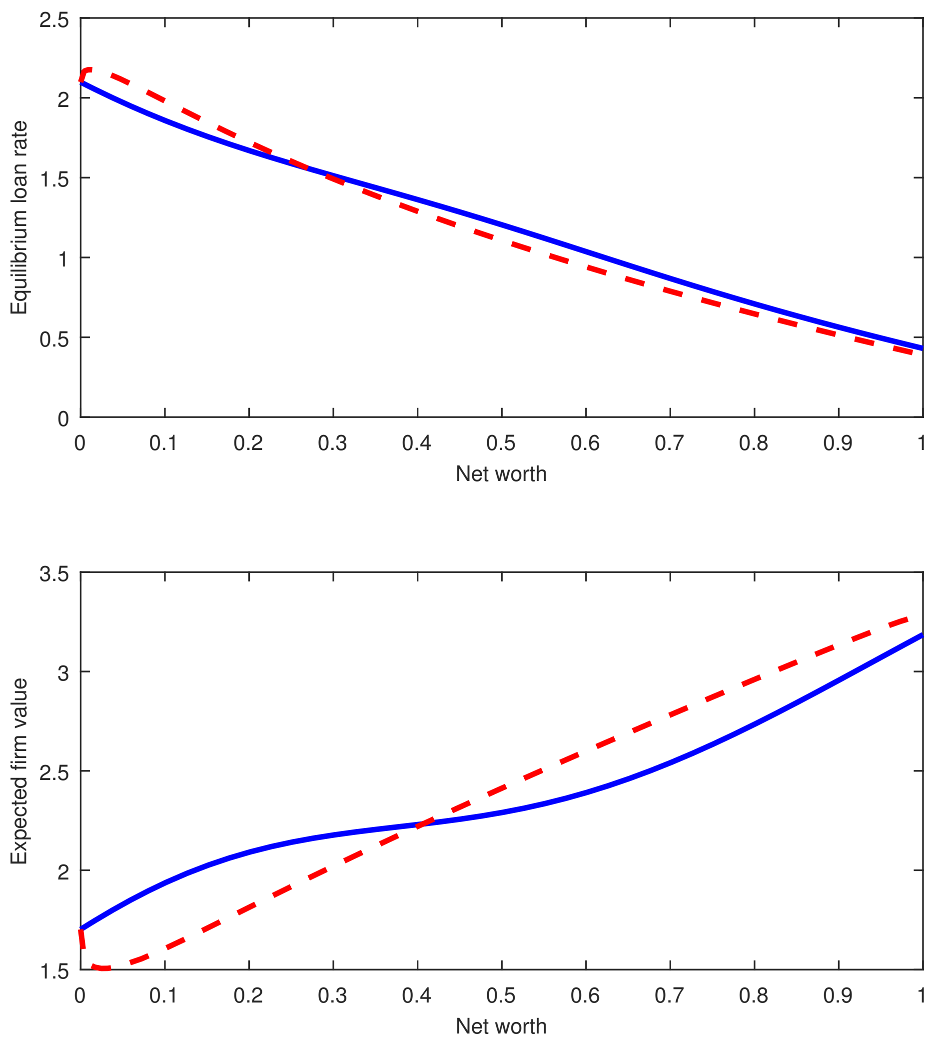

Figure 3: Effect of a change in variance of the distribution of net worth from $\mathrm{Var}[w \mid 3, 3]$ (the solid curve) to $\mathrm{Var}[w \mid 1/3, 1/3]$ (the dotted curve). Both distributions have the same mean. The upper panel describes the effect on the equilibrium loan rates and the lower panel describes the effect on the expected firm value.